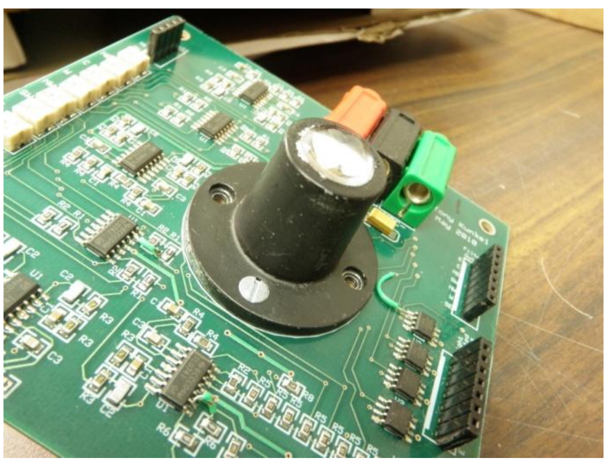
Fly-Eye Sensor Field of View and Trigonometry

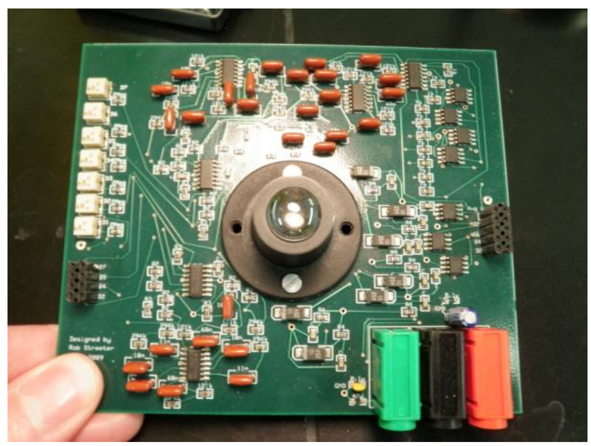
Description: Part of the research conducted in the Electrical and Computer Engineering Department at the University of Wyoming involves a sensor platform based on the function of a fly's vision system. As with any vision sensor platform, the device has a particular field of view (FoV) where it can "see". This FoV can be analyzed using trigonometry. This analysis is the basis for this lesson.

This applies to high school trigonometry.

Background: The sensor for which this lesson is modeled after is designed and built at the University of Wyoming. It seeks to emulate some very attractive properties present in the vision system of a house-fly (hence the name "fly-eye"). These properties include a hyper-sensitivity to motion, high functionality in low-contrast situations, and an instantaneous response.

This lesson *ideally* relies on the inspection of actual sensor prototypes, but that is not at all necessary to its success. The sensor utilizes a physical layout that involves "cartridges". These structures act as the "eyes" of the sensor. In the picture below, the black cylinder with the lens on top is a single cartridge. This image is property of the author and should not be redistributed outside the classroom or without this lesson plan.





The FoV is limited by the edge of the lens and the corresponding edge of the cartridge. If the sensors are available, the students can inspect this part of the sensor and come up with an angle limiting the FoV. If the sensors are not available, a sketch will suffice. Based on this physical layout, a series of calculations can be done. The emphasis of the sensor is on areas of overlap, thus, even though the "blind area" is a seemingly large percentage, the overlap remains crucial to the sensor's function. A suggested worksheet is provided.

Concepts: The following concepts apply to this lesson: right-angle triangle analysis, Law of Sines, percentages, triangle area, and angle relationships (alternate interior angles, etc.).

Vocabulary: These words are interesting and important words found throughout this lesson. Students may benefit from learning about these prior to the lesson, perhaps as a homework assignment.

Right-triangle - a triangle in which one angle is exactly 90°.

Percentage - a ratio of 100, wherein 100 represents the largest possible value and any smaller amount is between 0 and 100.

Law of Sines - a method used to analyze the relationships between the angles and side lengths of a triangle. $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Pythagorean Theorem - a method used to compute the side lengths of a right-triangle. In the following equation, *a* and *b* are the "legs" and *c* is the hypotenuse (longest side, always opposite the 90° angle): $a^2 + b^2 = c^2$

Field of View - the effective visual area of a sensor. A device cannot detect anything outside its field of view.

Congruent Triangles - these triangles have the same side lengths and the same angles. They may be translated or rotated however.

Similar Triangles - these triangles have the same length-to-angle ratios, but not the same values as in congruent triangles.

Bisection - the division of something into two equal parts.

