QUANTITATIVE REASONING & MATHEMATICAL MODELING IN STEM: A COMMENTARY

Richard A. Duschl
Pennsylvania State University

Current conceptualizations of scientific knowledge and mechanisms in philosophy, of cognition and learning in psychology, and of teaching and assessment in pedagogy represent, respectively, a narrative that is one of growth and development over time. Advances in cognitive and sociocultural psychology, recognition of the importance of disciplinary modeling and discourse practices in scientific inquiry, learning, and in the scaffolding of learning with tools and technologies, along with the adoption of ‘assessment for learning’ instructional strategies are factors, among others, that have led scholars to advance positions that learning ought to be coordinated and sequenced along conceptual trajectories (Driver, Leach, Scott, & Wood-Robinson, 1994), developmental corridors (Brown, 1997) and learning progressions (Duschl, Schweingruber, & Shouse, 2007; Duschl, Maeng & Sezen, 2011).

A reading of the Core Common Standards – Mathematics and the Next Generation Science Standards reveals that the focus for K-12 STEM education is now on the development of knowledge and engagement in practices that facilitate the attainment of conceptual, epistemic and social learning goals (Duschl, 2008). Conceptual goals are the core ideas (e.g., rational numbers, matter and energy, heredity) and crosscutting concepts (e.g., patterns, probability, mechanisms, system models) found in STEM disciplines. The epistemic goals are the scientific ‘what counts’ practices associated with questioning, measuring, modeling, and explaining. Social goals are the disciplinary talk, argumentation, and representations that are used to critique and/or communicate scientific and mathematical ideas, processes, and information.

Taking Science to School (TSTS) (Duschl, Schweingruber, & Shouse, 2007) echoes the mathematics education recommendations in Adding It All Up (NRC, 2001) that K-12 teaching and learning needs to focus on fewer curriculum topics, on using knowledge and on developing disciplinary proficiencies, literacies, and practices. The TSTS report identifies four proficiencies in science that all students should attain:

- Strand 1: Know, use, and interpret scientific explanations of the natural world.
- Strand 2: Generate and evaluate scientific evidence and explanation.
- Strand 3: Understand the nature and development of scientific knowledge.
- Strand 4: Participate productively in scientific practices and discourse.

The four strands of science proficiency emphasize 1) a move away from science as inquiry to science as a set of practices rooted in model-building and model-refining; and 2) a move toward taking science education out of its current silos of biology, chemistry, earth systems, and physics into a more integrated STEM approach focused on the application of science and mathematics content and practices in real-world contexts.

The NRC followed TSTS five years later with a framework report that would inform the design and development of the Next Generation Science Standards. That report is A Framework for K-12 Science Education: Practices, Crosscutting Concepts, and Core Ideas (NRC, 2012). The report proposes three dimensions around which STEM education should be coordinated (See Figure 1):
• Crosscutting Concepts that unify the study of science through common application across science fields and thereby represent the nature of science.
• Scientific Practices that frame the more nuanced ways inquiry and the growth of scientific knowledge occurs.
• Disciplinary Core Ideas represent the generative ideas found in physical sciences, life sciences, earth and space sciences, and engineering, technology, and the applications of science that support students learning.

**Figure 1.** Three Dimensions of the Framework. NRC (2012) *A Framework for K-12 Science Education: Crosscutting concepts, scientific practices and core ideas.*

Common to both the *TSTS* and *Framework* reports is a focus on the interdisciplinary study of real-world problems that emphasize key STEM understandings and practices. An important component of this integration is quantitative reasoning (QR), defined here as the application of mathematics and statistics in the design and analysis of science investigations and in the development of models. While all 8 of the practices are relevant to engaging in modeling, of the *Framework*’s eight ‘Scientific and Engineering Practices’, practices 2 through 5 are viewed as most relevant for QR. More specifically, the position I am taking here is that in K-12 middle grades (e.g., 4th to 9th) the QR focus should be on the longitudinal development of measurement, data and data modeling, statistics and inference, and probability. Each of these 4 QR domains I will argue are important for attaining Strand 4 of the science proficiencies, i.e., Participate productively in scientific practices and discourse.

