NEW PERSPECTIVES AND DIRECTIONS
FOR COLLABORATIVE RESEARCH
IN MATHEMATICS EDUCATION

PAPERS FROM A PLANNING CONFERENCE
FOR WISDOM'

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WISDOM® is a developing virtual Institute based at the University of Wyoming, consisting of a global consortium of individuals committed to collaborative research and scholarship related to specific targeted domains of inquiry in Mathematics Education: quantitative reasoning, mathematical modeling, technology tools and applications, and lived/living mathematical experience. A series of thematic Institute monographs will be published to promote study and communication related to these focal domains. Persons wishing to explore participation as a WISDOM® Affiliated Scholar are encouraged to contact Dr. Hatfield for further information.

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PREFACE

Mathematics is empowering and beautiful. Yet how many of America’s students would describe mathematics using these two words? How many parents and teachers would do so? More than any other major subject, mathematics resolves our citizenry into two populations: a small set of accomplished learners and capable users and a much larger set for whom mathematics is opaque and even scary.

Bridging this gulf is a daunting challenge. Our society forgives mathematical illiteracy all too easily. We treat mathematical ability as an accident of birth and genuine fascination with mathematics as aberrant behavior. In this milieu, complacency is even worse than irresponsible; it is the most deeply undemocratic posture an educator can adopt. For this reason, we owe many thanks to the scholars and teachers who have gathered to form the Wyoming Institute for the Study and Development of Mathematical Education. They represent a new vanguard in America’s efforts to bolster the teaching and learning of mathematics. There is no more important work to be done.

Myron B. Allen
Provost and Professor of Mathematics, University of Wyoming
August 28, 2011

Almost a year ago, I had the pleasure of greeting a distinguished group of scholars and advanced doctoral students who share a common interest in identifying and advancing interdisciplinary theories and research that will ground improvements in mathematical education. Not since the launch of Sputnik in 1957 has there been such a vocal public and political outcry regarding the quality of education in this nation as we are hearing today. The research areas advanced in this monograph emphasize the importance of collaboration and sharing across teachers, administrators, and those of us in higher education. The silos of educational research and the “real world” of the educational practitioner must be permeated. If we are to see real improvements in the quality of teaching, we must define our research around the measurable characteristics of improved instructional practices and the documentation of increased student learning…then we must develop the relationships necessary to move our findings off the printed page into the classroom.

The papers in this monograph are clearly focused on this dual responsibility. Led by Dr. Larry Hatfield, our Wyoming Excellence Chair in Mathematics Education, WISDOM* (Wyoming Institute for the Study and Development of Mathematical Education) shares this first monograph dedicated to supporting improvements in the teaching and learning of mathematics. We hope you find this and future volumes worthy!

Kay A. Persichitte, Ph.D.
Dean, College of Education, University of Wyoming
OVERVIEW AND ACKNOWLEDGEMENTS

This is the first volume of an anticipated monograph series being published in support of the activities of participants in the Wyoming Institute for the Study and Development of Mathematical Education (WISDOM©). As the inaugural volume, it is fitting that it is a report of some of the activities in the first year for the virtual Institute. Thus, most of this report provides a documentation of the invitational planning conference conducted during September 8-10, 2010 at the Hilton Garden/University of Wyoming (UW) Conference Center, Laramie, Wyoming. In addition, we have included a report arising from one important post-conference activity involving several conference in support of doctoral education.

The planning conference was organized to include five plenary presentations, based upon commissioned papers prepared by senior scholars (for further details, see Appendix B Conference Schedule). Drafts of these papers were received and distributed to all invited participants in advance of the conference, to allow each of the plenary authors to focus their plenary presentations upon an elaborative discussion based upon their audience having read the paper. Following each presentation, a brief commentary was provided by an invited discussant; thereafter was a time for overall discussions and questions from the audience engaging the presenter. These five sessions began in the opening evening session, and were completed by early afternoon of the first full conference day.

As a working meeting, both externally invited and UW faculty and students were organized into three Research Teams, these being named---

- Quantitative Reasoning and Mathematical Modeling (QRaMM; led by Robert Mayes and Larry Hatfield)
- Technology Tools and Applications in Mathematics Education (TTAME; led by Linda Hutchison and Larry Hatfield)
- Developing Investigations of Mathematics Experience (DIME; led by Larry Hatfield and Michelle Chamberlin)

The Research Teams met simultaneously yet separately two times during the rest of the conference, to undertake the formative work of stimulating and building collaborative research. In these working sessions, participants worked to become acquainted with each person’s research interests and plans. Out of these conversations, many ideas and suggestions for collaborative work were discussed, and there were various pairs and sub-groups that began to forge specific ideas for what might be undertaken. After the second breakout team meetings, brief reports from each team were presented that summarized major efforts and prospects up to that point.

