Consider two nonintersecting circles $\gamma$ and $\gamma'$, shown in green. A circle $\alpha_1$, tangent to both $\gamma$ and $\gamma'$, is chosen as shown, followed by a circle $\alpha_2$, tangent to $\alpha_1$ as well as $\gamma$ and $\gamma'$; then a circle $\alpha_3$, tangent to $\alpha_2$ as well as $\gamma$ and $\gamma'$; etc. resulting in a sequence of circles $\alpha_1, \alpha_2, \alpha_3, \ldots$ all of which are tangent to the original two circles.

After a finite number (say $n$) of circles, we eventually reach a circle $\alpha_n$ beyond which we cannot continue without overlapping $\alpha_1$. It may happen (either by design or by extreme luck) that $\alpha_n$ is in fact tangent to the original circle $\alpha_1$ in the sequence, as shown:
I say *extreme* luck (or design) because if \( \gamma \) were just a tiny bit larger, then all the \( \alpha \)'s would be a tiny bit smaller, resulting in a gap between \( \alpha_n \) and \( \alpha_1 \), like this:

![Diagram of nested circles with a gap](image1)

However, the position of the starting circle \( \alpha_1 \) has no effect on whether or not this works (i.e. results in the last circle \( \alpha_n \) being tangent to \( \alpha_1 \)); if we chose a different starting point, we would obtain another closed ring of tangent circles, as shown:

![Diagram of another closed ring of tangent circles](image2)

Why is this true? *Find* an inversion that simplifies the problem and *explain* how, with respect to the new (inverted) viewpoint, the problem becomes obvious.