Take Away Message: Fundamental trigonometric analysis techniques can be used to conduct analysis on the visual effectiveness of a sensor. Knowing how to apply knowledge about and information about trigonometry is useful to advanced research.

Lesson: How is trigonometry applicable to advanced machine vision sensor research? This lesson covers the following Wyoming State Math Standards and requires approximately 55 minutes.

MA 12.2.1 and 12.4.1

Materials: Single-cartridge sensor prototypes (optional), calculators.

About the Author: Rob Streeter is a graduate student at the University of Wyoming. He is currently pursuing a MS of Electrical Engineering. A Wyoming native, Streeter appreciates the need for Wyoming high school graduates to be aware of the extensive opportunities available to them. He genuinely hopes this lesson inspires interest in engineering and the University of Wyoming. Any feedback on this lesson would be appreciated and should be directed to Mr. Streeter (rstreete@uwyo.edu), NSF GK-12 EE-Nanotechnology Fellow, University of Wyoming, Laramie, Wyoming.

The following pages include a suggested worksheet and a solution to that worksheet.

Name:________________________________

Student Worksheet

“Fly-eye” Sensor Example –

This sensor is a machine vision sensor, designed to improve how a robot (or some other device) detects its surroundings. As with all vision sensors, this sensor has a particular “field of view” (FoV), that is, where the sensor can actually “see” things.

Inspect the sensor lens housing (called a cartridge) carefully. Guess what the FoV of the sensor is. Choose a few spots around the cartridge where you think the sensor can see and a few where you think it can’t. Sketch a figure that approximates the FoV of the sensor, and explain anything you put in the figure.

Did you label any angles? There is one angle that is particularly important: the one between the horizontal (x-axis) and the edge of the sensor’s FoV. Add this one into your sketch, if it’s not already there, and label it.

Besides the physical layout of the sensor, what things might play a role in limiting the effectiveness of the sensor? Try to come up with a few different factors.

Now, say there are two cartridges 2.5 cm apart. They are both pointing the same direction, at a picture on a wall 20 cm away. If each cartridge is 1 cm across, sketch, with dimensions, this set up.

Using what you know about trigonometry, compute the total area the sensors have within their FoV.

What percentage of that area is overlap between the two cartridges?

This overlap is what gives the sensor what we call "hyperacuity to motion," or a super-sensitivity to anything that moves. Without this, the sensor loses many of its unique advantages.

Lastly, if you consider all the area in front of the two sensors as the total area (more than just the FoV), what percentage of that is outside the FoV? In other words, of the total area, how much is the sensor blind to?

Name:________________________________

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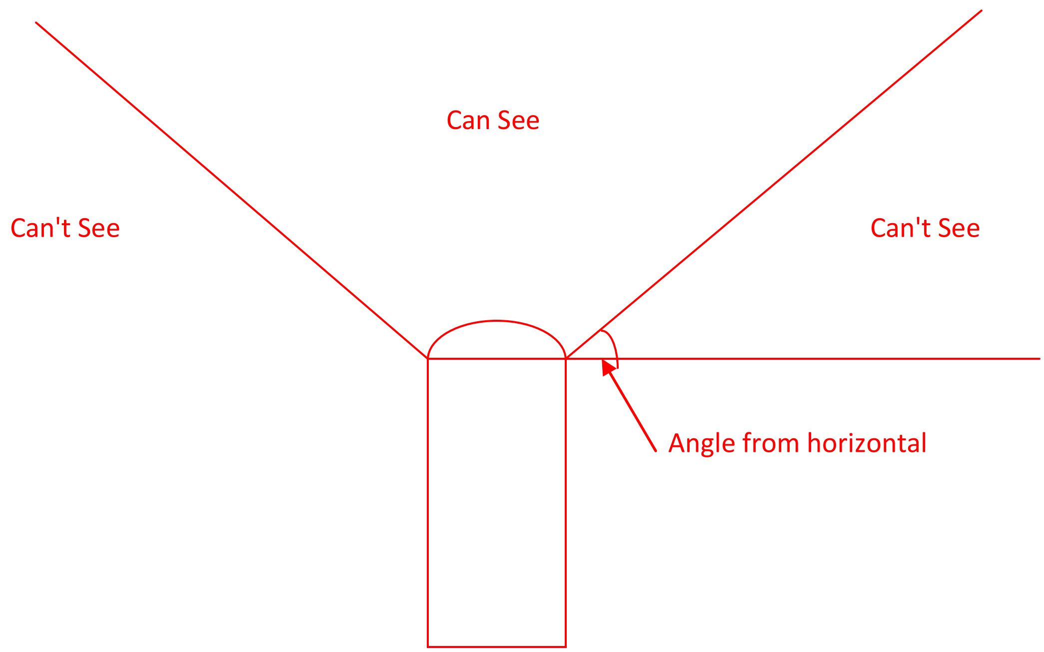