To foster these team efforts, each invited participant had prepared and distributed before the conference a brief essay discussion of their research experiences and interests, specifically addressing how their prior and anticipated research might connect with, and contribute to, the potential work of the team. These essays served to facilitate the conversations and the exchanges of ideas. Each of these nine papers is included as a “Participant Research Essay” in these proceedings in the section following the plenary paper and invited commentary for each of the three research focal domains.

Additionally, a special session to overview and explain the ideas for starting and building the virtual Institute was conducted; the opening essay in the proceedings documents these ideas. Finally, after the closing luncheon and the visitor departures, a final closing plenary session was conducted that involved only the UW participants to allow for a debriefing, summary, synthesis, and directions for next steps.
While many productive informal conversations and interactions also took place over shared meals and during other interim periods as well as in the open discussions following each plenary presentation, these proceedings are limited to our attempts to document at least some of the formal exchanges.

To demonstrate one post-conference productive activity that arose from the QRaMM team, we have chosen to include an additional essay. As noted in Hatfield’s essay about the WISDOM\textsuperscript{c} concepts, the research focal domains of the Research Teams are vitally anchored in our chosen doctoral programmatic emphases. Beyond four core Mathematics Education courses, we have designed our program to offer two joint courses to serve both Science and Mathematics Education doctoral students. The first of these joint courses was offered during Spring 2011 by Robert Mayes, within which fifteen weekly national seminars were held. These seminars were led by Institute QRaMM participants and other prominent educators concerned with quantitative reasoning and mathematical modeling in science and mathematics education. This essay provides a framework, and synopses from the seminars, all of which elaborates further potential collaborative work in this domain. [Note: Volume 2 from WISDOM\textsuperscript{c}, currently being developed, will be a more extensive discussion of issues, potentials, and research for investigating QRaMM and will feature papers prepared by many of these seminar presenters.]

All of the intentions and aspirations for this new collaboration of individuals would not be possible without the existence of professionals with expertise and commitment to research and development in mathematics education. These acts of professional commitment to the improvement of mathematical education are indicative of both the significance of our shared need to study and understand what is involved in the complex phenomena of our field, and of our shared struggle to improve the mathematical lives of students and teachers. We are profoundly grateful and respectful of the deep well of professionalism in the field of Mathematics Education.

We must express more specific gratitude to the many persons who actually made the planning conference succeed. These include our five plenary presenters, Professors John Olive, Wolff-Michael Roth, Bryan Shader, Les Steffe, and Pat Thompson, whose expertise and vision significantly helped all of us to shape this formative beginning and to be greatly stimulated by the potentials of collaborative research. Also, we thank the invited participants, both visitors and UW faculty and students, who made the interactions even more rich and valuable by their passion and commitment. [A listing of conference participants appears as Appendix A.] The conference would not have been possible, without the support and encouragement of University of Wyoming Provost Myron Allen and College of Education Dean Kay Perschkitte, both of whom attended the opening session to welcome and challenge all conferees in relation to the significance of building a research basis for a sound mathematical education for all students.

We also recognize the contributions of our UW colleagues and doctoral students who have reviewed and provided helpful feedback on the manuscripts. We also wish to thank the authors for their participation in revising their papers following the conference. A special note of thanks and recognition for Ms. Tana Stith, whose patience and insightful work to create the cover design (while we playfully found ways to incorporate representations of Euler’s amazing number, “e”!) were indeed amazing. Lastly, funding for the conference and the publication of this monograph came from the substantial annual fund provided by the generous, forward-thinking people of Wyoming to Larry Hatfield as the Wyoming Excellence Endowed Chair for Mathematics Education.

In conclusion, we offer a few clarifying comments about our vision for a WISDOM\textsuperscript{c} monograph series (see Hatfield’s essay for additional details). A basic purpose will be to contribute to the literature of Mathematics Education with an emphasis upon the particular problem domains
of quantitative reasoning, mathematical modeling, technology tools and applications, and lived/ living mathematical experience. The monographs will provide a context for the publication of Institute-related thematic manuscripts for which there may be no readily available outlet, where-in we hope to expand for our field the types of publication and dissemination opportunities. We hope to encourage the conceptualization and development of purposeful monographs, and will soon announce procedures for submitting proposals for such efforts. We are in the process of establishing a panel of Contributing Editors who will be constituted as external reviewers for subsequent monographs. An important aspect is that authors will retain their copyright, allowing them to decide later to submit their manuscript or a subsequent version for publication in other outlets without restraint. At this point, we will distribute the printed monographs without any cost, which will necessarily limit the extent of distribution. As we make operational the Institute website, we intend to offer electronic downloadable versions of the monographs. Overall, we intend that the monograph series will extend and strengthen the collaborative activities of the WISDOM Research Teams.

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