Isometries of the Euclidean plane:
transformations that preserve distance
(one-to-one and onto... why?)

Domain \( P \)

Range \( Q \)

Euclidean plane

subset of Euclidean plane
• The inverse of an isometry is an isometry.
• The composite of two isometries is an isometry.
• Isometries preserve angle.

\[ \text{(dilatation)} \]
\(\text{(NB. dilation preserves angle but not distance).}\)
Every isometry either preserves orientation or reverses orientation.
Classification of isometries of Euclidean plane

- translation i.e.

\[ (x, y) \mapsto (x + h, y + k) \]

where \( h, k \) are constants.

\( \text{(distance} = \sqrt{h^2 + k^2} \text{) } \)

- preserves orientation.
reflection

axis

reverses orientation.
rotation
preserves orientation
glide reflection

e.g. $(x, y) \rightarrow (x+1, -y)$

axis is the $x$-axis

reverses orientation; no fixed points.
<table>
<thead>
<tr>
<th>Computational Problem</th>
<th>Computable?</th>
<th>Efficiently Computable?</th>
</tr>
</thead>
<tbody>
<tr>
<td>testing primality</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>factoring integers</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>solving linear systems</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>solving polynomial systems</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>is a given set of tiles able to tile the plane?</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Given an equation, is there an integer solution?</td>
<td>no</td>
<td>no</td>
</tr>
</tbody>
</table>