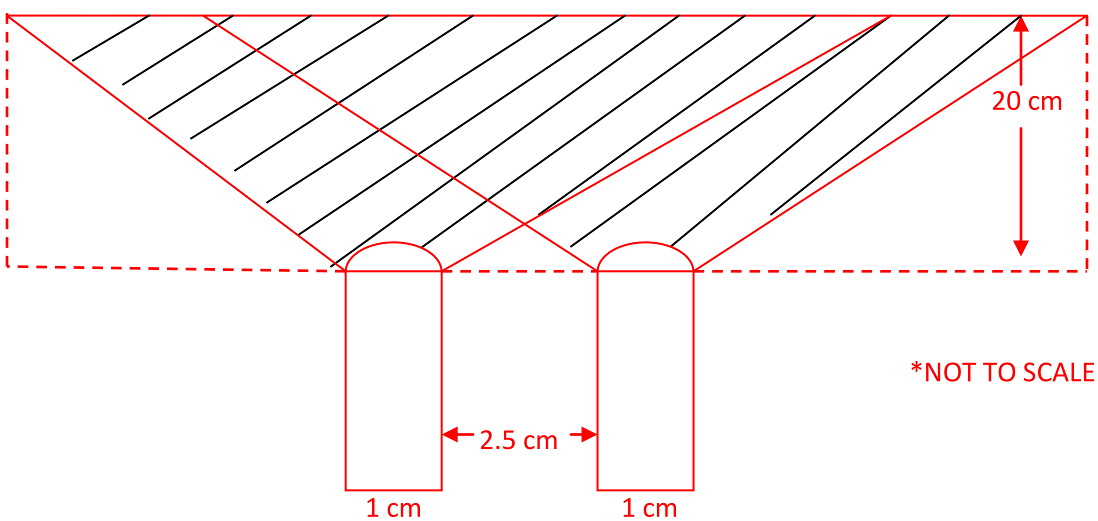
Did you label any angles? There is one angle that is particularly important: the one between the horizontal (x-axis) and the edge of the sensor's FoV. Add this one into your sketch, if it's not already there, and label it.

See the angle labeled above. This angle can be guessed at if the sensors are on hand, otherwise let it be 55°. This number will be used for the rest of the worksheet for computations.

Besides the physical layout of the sensor, what things might play a role in limiting the effectiveness of the sensor? Try to come up with a few different factors.

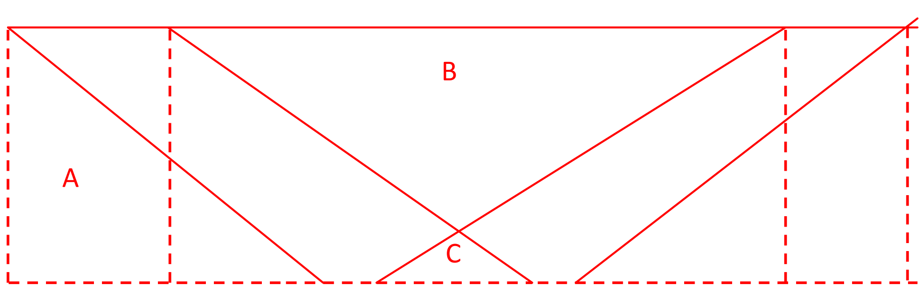
There are a host of answers to this open-ended question, some may include: strength of the lens on the cartridge, imperfections in the lens, speed of the object (it has to be very fast to not be detected), lighting conditions (low-light and low-contrast situations are generally overcome by the sensor, so this answer is less correct than others), resolution of the detecting components, ...

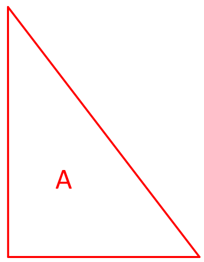
Now, say there are two cartridges 2.5 cm apart. They are both pointing the same direction, at a picture on a wall 20 cm away. If each cartridge is 1 cm across, sketch, with dimensions, this set up.



Using what you know about trigonometry, compute the total area the sensors have within their FoV.

This is the "meat" of this lesson. The area is the black hashed area indicated in the figure on the previous question. Refer to the figure below for further reference.





