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Understanding the Spin Correlation of Singlet State Pair Particles

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Abstract: *A proposal is given for a possible understanding of the unusual spin correlation that is found between two separated spin one-half particles, which originally formed a two-particle spin zero singlet quantum mechanical pair state. As the Bell inequality has shown that it is not possible to understand such spin correlations using a precise hidden variable theory, it was necessary to consider an underlying statistical theory dictated by quantum mechanics. However, in contrast to the need for invoking a nonlocal interpretation of quantum mechanics, the well-known spin correlation results can be obtained and understood using a local statistical theory. The underlying quantum spin reality emerging upon measurement can be understood in a similar fashion as one understands the infinitely complex character of nonlinear dissipative systems, which exhibit aspects of unpredictability associated with deterministic chaos.*

Keywords: *Quantum Entanglement, Quantum Foundations, Spin Correlation, Bell Inequality, Deterministic Chaos.*

1. Introduction

The unusual spin correlation between two separated spin one-half particles, which originally formed a two-particle spin zero singlet quantum mechanical pair state, is well known. The perfect anti-correlation, which occurs when there is zero degrees of separation angle between the spin measurements of each particle gives support for the desire to construct a successful and precise hidden variable theory for the spin of each particle. However, given the necessary condition that the spin of each particle must be random and independent, in combination with the well-known statistical correlation between the two particle spin measurements which is simply a function of the separation angle, makes the search for a successful spin model quite problematic. Ultimately, with the advent of the Bell inequality analysis [1],it was definitively shown that precise hidden variable theories for the spin construct of each particle cannot be found. This agreed with quantum mechanical theory and experiments [2-5]. The promulgated conclusion of the original Bell inequality analysis, as well as numerous subsequent analyses, was that a successful theory (which would produce exactly the quantum mechanical prediction) must have a grossly nonlocal structure. This has always been a very disturbing concept.

In the following, we provide a mechanism of understanding the unusual spin correlation of singlet state pair particles in a local framework. Curiously, the recent knowledge gained during the last several decades on the study of the seemingly unrelated topic of classical nonlinear dissipative systems exhibiting somewhat unpredictable features, makes this view of randomness as a representation of chaotic dynamics noteworthy. These ideas have also found application in a wide variety of fields, including ecological and social dynamics [6-9], electronic dynamics [10,11],as well as in general physical systems [12]. With this knowledge base in mind, we consider a model of the spin system, which exhibits deterministic chaos such that the results of any measurement of the system are highly sensitive to the initial conditions of the system in combination with the measuring apparatus. As a result of this chaotic behavior, we take the point of view that the result of any specific measurement is typically described as an emergent reality which occurs upon measurement of the system, but which cannot be predicted prior to the measurement. It should be noted that this concept is well known in the context of measuring the quantum spin of an individual particle or, more importantly, the correlation between the spin measurements of two separated particles which originally formed a spin zero singlet pair state. Consequently, we ask the reader to consider the spin measurement process as an emergent reality, which cannot be predicted in advance and thus no hidden variable theory of individual spins will be proposed. In the terminology of infinitely complex systems exhibiting deterministic chaos, sufficient knowledge of initial conditions will not be assumed in the model.

We limit our analysis to a statistical model of the spin correlation associated with the two separated spin one-half particles which originally formed a spin zero singlet pair state. Specifically, we will offer a model of the statistical expectation value of the spin product of the two separated particles, which is simply a function of the measurement angle of separation. This model will be required to be consistent with the quantum mechanical prediction, as well as with spin experiments. In section 2, we present a brief review of the quantum mechanical analysis of this spin problem. In section 3, we present a brief review of the hidden variable analysis of this spin problem, originally used to derive the Bell inequality, which was clearly shown to be inconsistent with quantum mechanics. In section 4, our spin correlation model of this spin problem will be presented and shown to agree with quantum mechanics. In section 5, a discussion of the results will be offered which clearly shows that the restriction of the Bell inequality does not apply and, furthermore, that the model provides a clear understanding of the results without the need for invoking a nonlocal structure. Finally, in section 6, conclusions will be offered which propose the need for development of more sophisticated and complex quantum mechanical spin models which are consistent with the results given here, but which will ultimately lead to deeper insight into quantum spin systems.

