MATH 5640 Final Project
Due by 5:00PM on May 8th

1. Consider the Minkowski metric in four dimensions defined by

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2.$$

(We follow the usual convention of choosing units for which the speed of light is numerically equal to 1.)

   (a) Consider a linear transformation of the form

$$T(t, x, y, z) = (t,\ a_{11}x+a_{12}y+a_{13}z,\ a_{21}x+a_{22}y+a_{23}z,\ a_{31}x+a_{32}y+a_{33}z).$$

   What are necessary and sufficient conditions on the real 3×3 matrix $A = \big(a_{ij}\big)$ such that T is an isometry (i.e. such that T preserves the metric)? Express you answer as a familiar condition on the matrix A, rather than trying to write out explicit conditions on the a_{ij}'s (which would be rather unmanageable).

   (b) Now consider a linear transformation of the form

$$S(t, x, y, z) = (\alpha t+\beta x,\ \gamma t+\delta x,\ y,\ z).$$

   Give an explicit parameterization of the set of all 4-tuples $(\alpha, \beta, \gamma, \delta)$ for which S is an isometry.

2. Consider the spherically symmetric Schwarzschild metric

$$ds^2 = e^{2\nu(r)}dt^2 - e^{2\lambda(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2).$$

Find the Christoffel symbols, then compute the curvature and the Ricci tensors. This does require some work; for those of you familiar with Maple, it contains a package that does it all for you once you input the metric. You are welcome to use it.