Fill in the blanks appropriately. If you cannot remember individually, work together.

**Affine Planes**

In an affine plane, any two points are joined by a unique _______________; given a point $P$ not on a line $\ell$, there is a unique _______________ through $P$ not meeting $\ell$.

The classical construction of an affine plane starts with a ____________ $F$. Here we take pairs $(x, y)$ as points where $x, y \in F$. Lines are point sets satisfying equations of the form _______________ or _______________ for any choice of the constants $m, b, a \in F$.

The most important example of an affine plane is the ______________________ plane which is constructed using $F = \______________$.

**Projective Planes**

In a projective plane, any two points are joined by a unique ______________; and any two lines meet in a unique ______________. The classical construction of a projective plane starts with a _____________ $F$. Not every projective plane has this form. The classical projective planes satisfy _______________’s Theorem; and conversely, every projective plane which satisfies _______________’s Theorem is a classical one, constructed from a ________________.

Every affine plane can be extended to a projective plane as follows:

a) For each _______________ class of lines, add a new ‘point at infinity’ which lies on every line in this _______________ class.

b) Add one new line, the ‘line at infinity’. Its points are just the ‘points at infinity’ introduced in (a).

For example if we extend the _______________ plane in this way, we obtain the real _______________ plane.

This process is easily reversed: From a projective plane, by deleting any _______________ (and all of its _______________ ) we obtain an affine plane.

Consider a conic $C$ and line $\ell$ in a classical projective plane. After deleting $\ell$, the conic $C$ (or what remains of it) becomes a conic in the resulting affine plane. This conic is

a) an ________________, if $\ell$ was a passant of the original conic $C$;

b) a _________________, if $\ell$ was a tangent of the original conic $C$; or

c) a _________________, if $\ell$ was a secant of the original conic $C$.

In the projective viewpoint, all conics may be treated uniformly.
Another evidence that the projective plane is more natural than the affine plane, is that it allows for a simple statement of Bezout’s Theorem, allowing us to exactly count points of __________________ of algebraic curves.

Yet another advantage of the projective viewpoint over the affine viewpoint, is the Principle of __________________ which interchanges the roles of points and lines. For example, if a theorem holds in a classical projective plane, then so does its __________________.

**Inversive Planes**

In an inversive plane, any three points are joined by a unique ______________________ .

Both the real inversive plane and the real projective plane are obtained as extensions of the __________________ plane. One major difference, however, is that to extend to the real inversive plane, we add one __________________ at __________________, whereas to obtain the real projective plane, we add many __________________ at __________________.

The real inversive plane admits special transformations called inversions. Inversion in a fixed circle γ maps points to __________________ and circles to __________________, while preserving __________________ and reversing __________________________. The process of inversion is a useful step in many constructions since it may be used to reduce apparently complicated situations to simpler ones.

**Finite Geometries versus Continuous Geometries**

Examples of the plane geometries listed above (affine, projective, and inversive) can be constructed using a variety of fields, leading to either finite or infinite examples. The most well-known examples are those coordinatized by the field of _______________ numbers but finite examples, coordinatized by finite ______________ , are useful in a number of applications.

For example, the projective plane of order 2 is used in mathematical ______________. It also gives rise to a code used for correcting ______________ which arise naturally during the transmission of binary data, due to unreliable transmission channels. We have described how the affine plane of order 3 can be used in the design of a statistical ______________ in order to balance effects from different factors, allowing the influence of irrelevant factors to cancel as much as possible. Algebraic curves in planes defined over finite fields, particularly certain cubic curves known as ______________ curves, are useful in cryptography and in algorithms for factorization of large integers.

Continuous plane geometries, such as the Euclidean plane, are much more difficult to axiomatize since in addition to the axioms of incidence we have studied, one must also include axioms that describe properties of ______________ and ______________.

While Euclidean geometry has a tradition of acceptance and therefore seems natural to us, it is mysterious in many ways. For example the Banach-Tarski Theorem shows that a ______________ in Euclidean 3-space can be decomposed into five pieces which can be reassembled (with no stretching) to form two ______________ of the same size as the original. This is not possible to perform with physical objects, or even with physical space itself.