2. Brief Review of the Quantum Mechanical Analysis of this Spin Problem

We use similar notation as in the original paper of Bell [1].The Bell inequality was derived in the context of Bohm's[13,14] simplified spin model used to elucidate the EPR[15] paradox. Here, we consider an initial singlet pair quantum state $|\psi\rangle$ of two spin ½ particles, which can be described using standard quantum mechanical spin state notation. Using a spin up $(+)$ and spin down $(-)$ basis, $|\pm\hat{\mathbf{a}}\rangle_1$, along the unit vector $\hat{\mathbf{a}}$ direction, for the first (labeled as 1) particle, and a similar basis for the second (labeled as 2) particle, the singlet state is

$$|\psi\rangle = \left(|+\hat{\mathbf{a}}\rangle_1 |-\hat{\mathbf{a}}\rangle_2 - |-\hat{\mathbf{a}}\rangle_1 |+\hat{\mathbf{a}}\rangle_2\right)/\sqrt{2}\,. \tag{1}$$

The two particles which form the original two-particle singlet pair state are assumed to be separated to a sufficient distance away from each other (without disturbing the spin structure of each particle), where simultaneous spin measurements are then made on the separated particles (beyond the light cone). Specifically, the first particle is measured in the $\hat{\mathbf{a}}$ direction, while the second particle is measured in the $\hat{\mathbf{b}}$ direction where $\hat{\mathbf{a}}\cdot\hat{\mathbf{b}} = \cos(\theta)$. With the use of standard quantum mechanical theory, the expectation value for the product of the two spin measurements can easily be derived, as follows. Denoting the normalized (to ± 1) spin measurement of the first particle as A, and the normalized spin measurement of the second particle as B, with the use of appropriate spin operators for the first particle $\boldsymbol{\sigma}_1$, and for the second particle as $\boldsymbol{\sigma}_2$, the quantum mechanical result for the spin product expectation value, $P_{QM}(\theta)$, is

$$P_{QM}(\theta) = \langle AB\rangle = \langle\psi|\left(\hat{\mathbf{a}}\cdot\boldsymbol{\sigma}_1\right)\left(\hat{\mathbf{b}}\cdot\boldsymbol{\sigma}_2\right)|\psi\rangle = -\hat{\mathbf{a}}\cdot\hat{\mathbf{b}} = -\cos(\theta)\,. \tag{2}$$

It is well known that this quantum mechanics result, which has been used in the Bell inequality analyses, is in agreement with all the spin (and polarization) experiments [2-5].

3. Brief Review of the Hidden Variable Analysis of this Spin Problem

As was done in section 2, we continue with the notation used in the original Bell inequality publication [1], in order to review that hidden variable spin proposal as well as the conclusions of that analysis. Specifically, associated with the singlet state pair of spin ½ particles, it was proposed that one should consider a precise hidden variable construct for the normalized spin of the first particle in the $\hat{\mathbf{a}}$ direction, and the spin of the second (separated) particle in the $\hat{\mathbf{b}}$ direction, given a hidden variable parameter value of λ. Here, it should be noted that the hidden variable spin of the first and second particles were precisely given by $A(\hat{\mathbf{a}},\lambda)$ and $B(\hat{\mathbf{b}},\lambda)$, respectively. To be consistent with the quantum mechanics, these functions can only take on plus or minus one values, i.e.

$$A(\hat{\mathbf{a}},\lambda)=\pm 1, B(\hat{\mathbf{b}},\lambda)=\pm 1. \tag{3}$$

In order to compare the quantum mechanically derived expectation value of the product of the spin measurements of the two particles with a hidden variable prediction, a general non-negative probability density, $\rho(\lambda)$, with $\rho(\lambda)\geq 0$, for occurrence of the hidden variable, λ, was proposed, where

$$\int d\lambda \rho(\lambda)=1. \tag{4}$$

Consequently, the hidden variable proposal for the spin product expectation value was given by

$$P(\hat{\mathbf{a}},\hat{\mathbf{b}})=\int d\lambda \rho(\lambda) A(\hat{\mathbf{a}},\lambda) B(\hat{\mathbf{b}},\lambda). \tag{5}$$