First, solve triangle "A". The distance from the sensors to the picture is given as 20 cm and the angle opposite that side (the vertical side) is given as 55°. Thus Law of Sines applies perfectly here. Solve for the length of the horizontal side.

$$\frac{x}{\sin{(90-55)}} = \frac{20}{\sin{(55)}} \qquad x = 14.004 \text{ cm}$$

By congruence, the triangles above and on the opposite side as A are all the same. Compute the area of this triangle with the dimensions known.

$$A_A = \frac{1}{2} * 14.004 * 20 \qquad A_A = 140.04 \text{ cm}^2$$

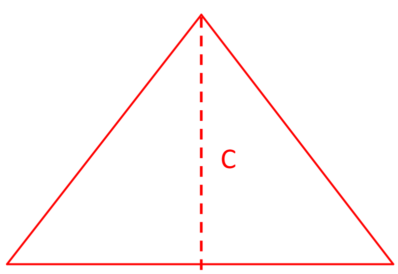
The length of the hypotenuse is also important later in the problem. Use the Pythagorean Theorem to compute this value.

$$\sqrt{20^2 + 14.004^2} = h \qquad h = 24.415 \text{ cm}$$

The area of C remains to be computed. Another application of the Law of Sines can be used here. However, it's necessary to consider the right-triangle formed by bisecting the angle at the "peak" of C. This triangle is similar to A. Either Law of Sines, or this similarity can be used to accurately compute the dimensions of C. The hypentenuous is also important, so compute that length as well.

$$\frac{0.5*2.5}{\sin{(90-55)}} = \frac{y}{\sin{(55)}} \qquad y = 1.7852 \text{ cm}$$

$$\sqrt{(2.5/2)^2 + 1.7852^2} = h \qquad h = 2.1793 \text{ cm}$$



Use this value in a computation of the area of C. Students may approach this by computing the area of one of the right-triangles then multiplying that by 2, but a more intelligent approach is to use *y* with the entire triangle at once.

$$A_C = \frac{1}{2} * 2.5 * 1.7852 \qquad A_C = 2.2315 \text{ cm}^2$$

Now consider the area direct in front (above in the model) of the sensors as a rectangle (in front and between). The area of this rectangle is found below.

$$(1 + 2.5 + 1) * 20 = A \qquad A = 90 \text{ cm}^2$$

Thus the total area in the sensor's field of view is the sum of the areas of this rectangle and 2 A triangles, with the area of C subtracted out.

$$2 * 140.04 + 90 - 2.2315 = A_{total} \qquad \boxed{A_{total} = 367.8485 \text{ cm}^2}$$

What percentage of that area is overlap between the two cartridges?

This requires the computation of the area of "B". The key here is to recognize that the "height" of B is related to the height of C that was already computed and that the base can be found using the length of the hypotenuses of A and C. Bear in mind that the result needs to be multiplied by two to account for both sections of B.

$$2 * \sqrt{(24.415 - 2.1793)^2 - (20 - 1.7852)^2} = b \quad b = 25.507 \text{ cm}$$

$$A_B = \frac{1}{2} * (25.507) * (20 - 1.7852)$$

$$A_B = 232.302 \text{ cm}^2$$

Thus the percentage overlap is found by dividing this result be the total area in the FoV and multiplying by 100%.

$$\frac{232.302}{367.8485} * 100\% = \%overlap \quad \boxed{\%overlap = 63.152\ \%}$$

This overlap is what gives the sensor what we call "hyperacuity to motion," or a super-sensitivity to anything that moves. Without this, the sensor loses many of its unique advantages.

Lastly, if you consider all the area in front of the two sensors as the total area (more than just the FoV), what percentage of that is outside the FoV? In other words, of the total area, how much is the sensor blind to?

After the last couple questions, this one is far simpler. The area inside the FoV is already known, so the total area "in front" of the sensors needs to be computed. Treat that area like a rectangle.

$$20 * (2 * 14.004 + 1 + 2.5 + 1) = A \quad A = 650.16 \text{ cm}^2$$

Now subtract the area within the FoV from this value to find the total "blind" area.

$$650.16 - 367.8485 = A_{blind} \quad A_{blind} = 282.3115 \text{ cm}^2$$

Thus the percentage of area the sensor is blind to is computed just like the overlap percentage above.

$$\frac{282.3115}{650.16} * 100\% = \%blind \quad \boxed{\%blind = 43.422\ \%}$$

Thus value is not as important as the overlap percentage, but allows for another computation and improves the overall understanding of the situation.