Mathematics 4340
Homework #3

Analytic Part (15%). Solve problems number 4.6 and 4.8 on page 190 in
the textbook.

Computational Part (85%). The goal of this part is to compare different
interpolants for the function

\[ f(x) = \frac{1}{1 + x^2} \]

on the interval \( x \in [-5, 5] \). Once the different interpolants are computed,
they should be plotted against the original function \( f(x) \). In order to un-
derstand the behavior, the plots must show the value of the interpolant and
the function for a much larger set of points than just the interpolation data
points, since at the data points the interpolant concides with the function.
To achieve this you can use the plotting capabilities of your software (i.e.
Matlab, Maple etc.) to plot the resulting interpolants (i.e. the polynomials)
specified by their coefficients (which you will obtain as a result of the inter-
polation procedure), or compute the value of these interpolants in a large
set of \( M \) points as explained below and then join these data with straight
lines.

(a) Compute the Lagrange interpolants based on \( N + 1 \) equi-distant nodes
that include the two ends of the interval, i.e. the nodes are given by
\[ x_j = -5 + \frac{10j}{N}, \ j = 0, 1, \ldots, N. \] Do this for three values of \( N \), \( N = 4, \)
\( N = 10 \) and \( N = 20 \); for each case plot the resulting polynomial versus
the function \( f \). To create the plots, compute the resulting interpolation
polynomial and the original function at \( (M + 1) \) equi-distant point locations
\( x_k, k = 0, 1, \ldots, M. \) Use \( M = 200 \) for all plots.

(b) Compute the Lagrange interpolants based on the \( N + 1 \) Chebyshev nodes
defined by \( x_j = -5 \cos \left( \frac{\pi j}{N} \right), \ for \ j = 0, 1, \ldots, N. \) Use the same three val-
ues of \( N \) as above and again create the three plots by computing the values
of these polynomials for the same set of \( M \) points.

(c) Compute the natural spline interpolant using the equi-distant nodes in
point (a) above for \( N = 10 \). Plot it as above.

Turn in a copy of your computer code, together with a separate plot for
each interpolant computed (i.e. three plots \( f \) versus the respective inter-
polant for point (a), three for point (b), etc.). Please indicate on each plot
what case it represents.