Ultimately, in the original publication it was shown that this hidden variable proposal for the spin product expectation value, equation (5), could not be used to reproduce the quantum mechanical result in equation (2), since

$$P(\hat{\mathbf{a}},\hat{\mathbf{b}})\neq -\hat{\mathbf{a}}\cdot\hat{\mathbf{b}}=P_{QM}(\theta). \tag{6}$$

Furthermore, for three general unit vector measurement directions, $\hat{\mathbf{a}},\hat{\mathbf{b}},\hat{\mathbf{c}}$, a Bell inequality was derived using the hidden variable proposal for the spin product expectation value, equation (5), which is

$$1+P(\hat{\mathbf{b}},\hat{\mathbf{c}})\geq \left|P(\hat{\mathbf{a}},\hat{\mathbf{b}})-P(\hat{\mathbf{a}},\hat{\mathbf{c}})\right|. \tag{7}$$

However, numerous experiments showed that the Bell inequalities such as that in equation (7) were violated [2-5]. Specifically, a violation is found by attempting to replace the hidden variable proposal of the spin product expectation value, equation (5), with the quantum mechanical result, equation (2), in the Bell inequality, equation (7), namely

$$P(\hat{\mathbf{a}},\hat{\mathbf{b}})\Rightarrow P_{QM}(\theta)=-\cos(\theta). \tag{8}$$

For example, if the three unit vector measurement directions, $\hat{\mathbf{a}},\hat{\mathbf{b}},\hat{\mathbf{c}}$, are in a plane and are each successively separated by sixty degrees, where

$$P(\hat{\mathbf{a}},\hat{\mathbf{b}})\Rightarrow -\cos(60^0)=-0.5, P(\hat{\mathbf{a}},\hat{\mathbf{c}})\Rightarrow -\cos(120^0)=0.5, P(\hat{\mathbf{b}},\hat{\mathbf{c}})\Rightarrow -\cos(60^0)=-0.5, \tag{9}$$

then the Bell inequality, equation (7), fails since the inequality is not correct, as shown here

$$1+P(\hat{\mathbf{b}},\hat{\mathbf{c}})\geq \left|P(\hat{\mathbf{a}},\hat{\mathbf{b}})-P(\hat{\mathbf{a}},\hat{\mathbf{c}})\right|\Rightarrow 0.5\geq 1.0. \tag{10}$$

Prior to proceeding to section 4 which describes our spin correlation model, that will be shown to agree with quantum mechanics, it is important to note that it is well known that the Bell inequality, equation (7), was correctly derived from the hidden variable proposal for the spin product expectation value in equation (5). Furthermore, it is important to note that the conclusion of the original Bell inequality publication[1] was that, in order for such a hidden variable spin construct, equation (5), to agree with quantum mechanics, equation (2), there must be a mechanism whereby the setting of one spin measuring device can influence the reading on the other device, such that the signal involved must propagate instantaneously (which is a nonlocal construct proposition). However, it should be emphasized that this conclusion does not indicate that quantum mechanical spin must be inherently nonlocal. It simply states that the proposed hidden variable model in equation (5) cannot agree with quantum mechanics unless the model includes a nonlocal construct. Nevertheless, since the advent of the Bell inequality there have been numerous searches for evidence of nonlocal spin interactions between separated particles [16-19]. In addition, and counter to the conclusion of the need for nonlocal spin constructs, it has also been published that the derivation of the Bell inequality does not include the possibility that the probability density for the hidden variable parameter could be a function of time (that is, different at each measurement location) [20], with the conclusion that the Bell inequality would not be applicable for more sophisticated hidden variable spin theories. Finally, in a recent special issue of the Journal of Physics A: Mathematical and Theoretical, which is devoted to 50 years of Bell's theorem, there is a published manuscript [21] which emphasizes that the Bell inequality is not just based on a locality assumption, but most importantly, the spin model assumes the existence of hidden variables, thus, the failure of the Bell inequality is not necessarily equivalent to the existence of nonlocal spin interactions. In any case, in the following, we take the position that a valid local hidden variable model of spin agreeing with the proposed spin product expectation value cannot be constructed. Consequently, in the following, we instead propose a way to understand the unusual spin correlation between singlet state pair particles through development of a local spin correlation model, which agrees with the quantum mechanical predictions.

