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**PARTICIPANT RESEARCH ESSAY FOR DIME RESEARCH TEAM**

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My research focuses on students’ mathematical thinking. I use teaching experiments (Steffe & Thompson, 2000) and scheme theory (von Glasersfeld, 1995) to build psychological models of students’ mathematics. I focus on students’ constructions of fractions schemes, in particular,
because this context allows me to rely on initial models developed by Steffe (2002) and Olive (1999), wherein I can refine those models and consider ways in which students’ conjecturing activity supports their learning. This narrow context broadens again as I consider ways to validate and disseminate the models, and as I relate conjecturing and other mathematical processes, such as problem solving and proving. The remainder of this statement will outline my efforts and contributions in this regard.

**Fractions Learning**

Although I intended my focus on students’ fractions learning as a context for studying conjecturing, my research has inevitably produced results related to the context itself. In particular, data from teaching experiments I have conducted with fifth and sixth-grade students have underscored the central importance of students’ splitting operations in their understanding of advanced fractions conceptions, such as improper fractions. In two publications in the *Journal for Research in Mathematics Education* I describe related findings. In the first publication (Norton, 2008) I describe ways in which students use splitting operations to form conjectures that result in the development of more advanced schemes for conceptualizing fractions. In the second (Norton & D’Ambrosio, 2008) I demonstrate the pedagogical consequences of working with students who have constructed splitting operations compared with those who have not.

**Assessing Schemes and Operations**

Data from teaching experiments such as mine have contributed to fine-grained models of students’ thinking, especially with regard to fractions. However, because they involve longitudinal and intensive interactions with small groups of students, they do not provide for the kind of large-scale hypothesis testing that Kilpatrick (2001) has advocated. In order to quantitatively test models and hypotheses resulting from teaching experiments, I have designed written instruments for assessing students’ schemes and operations, described in a *Journal of Mathematical Behavior* publication (Norton & Wilkins, 2009). Results from these tests not only affirm many of the hypotheses and models, but they provide a means of communicating results to a larger community of researchers who are unfamiliar with teaching experiment methodology. For example, results from written assessments of fifth and sixth-grade students indicate that students develop part-whole reasoning (e.g., 3/5 as 3 out of 5) for unit fractions and other proper fractions at the same time, and that most students at those grade levels conceptualize fractions as parts out of a whole. On the other hand, students develop partitive reasoning (3/5 as three iterations of 1/5, and 1/5 as a part that, when iterated five times, reproduces the whole) for unit fractions long before developing similar conceptions for other proper fractions.

**Conjecturing**

My research on students’ mathematical conjectures centers on answering, “whence and wherefore conjecture?” Norton (2008) demonstrated that students form productive conjectures when they use existing operations (such as splitting) in novel ways. The study demonstrated the productivity of those conjectures in terms of the new schemes students’ constructed as a result. This explanation for how students form conjectures also helps to resolve the ancient learning paradox, first posed by Plato: How can one construct more advanced ways of thinking on the basis on less advanced ways of thinking? Namely, when students use existing ways of operating in novel ways, new schemes are formed. Because students’ construct new operations through internalized action, possibilities for constructing new (and more powerful) schemes and operations are limitless. I elaborate on these arguments in a *For the Learning of Mathematics* publication (Norton, 2009).
For lack of a comprehensive literature review on students’ mathematical conjectures in existing literature, I have also endeavored to summarize research and theory about students’ conjecturing activity in a new manuscript submitted to the Journal for Research in Mathematics Education. The manuscript is intended to provide the kind of orienting perspective for research on conjecturing that existing work has provided for other mathematical processes, such as problem solving and proving.

**Teacher Education**

As a mathematics educator, I feel an obligation to consider potential impacts of research on practicing K-12 mathematics teachers. That obligation—combined with my own professional development as a teacher that has resulted from conducting teaching experiments—spurred me to investigate the value of engaging classroom teachers in teaching experiments with their own students. The *Fractions Recovery* project (Norton & McCloskey, 2008) did just that; over a two-year period, we engaged three classroom teachers in teaching experiments with pairs of students from their respective classrooms. Findings from that project indicate that teachers spend more time asking probing questions and listening to their students in the classroom, as a result of what they learn about students’ thinking during out-of-class teaching experiments. I have disseminated findings from this and other research projects in teacher journals, with two publications in each of the NCTM teacher journals.

As a mathematics educator, I am also responsible for the professional development of pre-service teachers. In order to recruit, prepare and retain more and better secondary school mathematics teachers, I serve as principal investigator on the *Virginia Teach* Noyce grant, which provides scholarship money for math majors who commit to teaching in high needs schools. Recruitment activities include internships for sophomores, so that they can have an early experience as teacher aids in high needs schools. The grant also includes several community building activities, such as VCTM conference travel and math education colloquia.

In an effort to focus pre-service teachers’ attention on students’ mathematical thinking, I have developed a letter-writing project that engages pre-service teachers in designing and posing tasks to high school students. The project has spanned four cohorts of pre-service teachers over a period of four years, and research from the project has produced numerous scholarly articles. Several of these articles were written by graduate students involved in the project.

I have served as either PI or co-PI on grants totaling nearly four million dollars, all of which include professional development components. The largest of these grants is an NSF-funded DR-K12 grant, *Iterative Model Building*, which seeks to transform math and science early field experiences for future elementary school teachers. It includes aspects of teaching experiments and it, too, focuses pre-service teachers’ attention on students’ mathematical (and scientific) thinking. As part of the project, I designed a video-based prediction assessment instrument to measure pre-service teachers’ abilities to model and predict students’ thinking. I have presented findings based on the design and use of the instrument at several conferences, including NCTM & AERA, and have submitted a related manuscript to the Journal for Mathematics Teacher Education.

**Summary of Plans for Continued Research**

Going forward, I intend to continue research on students’ mathematical thinking, with particular attention to students’ fractions knowledge and conjecturing activity. This work could help researchers identify which fractions schemes and operations contribute to algebraic reasoning and how conjecturing activity can support that transition. I have submitted a revised CAREER proposal to NSF with those goals in mind.
I will also continue build interdisciplinary bridges with researchers in other fields who are concerned with how students learn. For instance, I have proposed a new research cluster for the College of Science at Virginia Tech that would connect researchers across departments (and colleges) with the goal of advancing educational neuroscience. Currently, I am consulting some of those researchers on projects related to the design of educational technology. Findings on the effectiveness of such tools, along with results from neural imaging, can affirm and/or inform theories of learning that I am trying to build.

Written assessments of students’ ways of operating provide further triangulation of teaching experiment data concerning how students learn. I plan to continue refining and implementing written assessments of students’ fractions schemes with that goal in mind. If I do not receive a CAREER grant to continue this work, I intend to seek funding from other sources, such as an NSF DR-K12 grant. In any case, my primary focus will be to establish reliable learning trajectories from students’ existing ways of operating with fractions to students’ conjectural ways of operating with algebra. Once reliable learning trajectories are developed, I can then pursue NSF MSP grant money to work in classrooms across middles schools in the local school system.

References