Lecture 2: The Classical Model of the Economy

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Production Functions:

- Production functions describe how much output a set of given inputs (K and L) can create, given technology.
- Production functions have the following general properties:
  - no output without input
  - output is increasing in factor inputs used
  - as only one factor is increased, output increases at a decreasing rate (marginal products are diminishing in factors used)
This is a specific (mathematical) functional form often used to describe production

\[ Y = AK^{\alpha}L^{(1-\alpha)} \]

where:
- K is the amount of capital used
- L is the amount of labor used
- A is a parameter describing technology
Properties of the Cobb-Douglas Production Function

① Constant Returns to Scale (CRS)
- CRS technology refers to technology with the following property: if by changing L and K by a constant factor of proportionality, output also changes by the same proportion.
  - Example: double both L and K (inputs), then Y (output) doubles.
Constant Returns to Scale and Cobb-Douglas Production Functions

- Suppose both L and K increase by a factor \( z \)
- CRS technology implies that output also increases by \( z \)
  \[ F(zK, zL) = zY \]
- in a Cobb Douglas Function:
  \[ A(zK)^{\alpha}(zL)^{(1-\alpha)} = Az^{(\alpha+1-\alpha)}K^{\alpha}L^{(1-\alpha)} \]
  \[ = AzK^{\alpha}L^{(1-\alpha)} = zAK^{\alpha}L^{(1-\alpha)} = zY \]
Properties of the Cobb-Douglas Production Function

(2) Marginal Products are decreasing in their own factor and increasing in the other factor:

- Since \( Y = AK^\alpha L^{(1-\alpha)} \)
- labor:
  - \( \text{MP}_L = (1-\alpha)AK^\alpha L^{-\alpha} = [(1-\alpha)AK^\alpha]/L^{\alpha} \)
    (take a simple derivative)
  - If L increases, \( \text{MP}_L \) falls
  - If K increases, \( \text{MP}_L \) rises
Properties of the Cobb-Douglas Production Function

- capital:
  - $MP_K = \alpha AK^{(\alpha-1)}L^{(1-\alpha)} = [\alpha AL^{(1-\alpha)}]/K^{(1-\alpha)}$
- this is the derivative of $Y$ with respect to $K$, or in plain English, the change in output for a small change (like one unit) in $K$
  - If $K$ increases, $MP_K$ falls
  - If $L$ increases, $MP_K$ rises
**Useful properties of these MP’s:**

- Suppose we multiply the marginal product of labor by \( L/L = 1 \):
  
  \[
  \text{MP}_L \cdot \frac{L}{L} = \left[ (1-\alpha)AK^\alpha L^{-\alpha} \right] \frac{L}{L}
  \]

  using the rules for exponents:

  \[
  \text{MP}_L = \left[ (1-\alpha)AK^\alpha L^{-\alpha+1} \right] / L
  \]

  \[
  \text{MP}_L = \left[ (1-\alpha)AK^\alpha L^{1-\alpha} \right] / L
  \]

  Since \( Y = AK^\alpha L^{1-\alpha} \)

  \[
  \text{MP}_L = \left[ (1-\alpha)AK^\alpha L^{1-\alpha} \right] / L = (1-\alpha)Y/L
  \]

- therefore, \( \text{MP}_L = (1-\alpha)Y/L \)
Useful properties of these MP’s:

- Suppose we multiply the marginal product of capital by $K/K=1$:
  \[ MP_{K\times K/K} = [\alpha AK^{\alpha-1}L^{1-\alpha}]xK/K \]
  using the rules for exponents:
  \[ MP_K = [\alpha AK^{\alpha-1+1}L^{-\alpha+1}]/K \]
  \[ MP_K = [\alpha AK^{\alpha}L^{1-\alpha}]/K \]
  Since \( Y = AK^{\alpha}L^{1-\alpha} \)
  \[ MP_K = [\alpha AK^{\alpha}L^{1-\alpha}]/K = \alpha Y/K \]
- therefore, \( MP_K = \alpha Y/K \)
Useful properties of these MP’s:

- To recap:
  - $\text{MP}_L = (1-\alpha)Y/L$
  - $\text{MP}_K = \alpha Y/K$

- $Y/L$ and $Y/K$ are called the *average productivity of capital and labor* respectively.

- the marginal products of labor and capital are proportional to their average productivities.
Properties of the Cobb-Douglas Production Function

③ This function implies a constant proportion of total GDP is paid to each factor.