4. SPIN CORRELATION MODEL OF THIS SPIN PROBLEM

With the conclusion that it is not possible to construct a precise local hidden variable theory of spin for each particle that agrees with quantum mechanics when it is used in the spin product expectation value, we turn our attention in this section to an understanding of the statistical aspects of the spin correlation. To this end, we continue with the notion proposed in the introduction, section 1, on the possibility that there could be an underlying infinitely complex aspect of spin which emerges upon measurement. With this assumption in place, we do not propose a specific hidden variable construct for the spin of each particle which originally formed a spin zero singlet pair state, since the actual spin value along a general measurement direction cannot be predicted prior to measurement (which is identical to the prescription imposed by quantum mechanics). However, this does not restrict a proposal for a local statistical model of the spin correlation between the two separated particles.

In order to efficiently proceed, we employ knowledge of the well-known spin product expectation value dictated by quantum mechanics, being $P_{QM}(\theta) = -\cos(\theta)$ as shown in equation (2). Specifically, although we do not know the spin of either particle until measurements occur, for the situation that the separation angle between measurements is small, $\theta \to 0$, we do know prior to measurement that the spin measurements will be highly anti-correlated, namely $P_{QM} \to -1$. For example, if the spin result of the first particle is $+1$, then the spin result of the second particle will tend towards -1 on average and vice versa. Conversely, for a large separation angle, $\theta \to \pi$, then the spin measurements will be highly correlated, viz. $P_{QM} \to 1$. Finally, for a separation angle approaching an orthogonal measurement direction condition, $\theta \to \pi/2$, then the two spin measurements will tend towards being completely uncorrelated, that is $P_{QM} \to 0$. Consequently, although we will not invoke a nonlocal spin construct, as the spin measurement direction of one particle is not allowed to influence the spin measurement of the other particle, we can still use this spin correlation concept in the construction of our spin model such that the angle between measurements is brought forward as a critical component of the model. Here, it is important to keep in mind that the result of any spin measurement of either particle cannot be precisely known in advance, due to the proposed infinitely complex nature of the spin measurement process. However, as the two separated singlet state pair particles are essentially mirror images of each other (from a spin state perspective), it is reasonable that there is a high degree of anti-correlation or correlation as the angle between measurements approaches 0 or π, respectively. It should be noted that this is a statistical concept valid on average over numerous experiments. It can never be used to predict a precise result for any one set of measurements of the two separated particles.

With the above statistical concepts of the spin correlation structure for singlet state particles in mind, we propose an alternate type of hidden variable theory for the spin correlation model. At the outset of our model description, it is important to emphasize that this is not meant to be a precise hidden variable theory of spin, as was proposed in the original Bell inequality publication

[1], since a local theory of this type cannot be successfully constructed. Thus, in contrast to the usual hidden variable theory that attempts to specify a precise spin structure for each separated particle, this model will simply represent the statistical concepts associated with the spin product of the two separated particles. In the following, we use a similar notation as was done for the review of the standard hidden variable theory, given in section 3, with the concept of a hidden variable parameter introduced to represent a specific realization for the spin. It is important to again emphasize that the hidden variable realization parameter is not used here to represent a specific spin for each particle. Instead, it is simply used to represent the spin product of the two particles, which as usual can only take on ± 1normalized values. The statistical aspect of the spin model will then be entirely embedded in the probability density function, which is associated with a specific hidden variable parameter value as well as with the angle of separation between measurements. As noted above, it is necessary for the probability density function to incorporate the fact that there should be a significant anti-correlation (where the normalized spin product is -1) for small angles of separation, and a significant correlation (where the normalized spin product is $+1$) for large angles of separation. As the following spin correlation model for singlet state pair particles incorporates the notion that there could be an underlying infinitely complex aspect of spin, which emerges upon measurement, we coin the phrase Infinite Complexity Hidden Variable (ICHV) theory associated with our reduced spin correlation model. This will be used to obtain the spin product expectation value in agreement with quantum mechanics.

