Recall that a nondegenerate conic is either an ellipse, hyperbola or parabola. (This list includes circles as a special case.)

(a) If \( C \) is a nondegenerate conic and \( \ell \) is a line, how many points of intersection can \( C \) and \( \ell \) have? Justify your answer.

(b) If \( C \) and \( C' \) are distinct nondegenerate conics, how many points of intersection can \( C \) and \( C' \) have?

An algebraic curve of degree \( d \) is a curve defined by a polynomial equation of degree \( d \). We have been considering curves of degree 1 (i.e. lines) and curves of degree 2 (i.e. conics). (By contrast, a catenary has an equation of the form \( y = k(e^{ax} + e^{-ax}) \), which is not a polynomial; this curve is transcendental, and no degree can be designated for a catenary. The study of such curves belongs to analytic geometry rather than algebraic geometry.)

(c) Some examples of cubic curves (i.e. algebraic curves of degree three) are \( y = x^3 - x \) and \( x = y^3 - y \). What do these curves look like? How many points of intersection can a cubic curve have with a line? Or with a conic? Or with another cubic curve?

(d) The settings (a), (b) and (c) above are special cases of a more general result. Guess the statement of this result.