Another recommendation from Taking Science to School is that learning, assessment and curriculum designs be organized around learning progressions (LP) as a means of supporting learners’ development towards attaining the four proficiencies. My comments and recommendations for future work regarding embedding QR into STEM education will consider 1) how the 4 QR domains above can be coordinated for middle grades learning progressions and 2) the implications a QR focus in K-12 education has on the design of higher education disciplinary courses taken by preservice elementary and secondary teachers.
Quantitative Reasoning in the STEM Disciplines

Weaving together conceptual, epistemic and social learning goals so as to enable co-development of each is as outlined above needed to understand and participate in contemporary scientific and socio-scientific conversations. The complexity of this co-development can be seen in three geosciences reports. The National Academy of Sciences formed the ‘Committee on Grand Challenges in Environmental Sciences’ to address the question, What challenges exist for future generations of citizens? The outcome was the report Grand Challenges in Environmental Sciences (National Research Council, 2001a), which identified eight grand challenges for which there is a need for significant infusion of research over the next two decades. Challenges such as:

Biogeochemical cycles: understanding how human activity is perturbing the six nutrient cycles of carbon, oxygen, hydrogen, nitrogen, sulfur, and phosphorus which has impacts on climate change, CO₂ concentrations, acid rain, and chlorofluorocarbons (CFC).

Biological diversity and ecosystem functioning: understanding the regulation and functional consequences of biological diversity which has impacts on rates of species extinction, threats to biological diversity, and controls on biological diversity.

Hydrological forecasting: understanding and predicting changes in freshwater resources and the environment caused by floods, droughts, sedimentation, and contamination which threatens freshwater ecosystems.

In 2000 the Directorate for Geosciences at the National Science Foundation (NSF) reported on a long-range planning activity to evaluate opportunities and requirements for research, education and infrastructure. In the foreword of the report NSF Geoscience beyond 2000: Understanding and Predicting Earth’s Environment and Habitability, we learn that “the geosciences have enjoyed major advances in understanding the Earth systems and the complex interactions among the various elements: atmosphere, ocean, land surface and biosphere. These dramatic advances are now providing new and enhanced opportunities for geosciences, in combination with sister disciplines, to provide important services to the nation through predictions of potentially harmful or beneficial events” (NSF, 2000).

More recently, the NRC Board on Earth Sciences and Resources formed the ‘Committee on Strategic Directions for the Geographical Sciences in the Next Decade.’ The report – Understanding the Changing Planet: Strategic Directions for the Geographical Sciences (NRC, 2010) – which is organized around eleven Strategic Research Questions (e.g., How are we changing the physical environment of the Earth’s surface? How can we best preserve biological diversity and protect endangered ecosystems? How are climate and other environmental changes affecting the vulnerabilities of coupled Human-Environment systems?) also outlines a set of challenges, “Many of the central challenges of the 21st century are tied to changes to the spatial organizations and character of the landscapes and environments of Earth’s surface as populations move, natural resources are depleted, and climate shifts” (p1). The report also make very clear that “[t]echnological developments and changing research priorities have inspired the rapid growth of geographical sciences over the past two decades.” (p1) such that a broader set of researchers (e.g., economists, biologists, epidemiologists, geologists) now engage in the geographical sciences.