Let the combined normalized spin product of the two separated particles be given by $AB_{ICHV}\left(\lambda\right)$, where λ is a continuous hidden variable parameter value over the infinite domain,

$$-\infty < \lambda < \infty , \tag{11}$$

and

$$AB_{ICHV}\left(\lambda\right) = \pm 1 . \tag{12}$$

Note that while this is the spin product function, $AB_{ICHV}\left(\lambda\right)$is not the product of the individual spins. In addition to this spin product function, we propose an associated non-negative normalized probability density, $\rho_{ICHV}\left(\theta,\lambda\right) \geq 0$, for all separation angles of measurement, $0 \leq \theta \leq \pi$, where the hidden variable parameter is again λ and the probability density is properly normalized as

$$\int_{-\infty}^{\infty} d\lambda \rho_{ICHV}\left(\theta,\lambda\right) = 1. \tag{13}$$

Given this approach for the spin product, in combination with an appropriate probability density model, the proposed ICHV spin product expectation value, $P_{ICHV}\left(\theta\right)$, is

$$P_{ICHV}\left(\theta\right) = \int_{-\infty}^{\infty} d\lambda \rho_{ICHV}\left(\theta,\lambda\right) AB_{ICHV}\left(\lambda\right) . \tag{14}$$

Here, it should be noted that this expectation value construct appears similar to the original hidden variable theory approach, of equation (5); however, the significant difference here is that the probability density, $\rho_{ICHV}\left(\theta,\lambda\right)$, in equation (14), must be a function of the separation angle, θ, as well as the hidden variable, λ, and the spin product, $AB_{ICHV}\left(\lambda\right)$, is only a function of the hidden variable, λ, as it will take on only plus or minus one values, depending on the hidden variable realization parameter. Finally, as a point of reference with respect to the notion of an underlying infinitely complex emergent spin structure, one can think of the hidden variable parameter representing the infinite variety of initial conditions. Unlike the standard hidden variable theories, which utilize precise spin predictions, this hidden variable parameter cannot be used for precise spin predictions, as it is connected statistically through the probability density.

Our specific ICHV choices for the spin product and associated probability density functions will lead to a spin product expectation value that is consistent with quantum mechanics. Let the spin product function be

$$AB_{ICHV}(\lambda)=\begin{cases}+1, & 0<\lambda<+\infty \\ -1, & -\infty<\lambda<0\end{cases}, \tag{15}$$

where the probability density is

$$\rho_{ICHV}(\theta,\lambda)=\begin{cases}\dfrac{2}{\sqrt{\pi}}\exp\left\{-\left[\dfrac{\lambda}{\sin^2(\theta/2)}\right]^2\right\}, & 0<\lambda<+\infty \\ \dfrac{2}{\sqrt{\pi}}\exp\left\{-\left[\dfrac{\lambda}{\cos^2(\theta/2)}\right]^2\right\}, & -\infty<\lambda<0\end{cases}. \tag{16}$$

Here, it should be noted that our choice of the probability density, equation (16), for analytic simplicity, incorporates a Gaussian function structure, which can easily be integrated over the hidden variable. Clearly, other functional forms for the probability density function could be used if desired. Finally, in order to show that these choices for the spin product in combination with the probability density function will be consistent with the quantum mechanical predictions, it is necessary to check the probability density normalization as well as the spin product expectation value result. First, we show that this probability density is properly normalized.

$$\begin{aligned}\int_{-\infty}^{\infty} d\lambda\rho_{ICHV}(\theta,\lambda) &= \int_{0}^{\infty} d\lambda\frac{2}{\sqrt{\pi}}\exp\left\{-\left[\frac{\lambda}{\sin^2(\theta/2)}\right]^2\right\}+\int_{-\infty}^{0} d\lambda\frac{2}{\sqrt{\pi}}\exp\left\{-\left[\frac{\lambda}{\cos^2(\theta/2)}\right]^2\right\}. \\ &=\sin^2(\theta/2)+\cos^2(\theta/2)=1\end{aligned} \tag{17}$$

Second, we show that the spin product expectation value, using the ICHV model, equation (14), agrees with quantum mechanics, as given in equation (2).

$$\begin{aligned}P_{ICHV}(\theta) &= \int_{-\infty}^{\infty} d\lambda\rho_{ICHV}(\theta,\lambda)AB_{ICHV}(\lambda)=(+1)\int_{0}^{\infty} d\lambda\rho_{ICHV}(\theta,\lambda)+(-1)\int_{-\infty}^{0} d\lambda\rho_{ICHV}(\theta,\lambda) \\ &=\sin^2(\theta/2)-\cos^2(\theta/2)=1-2\cos^2(\theta/2)=-\cos(\theta)=P_{QM}(\theta)\end{aligned}. \tag{18}$$

