#1. Using Maple we find \( \gcd(4321, p-1) = 1 = 4321r + (p-1)s \)
where \( r = -11108539689887, \ s = 48 \). Now \( a^{p-1} \equiv 1 \pmod{p} \)
by Fermat's Little Theorem so
\[
a = a^{4321r + (p-1)s} = (a^{4321})^r (a^{p-1})^s = (a^{4321})^s = 418910750415330^s \equiv 12345678901234 \pmod{p}.
\]
One integer solution is \( 12345678901234 \).

#2. Using Maple we get \( \gcd(1234, 4321) = 1 = 1234r + 4321s \)
where \( r = -1082, \ s = 309 \) so
\[
b = b^{1234r + 4321s} = (b^{1234})^r (b^{4321})^s = 24630086807320^s \times 58463134801742^s \equiv 801234567890123 \pmod{p}.
\]
One integer solution is \( 801234567890123 \).

#3. Using Maple, we find \( 2^{p-1} \not\equiv 1 \pmod{a} \). See the accompanying worksheet which finds the
remainder (residue) of \( 2^{p-1} \pmod{a} \); however, this
gives no direct information as to the prime
factorization of \( n \). The only information we
obtain (from Fermat's Little Theorem) is that \( n 
\) is not prime.
4. Given that \( n = pq \) where \( p, q \) are distinct primes, we have \( \phi(n) = (p-1)(q-1) = pq - p - q + 1 \) so \( p, q \) are the roots of the quadratic polynomial

\[
x^2 - (p+q)x + pq = x^2 - (n - \phi(n) + 1)x + n.
\]

Using Maple, the roots are given by

\[
p = \frac{-b + \sqrt{\Delta}}{2} = 10^{10} + 3,
q = \frac{-b - \sqrt{\Delta}}{2} = 10^{10} + 267
\]

where \( b = -(n - \phi(n) + 1) \), \( \Delta = b^2 - 4n \) (the discriminant).
Question #1.
> p := 10^15 + 37;
> igcdex(4321, p-1, 'r', 's');
> r, s;
> 418910750415330 &^ r \mod p;
> 12345678901234

If you are uncomfortable with the negative exponent here, one can first reduce \( r \mod p - 1 \); but of course this yields the same answer:
> r := r \mod (p-1);
> 418910750415330 &^ r \mod p;
> 12345678901234

Question #2.
> igcdex(1234, 4321, 'r', 's');
> r, s;
> (256360808697320 &^ r) * (584631348017142 &^ s) \mod p;
> 801234567890123

Just as in #1, the negative exponent can be avoided by first reducing \( r \mod p - 1 \); and once again, this does not change the answer:
> r := r \mod (p-1);
> (256360808697320 &^ r) * (584631348017142 &^ s) \mod p;
> 801234567890123

Question #3.
> n := (10^101 + 2673) * 10^100 + 801;
> 2 &^ (n-1) \mod n;
> 30170771928057262249964177081596257824747799663986032536464164761629828439455\ 65669711673713642488762889260049104219389222304156796614080230557691384405\ 46090053001246555810330988148855055118765117194403

Question #4.
> phi := (10^101 + 2662) * 10^100 + 532;
> b := -(n - phi + 1);
\[ \Delta := b^2 - 4n; \]
\[ p := \frac{-b + \sqrt{\Delta}}{2}; \]
\[ q := \frac{-b - \sqrt{\Delta}}{2}; \]

Check that \( pq = n \):
\[ p * q; \]

Also check that \( (p - 1)(q - 1) = \phi(n) \):
\[ (p-1)(q-1); \]