What these three reports confirm is that access to new data and new forms of data that geographical information systems (GIS) provide has contributed to new conceptualizations of
the mechanics of Earth systems. Changes in the, geosciences and geographical science indeed have been dramatic in the last 20 years. The reconceptualization of Earth science as Earth systems science is a very recent development. Early on the pursuit of geographical and geoscience knowledge was closely aligned with the applied problems of commerce and industry and with a keen interest in obtaining an accurate history of the earth. In the US, state and national geological surveys showed the way with geologists in the field getting up close and personal with rocks and structures (McPhee, 1998). Mining and mapping the resources and terrains of regions and establishing the places and path for trains, ships and trucks to reach the resources needed for an industrial and increasingly global society was the pragmatic goal that once dominated the geosciences. Such an orientation to the geosciences, I would argue, continues to dominate the curriculum materials of K-12 earth science programs and courses of study.

Advances in technology, especially computing power for simulations, visualizations, and sensors, and in scientific understanding coupled with a growing global sense of responsibility have shifted both the focus and the methods of the Earth sciences. With respect to focus, the trend for several decades has been toward a systems analysis of the Earth. With respect to methods, the trend is toward model-based science. Models are increasingly being used for exploring and explaining the complex dynamics and structures of the earth’s surface.

With respect to grand challenges for our education system, there are two agendas. First, the education system seeks to sustain a STEM workforce ‘pipeline’ of students who serve as the next generation of technicians, scientists, engineers, and mathematicians that will help, for example, research the grand challenges. But just as important, the education system also seeks to produce a scientifically literate citizenry that can make informed decisions about grand challenges. Economic, policy, ethical, and social issues will converge around the grand challenges forcing citizens to make decisions that will impact the future availability of resources. These two seemingly compatible agendas will require that STEM education for all students have deeper learning experiences facilitating the understanding and use of key scientific concepts and scientific practices to interpret, evaluate and participate in the conversations examining real world situations. Increasingly these conversations are embedded in QR contexts.

Robert Mayes and Bryan Shader (this volume) use cases from computational science and from biology and ecology to show how QR is central to the work of scientists, applied mathematicians, and statisticians as they model phenomenon and address grand challenges about environments and energy needs. Shader’s presentation on QR and Modeling in Computational Sciences – a Mathematician’s Perspective signals the arrival of two new scientific paradigms: (1) Scientific Computing or Computational Science; (2) Data-Intensive Science or Data-Centric Science. Some refer to this form of scientific inquiry as ‘Discovery Science’ inasmuch as the investigation does not begin with posing a focused question or a testable hypothesis. Rather, the method is to search large datasets to discover interesting patterns that then require explanations. Some argue that this science is a theoretical.

Thus, a concern that comes up from the Mayes and Shader chapter is how these two new “legs of science” can or even must coordinate with established legs of ‘empirical method and theory’. Russell Hulse (2007) in his NSF Colloquium ‘The Nature of Scientific Knowledge’ uses his personal experiences of discovering binary pulsars, which won he and his mentor Joe Taylor the 1993 Nobel Prize in Physics, to challenge the notion that good science is hypothesis-driven science. Hulse sees as a central educational problem that hypothesis testing as in the “The Scientific Method” is still being taught in the K-16 curriculum. He noted, the bedrock of science is the objective testing of ideas, this can easily lead to the misconception that all scientific research - or at least “good” research - is testing. Hulse contends that conducting good, valuable scientific research spans the range of “having a good look around” to rigorous hypothesis
testing. Scientific method for Hulse is better-understood as a “tool kit” from which one draws in relation to the problem situation of the research. Further, Hulse pointed to the importance of models - mental, physical, and computational - in conducting scientific research, and called computational models “repositories of knowledge in today’s science.”

Mayes and Shader (this volume) state that the new paradigm of scientific computing impacts our thinking about teaching in four distinct ways:

- **Profoundly** given the short timeframe with which scientific computing emerged;
- **Systemically** given the interdisciplinary bases the sciences and engineering fields and the symbiotic relationship mathematics has with science and engineering;
- **Vertically** given years-long developmental corridors for obtaining proficiency with scientific computing;
- **Wisely** given that the emphasis on programming and computer engineering needs to focus on the quality of experiences, not the quantity.