- If factors of production earn the value of their marginal products, the exponents on the Cobb-Douglas function describe the share of GDP labor and capital receive.
Properties of the Cobb-Douglas Production Function

- Example: \( Y = AK^\alpha L^{1-\alpha} \)
  - \( \alpha \) % of GDP goes to \( K \), (1 - \( \alpha \)) % of GDP goes to \( L \)

- Total factor payments to \( K \) are expressed as
  \[
  \frac{R}{P} \times K = MP_K \times K \]
  \[
  \frac{\alpha Y}{K} \times K = \alpha Y
  \]

- Similarly:
  \[
  \frac{w}{P} \times L = MP_L \times L
  \]
  \[
  (1-\alpha) \frac{Y}{L} \times L = (1-\alpha)Y
  \]
Constant Returns to Scale in the Economy

- It is easy to show then, that the total payments to factors are
  \[ = \left( \frac{R}{P} \times K \right) + \left( \frac{w}{P} \times L \right) \]
  \[ = \alpha Y + (1-\alpha)Y = (\alpha+1-\alpha)Y = Y \]

- This is a general property of an economy with CRS technology: *the value total factor payments must equal the value of output* *(total profits must be zero)*
Comparative Statics

- Imagine an exogenous change occurs in the economy.
- Comparative statics evaluate such a change as it impacts the total economy.
- Using comparative statics allows us to determine how the model predicts the economy will be affected by such a change.
For example, suppose that the population decreases suddenly, as it did during the Plague.

- How does this impact the labor market?
- How will this affect production?
- How will this affect prices?
- How will these, in turn affect the labor market?
\[ Y = F(L, \bar{K}) \]

\[ Y = F(\bar{L}, K) \]

\[ w_2 \]

\[ w_1 \]

\[ \bar{L} \]

\[ \bar{K} \]

\[ L \]

\[ K \]

\[ w \]

\[ P \]

\[ P \cdot MP_L \]

\[ P \cdot MP_K \]

\[ Y \]

\[ Y = Y \]

\[ \text{Labor Market} \]

\[ \text{Capital Market} \]

\[ \text{Goods Market} \]

\[ \text{AD} \]
Comparative Statics

- From the diagram we can see that the fall in L increases the nominal wage.
- The fall in L also reduces output as a function of capital \( Y = F(K, L) \) where L is fixed) as there is less labor at all levels of capital. Since production is increasing in factors of production, the fall in L lowers output and the \( MP_K \).
Comparative Statics

- Since the capital market clears, and all K is used, the fall in L reduces output and lowers Aggregate Supply.
- This effect increases the price level.
Comparative Statics

- The increase in the price level effects the demand for K and L.
  - $P \times MP_L$ shifts upward, increasing the nominal wage.
  - Less L implies a higher $MP_L$, thus $w/P$ must rise.
  - The same shift is occurring in the K market, but it is offset by the fall in $MP_K$ due to the fall in L.
  - Diagrammatically, this change cannot be determined.
Overall then, the fall in L caused
- a fall in $MP_K$
- a fall in total output $Y$
- an increase in $P$
- an increase in MPL
- an increase in nominal and real wages.
- An indeterminate change in nominal and real rental rates of $K$
Comparative Statics

- Instead of using the diagram, we can also use the equilibrium conditions that must be true after the economy reacts to the exogenous change in L.
- To do so, we must identify the equilibrium conditions that are true in each market.
In the factor markets, the value of the wage (a firm’s marginal cost) must be equal to the value of the benefit the firm gets from using that factor $P \times MP$

$$w = P \times MP_L, \quad R = P \times MP_K$$

or

$$\frac{w}{P} = MP_L, \quad \frac{R}{P} = MP_K$$
Factor Market Equilibrium

- In the labor market, L available has fallen, therefore $MP_L$ will rise
  - from the equilibrium condition in the labor market this implies the change in L results in an increase in the real wage.

- In the capital market, L available has fallen, therefore $MP_K$ will fall (assuming Cobb-Douglas production)
  - this implies $R/P$ must fall to maintain equilibrium
Goods Market Equilibrium

- Reduced L implies reduced Y
- Since \( Y = C + I + G \), AD must fall to maintain equilibrium.
  - With no change in AD, this implies an increase in Price level in the economy.
- Using the equilibrium conditions we have now described all of the outcomes the diagram could tell us, plus the indeterminant one in the K market.
Fiscal Policy Changes in the Long Run

- What if G is increased?
- To evaluate this change we consider the effect it has in the Goods market in equilibrium:

\[ Y = C + I + G \]
\[ C = C(Y - T) \]
\[ I = I(r) \]
\[ G = G \]
Fiscal Policy Changes in the Long Run

- Previously we have shown that this can be re-arranged as
  \[ Y-C(Y-T)-G = I(r) \]
- since all terms on the left-hand side are exogenously determined, this implies that, if G increases, I(r) must decrease by an equivalent amount to maintain equilibrium.
- This is called “complete crowding out”.
Fiscal Policy Changes in the Long Run

- In the long run, the classical model predicts that if government expenditure increases, it will reduce private investment by an equivalent amount.
- This makes sense because total output is determined only by the production technology and the amount of factors available, not expenditure.
Classical Dichotomy

- This refers to the fact that the real side of the economy (technology, factors of production available) determines the level of real output and real productivity.

- Nominal values in the economy are determined by demand (i.e. level of AD determines the price level given AS).
The Classical Model

- This is the basic model of the long run relationships in the economy.
- It tells us real output is determined only by technology and the factors of production available.
- It also tells us how income is distributed.
- It tells us how price levels are determined in the economy.
The Classical Model

- Because this is a long run model, it cannot tell us how recessions and booms are caused (why?).
- It cannot explain business cycles (why?).
- It cannot explain persistent unemployment above the natural rate (why?).