My Favourite Problem
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Infinitely many participants are gathered for a game in which I place on every participant’s head a cap which is either red or blue.

Everyone can see the caps on the other participants’ heads but cannot see their own cap. They must guess the colour of their own cap. If most (all but a finite number) of participants guess correctly, I suitably reward all participants. Otherwise (if infinitely many participants guess wrongly) I punish all participants by death.

Participants may plan a strategy before the contest starts but they are not permitted to communicate after I place the caps on their heads.

What strategy guarantees that the participants succeed?

Note: Each participant’s guess can be based only on the information available to him/her, i.e. the colors of the hats he/she sees. As part of a strategy, for example, participants may agree that if Ed sees a blue hat on Jed and a red hat on Ned, then Ed should guess ‘red’, and otherwise guess ‘blue’ for his hat color.