Consider a set of \( n \) distinct points \( \{P_1, P_2, \ldots, P_n\} \) in the Euclidean plane, not all collinear. A connecting line is a line joining two points of the set. Each point is on at most \( n - 1 \) connecting lines (maybe fewer, since some connecting lines may contain more than two \( P_i \)’s). We investigate the following open (i.e. unsolved) problem:

**Must one of the points \( P_i \) lie on at least \( \frac{n}{3} \) connecting lines?**

Our goal here is not to resolve any unsolved problems, but merely to acquire a sense of the nature and difficulty of certain problems in Euclidean plane geometry that can be stated in quite elementary terms.

**For class discussion**

Suppose \( n = 5 \) points are given, not all collinear.

- Must one of the points lie on at least 2 connecting lines?
- Must one of the points lie on at least 3 connecting lines?
- Must one of the points lie on at least 4 connecting lines?

Whenever the answer is ‘no’, sketch a counterexample.

**HW13**

a) Suppose \( n = 6 \) points are given, not all collinear. Must one of the points lie on at least 2 connecting lines? Must one of the points lie on at least 3 connecting lines? Must one of the points lie on at least 4 connecting lines? Whenever the answer is ‘no’, sketch a counterexample.

b) Suppose \( n = 7 \) points are given, not all collinear. Must one of the points lie on at least 3 connecting lines? Must one of the points lie on at least 4 connecting lines? Must one of the points lie on at least 5 connecting lines? Whenever the answer is ‘no’, sketch a counterexample.

c) Suppose \( n = 8 \) points are given, not all collinear. Must one of the points lie on at least 3 connecting lines? Must one of the points lie on at least 4 connecting lines? Must one of the points lie on at least 5 connecting lines? Whenever the answer is ‘no’, sketch a counterexample.

d) Suppose \( n = 9 \) points are given, not all collinear. Must one of the points lie on at least 3 connecting lines? Must one of the points lie on at least 4 connecting lines? Must one of the points lie on at least 5 connecting lines? Whenever the answer is ‘no’, sketch a counterexample.