Jennifer Schuttlefield, Marjorie MacGregor, and John Moore (this volume) argue for the centrality of QR in science education through the consideration of socio-scientific issues and problems that are based on important environmental challenges. For them, the challenges include elements of biological diversity and ecosystem functioning, land-use dynamics, and climate variability and its effects on humans and natural systems. They build their case for QR around Grawe’s (2011) four facets:

1. Commanding mathematical skills through use of mathematical tools.
2. Application in context by applying mathematical tools to solve problems.
3. Communication through correlating mathematical transformations into data-based arguments.
4. Determining the limitations and strengths of the data presented.

They claim evidence from college teaching studies suggests that developing and using QR in science courses brings about gains in students’ scientific knowledge. Schuttlefield et al (this volume) identify three major themes in these studies – models, statistics, and scaling – and draw on the Mayes, Bonilla and Peterson (2012) framework that partitions QR into Quantitative literacy (QL), interpretation (QI), and modeling (QM) to “classify the math behind the science”. (See Table 1 in Schuttlefield, et al, this volume).

The seven vertical strands of scientific computing put forth by Mayes and Shader and the Schuttlefield et al three themes capture many of Hulse’s “having a good look around” intuitions about the nature of scientific knowledge; in particular, modeling, pattern recognition, and abstraction. The ‘vertical’ impact on K-12 teaching and the ‘themes’ for college science courses raises important questions about accessibility to and affordances for QR and mathematical modeling in science and engineering. Where in the curriculum and in the sequence of instruction? When to begin, in which course, topic or grade band? How to teach and assess?

Professor Hulse’s presentation of his Nobel Prize research serves as an example that there are alternatives to hypothesis driven methods of research. In particular, as his lecture title suggests, one alternative is “The Process of Scientific Discovery”. Scientific processes and practices differ, Hulse argues, according to difference within disciplines on the following dimensions, among others:
The extent to which science domains are mathematical or quantitative.

- The range of experimental or observational techniques, which are available.

- The possibility to have strictly controlled experiments.

- The maturity of knowledge within the discipline or field; e.g., refining knowledge or casting about to obtain knowledge.

- The discipline having first principles understanding of mechanisms or trying to get to first principles.

- The being a part of big science with teams of scientists or individual science carried out by one person.

Mayes and Shader (this volume) and Schuttlefield et al (this volume) both turn to ecosystems as exemplars for infusing QR. The complex systems involved represent viable contexts for situating QR development and learning. Hulse points to the molecular biological sciences and the high performance computing to deal with the wealth of data like that found in the Human Genome Project. Here one finds new techniques and new technologies contributing to the development of molecular level tools and understandings. The developments in the study of Earth systems, ecosystems and molecular biology are pushing biological sciences and geosciences to more quantitative inquiries and to methods of data analysis that are not hypothesis driven. Some critics call them fishing expeditions because there are no clear outcomes of what the conclusion will draw. And, this is precisely what Professor Hulse did in his Nobel Prize research on the discovery of binary pulsars; pointing the radio telescope to regions of the sky, collecting data, looking for anomalies in the patterns. No formal hypothesis in hand. An important question for both K-12 and post-secondary STEM education is how to establish the QR ‘verticality’ and the ‘themes’ so as to facilitate the development of coherent sequences of learning. A partial answer will reside in the design of teaching sequence, learning progressions and assessments that inform adaptive teaching models.

**Teaching Sequences, Learning Progressions and QR Challenges**

The commentary began by pointing out that the growth and development of knowledge has been a cornerstone to our current thinking about learning and teaching. The recommendation is that science/math learning be connected through longer sequences of instruction (e.g., immersion units; teaching sequences, LPs; LTs) that function vertically across grades/years and horizontally within a given school year. The rationale is to facilitate the learning of core knowledge and practices that are critical for development of science/math knowledge and reasoning. The research synthesis found in *Taking Science to School* (Duschl, et al, 2007) informs us that developing rich, conceptual knowledge takes time and requires instructional support employing sound assessment practices. The LPs approach to the design and alignment of curriculum, instruction and assessment is grounded in core knowledge theories of cognitive development and learning. The focus is for LPs to be built around the most generative and core ideas/practices that are central to the discipline and that support students’ learning.

