Perspectives and Collaborations for Quantitative Reasoning in Mathematics Education and Science Education

Larry L. Hatfield
University of Wyoming

Overview

As the technological revolution in the 21st century continues to transform all societies, our capacities to share and access an increasingly *quantitative conversation and approach* continues to transform the ways by which we citizens of the world can function in an ever-widening variety of contexts. An informed citizen now has direct access to a powerful array of informational, computational, communicational, and decision-making tools and methods for addressing the problematic situations that confront their needs, choices, and participations wherein important individual, familial, societal, and global humankind decisions must be made.

For more than a century, one of the most important (and still largely unfulfilled) aims and purposes of a sound mathematical or science education for citizenship has implicitly (but more rarely explicitly) involved an increasing functional quantitative literacy. Often shrouded or hidden within other popularized philosophical or pedagogical rhetoric, yet always present within an amazingly consistent array of curricular "topics" mostly perceived and treated in relative isolation, there still can be found a pervasive fundamental focus on educating in relation to performing with quantities. This is so because the scientific and mathematical disciplines of formal inquiry mirrored in our educational practices are being driven both by needs of intellectual understanding of ever-more penetrating sets of quantitative problems, as well as increasingly sophisticated needs from challenging quantitative applied problems that require ever more powerful and expansive forms of quantitative reasoning (QR). Indeed, quantitative problems become ever more critical to our wellbeing and very survival. In this, the demands for appropriate forms of such reasoning (well beyond a basic or simple quantitative literacy) by informed citizenship seem also to be ever increasing, perhaps even with an exponential growth.

We appear now to be in a period in which scholar-leaders from both Mathematics Education and Science Education are developing an explicitly stronger attention to quantitative aspects of curricula, teaching, learning, and assessment. Again, this mirrors the contemporary scientific disciplines and transformative societal contexts that fuel the basis for our educational perspectives. The Quantitative Reasoning Symposium held in Savannah, Georgia in Summer 2012 was formulated upon a recognition and appreciation of this backdrop and these potentials for stimulating and impacting upon our educational approaches in ways that will address these global, societal transformations.

We chose to conceptualize this Symposium to bring together some of the most active scholars addressing the educational potentials of an enhanced attention to, and greater focused study of, quantitative thinking to deepen our understandings to improve science and mathematics education for all students and teachers. In this Symposium, we sought to gain from the cross-fertilization of ideas for addressing this focal domain of research and application to educational practices. Yet, we also must honor the distinct cultures and practices that frame the ways that thinkers from mathematics, the sciences, mathematics education, and science education bring to this focus upon quantitative reasoning. Even so, it is a fundamental goal of WISDOMe, a co-sponsor of the conference, to foster collaborative work among typically isolated scholars to amplify what we can understand and undertake in relation to "QR," by gaining from each other.

Some Framing Perspectives from Personal Experiences

What are some of the perspectives that we might bring to this focus upon quantitative reasoning within the STEM disciplines? In this essay, I will not attempt to review or analyze what may be found in our literatures. Indeed, in the presentations at the Symposium and in the essays of this monograph, a considerable variety of perspectives have been reviewed and proposed. Rather, I suggest my idiosyncratic perspectives, derived from my more than five decades of personal experiences working in both mathematics education and science education in a wide variety of contexts.

Early Teaching of Quantities

These include my early time (late Fifties-early Sixties), studying astrophysics and mathematics while working as an undergraduate research assistant in astronomy, where I was guided to assist in measurement studies of proper motions of our galactic stars and in accidental discoveries of stars classified as white dwarfs. It was then that I also experienced my first efforts to teach astronomical concepts to others, which revealed many different ways in which students struggled to think clearly in contexts that involved spatial motions or positional quantities that varied and co-varied.