Ultimately, to gain further insight into the ICHV spin correlation model, it is useful to show that the spin product function, equation (15), in combination with the probability density function, equation (16), achieve the desired spin correlation results depending on the separation angle between measurements, as described above. Here, we show that a precise anti-correlation occurs where the spin product is negative one, $AB=-1$, in the limiting case of a zero angle of separation, $\theta=0$; a precise correlation occurs where the spin product is plus one, $AB=+1$, in the case of one-hundred and eighty degrees of separation, $\theta=\pi$; and that there is no correlation for the spin product, which is zero upon average as $\langle AB\rangle=0$, in the case of ninety degrees of separation, $\theta=\pi/2$. In fact, all of these boundary conditions for the spin product are satisfied, as noted by taking the appropriate separation angle limits, given as

$$AB_{ICHV}(\lambda)=-1,\quad -\infty<\lambda<0 \text{ with } \lim_{\theta\to 0}\rho_{ICHV}(\theta,\lambda)\to\begin{cases}0, & 0<\lambda<+\infty \\ \dfrac{2}{\sqrt{\pi}}\exp(-\lambda^2), & -\infty<\lambda<0\end{cases}; \tag{19}$$

$$AB_{ICHV}(\lambda)=+1,\quad 0<\lambda<+\infty \text{ with } \lim_{\theta\to\pi}\rho_{ICHV}(\theta,\lambda)\to\begin{cases}\dfrac{2}{\sqrt{\pi}}\exp(-\lambda^2), & 0<\lambda<+\infty \\ 0, & -\infty<\lambda<0\end{cases}; \tag{20}$$

and

$$AB_{ICHV}\left(\lambda\right)=\begin{cases}+1, & 0<\lambda<+\infty\\ -1, & -\infty<\lambda<0\end{cases} \text{ with } \lim_{\theta\to\pi/2}\rho_{ICHV}\left(\theta,\lambda\right)=\begin{cases}\frac{2}{\sqrt{\pi}}\exp\left(-2\lambda^2\right), & 0<\lambda<+\infty\\ \frac{2}{\sqrt{\pi}}\exp\left(-2\lambda^2\right), & -\infty<\lambda<0\end{cases}. \quad (21)$$

5. Results and Discussion

It is important to review the lessons learned and understanding gained from the development of this spin correlation model for singlet state pair particles. First, recall that after the advent of the original Bell inequality analysis, where it was shown that a precise local hidden variable model of spin, equation (3), cannot be used to reproduce the unusual spin product expectation value quantum mechanical result, equation (2), much of the physics community was led to the assumption that a nonlocal ("spooky action at a distance") spin structure must be responsible for the quantum mechanical results. However, now it should be clear that such a nonlocal structure is not necessary to achieve the quantum mechanical spin results. Instead, this can be achieved by using a strategically chosen statistical model of the spin structure, which incorporates a probability density, equation (16), which is a function of the separation angle between measurements, θ. It should be emphasized that this spin model is fundamentally local, as it is built into the spin structure at the outset prior to measurement of either particle. Ultimately, in combination with the ± 1normalized spin product values, equation (15), and the associated probability density, equation (16), the ICHV local spin model leads to the spin product expectation value, equation (14), which reproduces precisely the quantum mechanical result $P_{ICHV}\left(\theta\right)=-\cos\left(\theta\right)$.