The two chapters here have provided some useful insights on where to begin but a great deal more ‘having a good look around’ needs to take place. If we want QR in the education of our K-12 children, then we also need think about teaching teachers and teaching the teachers of teachers. What will be the LPs for QR and mathematical modeling for our elementary and secondary teachers? How will college level courses be ‘bundled’ to promote/establish a QR learning progression not just for STEM majors but non-STEM major
as well? Can QR learning goals have priority over or even equal status with conceptual learning goals? What science/engineering contexts and challenges will best afford QR development in K-12 middle grades?

LPs are generally viewed by researchers as conjectural or hypothetical model pathways of learning over periods of time that have been empirically validated. Simon (1995) writing about mathematics learning trajectories (LT) suggests that LPs include “the learning goal, the learning activities, and the thinking and learning which students might engage” (p. 133). The interest in LP research represents a shift in emphasis from partitioned teaching of independent units/modules of instruction which focus on what we know (e.g., facts and skills) to coordinated sequential teaching that focuses on developing scientific and mathematic knowledge with accompanying cognitive and metacognitive practices.

Why learning progressions? Taking Science To School (Duschl, et al, 2007) makes the recommendation that science learning needs to be strongly grounded in model-based approaches that focus on the use and consideration of evidence for posing, building and refining models. Within the model-based learning/teaching approaches data modeling, quantitative reasoning and conceptual understandings can, along with other science and engineering practices, develop. Science and engineering practices are one of the three NRC Framework Dimensions (See Figure 1) and are an important component wherein the critique and communication discourse practices reside. Like the Core Ideas and Crosscutting concepts, the Science and Engineering Practices require time to develop.

Thus, the recommendation in TSTS, in the NRC (2011) framework for science standards and in the NRC (2009) report Engineering in K-12 Education is that science learning be organized into longer sequences – learning progressions - that serve as vertical pathways of learning across grade levels and as teaching sequences horizontally within any instructional year. The rationale is that facilitating the learning of core knowledge and practices that are critical for development of scientific knowledge and scientific reasoning is complex, takes time, and requires instruction-assisted development grounded in sound assessment practices. Thus, the content of LPs includes the core knowledge, the epistemic practices (e.g., science talk and argumentation) and the social practices (e.g., critique, communication and representation) that characterize a domain of science and/or engineering. The recommendation from TSTS is that:

The core concepts used in this practice [learning progressions] would be dramatically fewer in number than those currently focused on or included in standards and curriculum documents…a grade-level teacher would need to be concerned not only with the relevant “slice” of a given core idea in her particular grade, but also with the longer continuum of learning that K-8 students experience. Thus, teachers and science teacher educators…would need to build structures and social processes to support the exchange of knowledge and information related to core concepts across grade levels. (Duschl et al, 2007, p. 61)

The LPs approach to the design and alignment of curriculum, instruction and assessment is grounded in domain-specific or core knowledge theories of cognitive development and learning as documented in recent National Research Council reports (NRC 1999; 2001b). Corcoran, Mosher, and Rogat (2009) convened several workshops of experts exploring LPs to look at two questions:

What promise might LPs have for improving instruction in schools?

What further might be required to make the promise real?
LPs are seen as empirically grounded and testable hypotheses about how students’ understandings of and abilities to use core ideas grow and become more sophisticated over time. In an early review of LPs, 4 features were found to characterize LPs (Corcoran et al., 2009):

1. Targeting core and generative disciplinary understandings and practices that merge science content with science practices;
2. Lower and upper boundaries that describe entry assumptions and exiting expectations for knowing and doing;
3. Descriptions of LPs that inform progress levels or steps of achievement; and
4. Purposeful curriculum and instruction that mediates targeted student outcomes.