Subsequently, my long career in mathematics education began first as a classroom teacher, yet framed by team and personal research on very early (mid-Sixties) uses of computers by young (grade seven) students. Our team approach engaged "student programming" as we guided them to construct, test, correct and refine their mathematical algorithms with purposes of exploring mathematics by modeling key curricular concepts, investigating powerful mathematical questions, and solving non-routine problems---all of which involved many variations of quantities and forms of quantitative reasoning. From this first decade of experiences as a mathematics educator, I began to recognize the powerful benefits from students engaging in quantitative reasoning. But I also saw quite a vast array of thinking challenges and difficulties my students appeared to experience in such situations. Many important aspects of these difficulties involved a constellation of complex, and seemingly subtle, components of conceptual thinking with quantities, and their acting upon and with those quantities to understand ideas, to solve problematic situations, and to become mathematically generative.

Quantities in Some of My Research

Later, as I focused on investigations of mathematical problem solving, the central role of interpreting problematic situations in terms of the implied quantities arose as an important issue. Indeed, the solver's capacity to interpret, model, and mentally coordinate and manipulate those quantities in relation to the posed conditions was identified as one of the most important constraints, or predictors, for producing successful solutions, or even making progress with the problem.

In a study of grade two students engaged in constructing arithmetic as "child-generated algorithms," I again witnessed how their numerical reasoning, supported by represented (and re-presented) numerical quantities upon which they could constructively act, appeared to be a primary factor in the quality of what they were able to produce as "thinking strategies" for finding solutions with their own invented computational methods. Here again, I emphasize that early meaningful thinking with numbers necessarily involves constructs that seem to require young minds to model (re-present) particular numbers as intentional quantities of "things." It is upon such constructions that they can concretely act in ways that engage conceptual mechanisms and operations (e.g., various counting strategies, or restructuring numerical quantities through decomposing and re-composing mental actions which may involve using multiple numerical units). From such quantitative reasoning they may be able to discern patterns of modeling and acting that can lead to repeatable systematic thinking strategies that can properly be called "the child's algorithm" as a viable, but often non-standard, method.

Quantities in Professional Development Strategies

Even later, as I worked across many decades with practicing elementary and middle school teachers to find and model teaching approaches to foster the construction of clear conceptual meanings by their students, the pervasive roles of quantitative representations, and acts of quantification and mathematical modeling to foster re-presentations that stimulated and supported student reasoning, was made even clearer. For example, my guidance of students to build-up "sense making" for fractions as numbers, and for constructing fractional operations, was always supported by student modeling of fractional numerical quantities (e.g., lengths, areas, volumes) upon which they could think and act as they engaged in exploring and discovering rational-number-defining behaviors and properties.

Today, as I ponder what appears to be a widespread lack of student constructions and reconstructions of deeply owned conceptual meanings across a wide variety of mathematical concepts and situations, I am led to identify a pervasive continuing lack of appropriate and sustained experiences in thinking and reasoning with quantities, models and their mental actions in ways that could stimulate and guide development of such deep conceptual meanings and reasoning with quantities. It appears that in today's standards-based curricula and teaching practices, driven by seemingly inappropriate assessment demands, too many students are not getting sufficient opportunities to experience critically important thinking and reasoning that might engender such deeply owned conceptual meanings involving fundamental quantities and quantitative reasoning.

Some Experiential Aspects to Promote Thinking with Quantities

What, then, might be the nature of experiences to engender powerful forms of thinking and reasoning with quantities? Perhaps this question may be one of the starting points for programs of research aimed at investigating the development of student constructions for quantitative reasoning. It is a question from which readers might consider the visions and perspectives presented in each of the essays of this monograph, as well as those also offered in preceding WISDOM[e] monographs (Thompson, 2011; Mayes, Hatfield, & Belbase, 2012; Hatfield, 2012). Of particular note are the illuminating comments provided by Thompson (2012) as he discusses the ways the six preceding authors addressed acts of quantification, while he states his own further clarifications of what is conceptually required in quantifying, reinforcing his central tenet that "a quantity is in a mind, it is not in the world" (Thompson, 2011, p. 33).