Second, it is important to note that, although the spin correlation model given here is equivalent to standard quantum theory after integration over the hidden variable, the ICHV model is offered as an approach to understand a possible explanation for the unusual quantum mechanical results. Specifically, the hidden variable, λ, is utilized in order to incorporate the needed statistics for the spin model so that it matches the quantum mechanical spin product expectation value result without employing exotic concepts, such as the need for a nonlocal spin mechanism. Furthermore, recall that the spin measurement concept employed here is that the spin system in combination with the measuring device is sufficiently complex such that the spin states cannot be predicted precisely in advance. It emerges in the process of measurement (for example, as found in systems which exhibit deterministic chaos), while statistical spin results can be consistently predicted for the pair of singlet state particles as a function of the separation angle of measurement, θ. Here, it should be noted that it is the hidden variable parameter in the ICHV spin correlation model that mathematically incorporates the unpredictability for a specific spin measurement realization result, representing the infinite variety of initial conditions. Consequently, with the ICHV spin model in mind, one may gain further understanding of the unusual quantum mechanical spin result; however, if the hidden variable is integrated over all the possible realizations, the usual and somewhat mystical quantum mechanical probability theory result appears. This can be demonstrated by noting that the quantum mechanical probability for the spin product to be one (that is $AB=+1$) is $\sin^2\left(\theta/2\right)$, which is found by integrating the ICHV probability density, equation (16), over the positive hidden variable parameter domain, $0<\lambda<+\infty$, where

$$\Pr\left(AB=+1\right)=\int_0^\infty d\lambda\rho_{ICHV}\left(\theta,\lambda\right)=\sin^2\left(\theta/2\right), \quad (22)$$

and the quantum mechanical probability for the spin product to be minus one (that is $AB=-1$) is $\cos^2\left(\theta/2\right)$, which is found by integrating the ICHV probability density, equation (16), over the negative hidden variable parameter domain, $-\infty<\lambda<0$, where

$$\Pr\left(AB=-1\right)=\int_{-\infty}^0 d\lambda\rho_{ICHV}\left(\theta,\lambda\right)=\cos^2\left(\theta/2\right). \quad (23)$$

Therefore, the spin product expectation value from the quantum mechanical perspective, equation (2), is identical to the integrated version of the ICHV spin model, equation (18), as it should be, which is shown by using these quantum mechanical probability results, equations (22) and (23).

$$\begin{aligned} P_{QM}(\theta) &= (AB=+1)\Pr(AB=+1)+(AB=-1)\Pr(AB=-1) \\ &= (+1)\sin^2(\theta/2)+(-1)\cos^2(\theta/2) = -\cos(\theta) = P_{ICHV}(\theta) \end{aligned}. \quad (24)$$

Nevertheless, it should be clear that the detailed unpredictability and the associated statistical interpretation of this spin correlation model is lost when integration is performed over the hidden variable.

Finally, it is important to note that the Bell inequality restriction, which was applicable to the original and precise hidden variable spin construct, is not applicable to this statistical spin model. To see this, recall that the Bell inequality analysis associated with three distinct vector measurement directions, $\hat{\mathbf{a}},\hat{\mathbf{b}},\hat{\mathbf{c}}$, and shown in equation (7), was based on the precise hidden variable construct for the spin of each particle, equation (3), in combination with the spin product expectation value calculation. However, one should also recall that the spin correlation model presented here, which correctly predicts the quantum mechanical result for the spin product expectation value, is obtained through the integral construct found in equation (14). The spin product expectation value explored here has no detailed functional dependence on any distinct vector measurement directions, so that Bell inequality type of analyses cannot be constructed for this spin correlation model.

6. CONCLUSION

This model allows one to gain further insight into the complexity of the quantum spin process through an alternate use of a hidden variable parameter. By using a purely local framework and a statistical spin model, this model produces all the critical features of quantum mechanics and spin experiments. Given the success in obtaining the quantum spin product expectation value, we propose that quantum unpredictability of individual measurements can be associated with an underlying complexity (modeled using a hidden variable) that can be thought of as representing an infinite variety of initial conditions. Thus, we suggest that exploration of more sophisticated quantum spin models which exhibit these complex characteristics is warranted. Specifically, we would like to encourage those with expertise associated with the study of classical nonlinear dissipative systems (exhibiting deterministic chaos due to extreme sensitivity on initial conditions) to explore the construction of more sophisticated quantum spin models. It is assumed that these complex spin models will not only achieve the needed unpredictability, but that they will also predict the well-known statistical quantum spin results. Ultimately, using such exploratory approaches for the construction of complex spin models, there is a possibility of achieving a deeper insight into the quantum spin process, which may exhibit a more deterministic dynamics than traditional quantum mechanics, but which will still exhibit the usual unpredictability found in traditional quantum mechanics.

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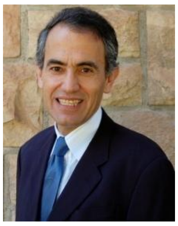

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