The consideration of LPs represents a shift in emphasis away from teaching that focuses on what we know (e.g., facts and skills) to teaching that focuses on how did we come to know and develop scientific knowledge and on why we believe what we know over alternatives. Within the how and why approaches reside ‘assessment for learning’ opportunities which make thinking visible as students engage in talk and argumentation and in modeling and representation. The report by Corcoran et al. (2009) states “progressions can play a central role in supporting the needed shift toward adaptive instruction” (p. 9) and that the following are seen as possible learning outcome benefits of establishing LPs:

- Providing a basis for setting standards that are tighter and more clearly tied to instruction;
- Providing reference points for assessment to report on levels of progress and thereby facilitate teacher interventions and instruction-assisted development;
- Informing the design of curricula that are aligned with progressing students (e.g., assessments for learning).

The Consortium for Policy Research in Education (CPRE) report Learning Progressions in Science: An Evidence-based Approach to Reform (Corcoran, Mosher, & Rogat, 2009) identified learning progressions as a promising model that can advance effective adaptive instruction teaching techniques and thereby change the norms of practice in schools. The QR agenda is one that needs to study the theoretical, foundational assumptions of learning progressions for quantitative reasoning. The papers here by Mayes and Shader and by Schuttefeld et al hypothesize that QR is an essential component of all sciences. The goal for building on the study of scientific QR practices is to establish the theoretical bases that will inform the design of learning progressions for quantitative reasoning in science as model-building. A key component of this research and development project will be designing learning performance assessments that when implemented inform the learning progressions and the adaptive instruction strategies for teachers.

**Closing Commentary**

I close this commentary by presenting two examples for conceptualizing the organization of teaching sequences and learning progressions that emphasize QR and modeling. The first example considers Earth Systems Science and Complex Systems Thinking. The second example focuses on scientific practices. The recommendation for LPs represents a shift in emphasis from teaching that focuses on what we know (e.g., facts and skills) to teaching that focuses on how do we know and on why we believe what we know over competing scientific claims. This, in turn, leads to recommendations that science learning be connected through longer sequences of
instruction (e.g., immersion units; LPs) that function vertically across grade bands and horizontally within a grade level. The rationale is to facilitate the learning of core science knowledge and practices that are critical for development of scientific knowledge and reasoning. The position being advanced is the recognition that developing rich, conceptual knowledge takes time and requires instructional support and mediation via sound assessment practices.

**Earth Systems Science**
As part of a project for an NSF MSP involving 5 city school districts, partnerships between scientists and learning scientists were established and asked to outline the core ideas and core inquiry practices for biology, physics, chemistry and Earth science. I was asked to join with Bruce Herbert, Texas A&M University. We framed our Earth Science report in terms of 1) three fundamental challenges in the learning of Earth sciences and 2) core ideas linked to the study of biogeochemical cycles. The proposed framework serves as an example of how educational efforts need to be construed and decomposed to attend to the big ideas and science practices that are representative of Earth Systems Science (ESS).

**ESS Learning challenges.** The learning challenges can be seen as the cognitive frameworks within which Earth systems scientists do science. The first learning challenge is the ability to conceptualize natural Earth environments as having boundaries and mechanisms for transferring and manipulating matter and energy within and across Earth systems. The second learning challenge is describing and explaining the dynamic nature of Earth systems through a characterization of system states over space and time with a focus on both steady-state and non-equilibrium conditions. The third learning challenge is understanding complexity and the practices geoscientists use to study complexity. Thus, geosciences inquiry synthesizes across at least three principal sets of practices:

1. Investigations involving simulations,
2. Investigations involving characterization of the properties and dynamics of natural systems, and
3. Investigations involving laboratory experimentation where conditions can be controlled and causal relationships established.