To stimulate further discussion of this question, in what follows I briefly offer some possible experiential aspects I consider important, as a teacher and a teacher-researcher, to promote student engagement and thinking with quantities. First, mirroring the tenet quoted above, to be engendering of potentially powerful forms of thinking and reasoning with quantities, quality "real-world" experiences must evoke in the person's interiorized experiences "sense-making" dispositions and mental efforts. As a teacher, I have no control over such thoughts, but I subscribe to the view that I can stimulate and guide the mind-of-the-other to recognize and adopt modes of responding through engagement that expects and fosters such thinking. In my "teaching as art" (teacher-artist), I do this within questioning interactions that seek "sense-making" efforts and explanations, and when possible provokes analytical thinking aimed at engaging the mind-of-the-other to be reflective about the experiential stream of activity.

Secondly, as a teacher-artist I typically begin an episode by posing a problematic context (e.g., see my narrative of the "view tube" episode; Hatfield, 2012) chosen because my own understandings of the situation include an awareness that the kinds of quantities and potential quantitative thinking I seek to stimulate may be possible (a first-order self-analysis of my own knowing). That is, I engage in anticipatory visions of "zones of potential construction" (Steffe & Olive, 2010, p. 17) as stimulated potentialities of student experiences. In this, I am clearly engaging in psychological projection, and I conceptualize my intended teaching actions and their potential experiential effects within such projections.

A key aspect of possible quantitative reasoning experiences can rest upon the quality of *problematic provocation*---resulting in the student seeking to interpret the situation in relation to measureable attributes, then to employ her measurement schemes. As a teacher, I believe engendering many such experiences across time and contexts can serve to instill the critical perceptual predisposition by the student to interpret and approach situations by looking for quantifiable elements of the problem as a way of building or deepening one's understanding through analysis.

Some further clarifications for a problematic approach are important. Traditionally, mathematics instruction has primarily placed contextualized problems and applications *after* the teaching of new concepts, principles and generalizations, and procedures---seen as "uses" of what has just been taught. This can also be found in the curricular organization of science education. As a further experiential provocation for promoting QR, I intentionally employ a problematic approach when guiding students to develop new concepts, to discover new generalizations, and to allow students to explore and search for new methods and procedures (forms of mathematical practices). Within student searching, a deep engagement of thinking about, and with, quantities can occur, and it instills the important disposition to approach subject matter content with an exploratory attitude. Such a problematic approach can be the daily lesson lens through which essentially all of the content can be approached.

Additionally, I often pose a complex problematic situation as the "advanced organizer" at the start of a new curricular unit. While the students are not able to "solve" that problem, by discussing it I am able to foreshadow important ideas to be studied that will enable them to produce results for the situation. In this way, new concepts they will be experiencing are motivated as important to a problem context, and thereafter we may "return" to the posed situation to make connections to the ideas from a lesson. In building up a solution for the posed problem we may discover it involves something new (e.g., the hyperbola produced by the data in the "view tube" episode was a new curvilinear graph for the students; Hatfield, 2012)---such can be the nature of unexpected original discoveries in mathematics and sciences!

Measuring and measures can lead to experiences in analysis of the measures as data, aimed in part to experience the "behavior" of the values, to try to see forms of variation or perhaps co-variation between or among measurable constructs. Of course, constructed graphs of measured data can be helpful, and developmental experiences can foster an awareness and eventual predisposition to make and use such graphs. Direct comparisons, such as extending a table of data pairs with computed ratios, may also provide insights into numerical relationships, so experiences in exploring ratios can develop orientations to use those in new situations. Numerical analyses to grasp such variation can be a more concretely based experience to anchor more abstract formulations, such as statistically derived values or graphs or algebraic formulations.

Finally for me, attention in the flow of experiences to be sensitive to the student's apparent stage or level of progressive abstractions attends to readiness factors; experiences outside (beyond) the student's "zone of potential construction" can produce a variety of counter-productive or negative results. In the artful approaches of quality teaching for QR we would surely pay attention to such factors, while maintaining the fundamental intentionality for progressive building-up of the holistic schemas that may come to constitute the thinking and knowing to reason with quantities.

