circle \quad x^2 + y^2 = 1

disk \quad x^2 + y^2 \leq 1
Modern Geometry

Day 3 Review
Wednesday, June 10, 2013

Fill in the blanks appropriately. If you cannot remember individually, work together.

Point-Line Duality

Duality interchanges points and lines, in any statement regarding points and lines. For example, if we start with the statement “Points A, B and C lie on the line m” then the dual statement is 

Lines A, B and C go through the point m.

Nondegenerate Conics

A nondegenerate conic is either a ________, a ________ or a ________. These three cases may often be treated uniformly, since in the projective setting, all three types of nondegenerate conic are the same.

Let C be a conic. Every line meets C in 0, 1 or 2 points. We say a line ℓ is a ________, a ________ or a ________ with respect to C according as ℓ meets C in 0, 1 or 2 points. Dually, every point lies on 0, 1 or 2 tangents. We say a point P is ________, interior, or ________ according as P lies on 0, 1 or 2 tangents.

Pascal’s Theorem

Consider a hexagon inscribed in a nondegenerate conic. Let P be the point of intersection of two opposite sides of the hexagon. Similarly, the other two pairs of opposite sides intersect in points Q and R. Then the points P, Q and R are ________.

Pascal’s Theorem does not strictly hold in the Euclidean plane simply because opposite sides of the inscribed hexagon may be ________. We may accommodate these exceptional cases if we are careful. Another way to resolve the exceptional cases is to allow points at ________, i.e. to work in the projective plane rather than in the Euclidean plane.

One may replace the nondegenerate conic with a pair of lines, this being a degenerate conic. The analogue of Pascal’s Theorem in this case is known as ________ Theorem.

Algebraic Plane Curves

A curve of degree 1 is a ________. A curve of degree 2 is a ________.

A curve of degree m intersects a curve of degree n in at most ________ points. This maximum number of intersection points is attained if we

(a) allow points with ________ coordinates;
(b) include points at ________; and
(c) count points with appropriate ________.


interior points
Exterior points form a Möbius strip. Interior points.
Real Projective Plane

= Euclidean plane

+ points at infinity

= disk + Möbius strip
= cylinder

= Möbius strip
Torus (surface of donut)
Topology

= Sphere
= projective plane
"B is between A and C."

This statement is valid in the Euclidean plane.
Axioms of (Euclidean) plane geometry

Affine

2 distinct points lie on a unique line.
If a point $P$ is not on a line $l$, then there is a unique line through $P$ parallel to $l$. 
If lines \( l \) and \( m \) are not parallel, then they meet at a unique point.