Together the 3 sets of inquiry practices help develop quantitative and conceptual models of Earth systems. Hence, quantitative reasoning and model-based reasoning as conducted through the building and refining of models becomes central to understanding the essential principles in the geosciences literacy documents.

**Crosscutting Concepts.** the core concepts that are fundamental to reasoning about and understanding the actions in and on the biogeochemical cycles (e.g., water, carbon, nitrogen, rock) that occur in Earth systems include:

- **scale**: deep time and deep space,
- **energy**: gravitational, thermal, tidal and solar sources (measured by temperature/pressure conditions), and
- **matter transformations** – physical and chemical.

Thus, when considering QR and modeling in the Earth science education curricula learning goals need to address the development of increasing sophistication with knowing and using the tenets of deep space and time, of Earth systems science, and of boundaries within biogeochemical cycles. In addition, the emergence of new tools and technologies (e.g., GIS) for simulations, visualizations and modeling have significant implications for developing a framework to guide...
the design of immersion units that function across the K-16 science experience.

**Scientific Practices**

Lehrer and Schauble (2004, 2007) maintain that through thoughtful and reasoned implementation of instruction-assisted inquiry, the scientific practices of modeling and reasoning can support (a) sustained engagement with epistemic and social practices, and (b) the construction of mathematical representational forms that afford quantification and investigation of relations among quantities. Lehrer and Schauble (2006, 2002) see engagement in resemblance representation tasks as an entrée to modeling. They have demonstrated from a longitudinal study of teaching examples around data modeling and students’ engagements in data representation tasks that modeling data is a powerful basis for developing QR practices. The instruction-assisted-development teaching sequences have students using and learning from data modeling, bridging mathematics and science, engaging in inquiry studies and using emergent representational forms.

Lehrer and Schauble’s (2011) LP provides an example of how instructional design can be a coordinated investigation of student learning. Beginning with student investigations of ecosystems they situated instruction-assisted learning about variability, change and data modeling. The design of instruction was guided by several generative principles: posing and revising questions, comparing questions and investigations, participating in the design of investigations for answering questions, inventing measurements and representations, and frequent whole class participation in research meetings. Working from the instruction-assisted learning design, Lehrer and Schauble develop several conjectured learning trajectory levels based on descriptions and expressions of students’ learning performances. For example, understanding variability consisted of ‘difference described’ (level 1), ‘difference measured’ (level 2), ‘distribution making’ (level 3), ‘modeling the distribution’ (level 4), and ‘competition of models’ (level 5).

Looking across the LP research (Duschl et al., 2011) one finds that the instruction-assisted episodes are linked with benchmarks or steps of understanding. For Lehrer and Schauble, students’ level 1 understanding is obtained via posing and revising questions, comparing with other investigations, and participating in the development of ways to answer questions. Level 2 and 3 are achieved through the additional inclusion of inventing measurements and representations. Level 4 is achieved through participation in the design of investigations. Finally, level 5 is achieved through students’ participation in the research meetings.

In this LP, learning goals are not based on attainment of scientifically correct or canonical knowledge, but reasoning with and about data and information. Students move toward the higher levels using the knowledge and practices established at lower levels. Thus, Lehrer and Schauble adopt an instruction-assisted learning design that serves to mediate learners’ pathway of development of understanding. Duschl et al. (2011) regard LPs with such features as ‘Evolutionary LP’ because they deepen and broaden as iterative designs of instruction and instructional tasks inform the learning pathways.

The challenge then to QR researchers is one of outlining the learning pathways through the development and refinement of Evolutionary QR practices LPs. However, this is a two-fold problem – one problem plays out in K-12 classrooms and one plays out in post-secondary classrooms and programs that are preparing our K-12 teachers. QR and mathematical modeling need to become frequent conversations at both educational levels.
References