There are many other potential responses to the question posed first in this section, and for framing and articulating both research and teaching practices I suggest that our conversations for characterizing student experiences that can progressively lead developmentally to quantitative reasoning be continued. Moreover, I conclude this brief discussion by emphasizing the fundamental point---it is within the lived and living experiences of the student that our visions and goals for powerful forms of educating for, and with, "thinking with quantities" must occur. While curricular frameworks, or pedagogical advocacies, or theories of knowledge and knowing, or assessment plans and results may all seek to shape new purposes or expectations related to QR, these will only materialize to become operational if our students directly experience, and thereby enact their constructive mechanisms, in ways that deeply affect the inherently powerful qualities of such thinking and knowing.

Possible Collaborative Roles in Educational Practices and Research

Though there are many important distinctions that characterize the ways of promoting learning and student thinking to be found in Science Education and Mathematics Education, on many critically important aspects there has been, and is today, remarkable concurrences of values, beliefs, goals, and approaches in the advocacies for educational practices. Both fields advocate for the development of students as thinking, reasoning, discerning minds; to become able to explain concepts, phenomenal situations, and problematic solutions with understanding; to become disposed to approach content in ways that expect and proceed toward "sense making," all done in ways that promote knowing through direct engagements and actions that value exploratory "processes" or disciplinary investigative practices as core approaches and outcomes and that seek to stimulate and guide the student to become productive and even generative when dealing with new contexts or problems.

Yet, there are also important differences, and it may be in those that we may find the critically important new ways that our treatments of quantities to build-up student knowing might be differentiated, yet mutually collaborative. About these I offer some speculations and questions.

One assumption we might make is that coming to engage in sound quantitative reasoning by most students will require prolonged developmental experiences across time, various content, and many contexts. Given the reality that mathematics is now considered to be a required subject for study and learning across PreK-13, while science education is still given much less consistent attention or curricular position (and is often seriously diminished or largely ignored at elementary school levels), it would be sensible to place much of the fundamental conceptual burden for addressing and developing at least formative quantitative thinking upon mathematical education.

Perhaps this presumption can be supported further in the nature of the ways that mathematics educators, compared to science educators, appear to approach the challenges. When analyzing or rationalizing the investigation of quantitative reasoning, mathematics educators appear to emphasize a more psychological or developmental perspective, while science educators appear to emphasize a more applied curricular or pedagogical approach? Is there within mathematics education more focus on underlying psychological mechanisms, compared to more need to structure carefully (as a learning trajectory or progression) the intended experiences within particular contexts aimed at fostering the thinking that will be used in those rich situations? Perhaps this role contrast can be supported further by what may be contrasting views of epistemology, emphases in research questions, and dominant research methods.

Yet, as a consequence of coincident shifts toward increased attention to quantities, quantifying, interpreting quantification, and quantitative reasoning, we might come to recognize this as an important context for interdisciplinary collaborations. Can we begin to think of revisions toward a more integrated mathematics curriculum that would give an enhanced attention to conceptual development for QR at all levels, while incorporating a stronger, perhaps pervasive, use of science-driven contexts? Contexts where students could simultaneously experience uses of their mathematical QR knowing while experiencing the ways that science concepts, models, and QR are used?

In formulating such re-visions, we must leave our "academic silos" so we can collaborate on basic issues and understandings. If there are important differences in the very ways we think about "quantities," or how we think about the nature of thought that understands and uses quantities, or what aims and purposes we have for emphasizing QR, then we must continue to develop venues for continued interactions toward greater resolutions, or at least understandings, of such differences. While conferences can be viable events for promoting such interactions, I would submit that sustained and detailed interactions are needed. Within WISDOM^e we have a vision for collaborative research teams for certain targeted themes, and QRaMM is one we have chosen. Perhaps we need to broaden our intentions for Mathematics Education to be more inclusive of Science Education scholars. To me, it can be within the intense efforts of conceptualizing, planning, conducting, interpreting, and reporting research that we could engage in the kinds of sustained interactions to help us clarify the many fundamental issues and differing understandings. Such sustained collaborations could also lead to increasing the potentials of experimental forms of more integrated curricula.

References

Hatfield, L. (2012). Some pedagogical and psychological aspects of mathematical modeling in school mathematics. In Mayes, R., Hatfield, L. & Belbase, S. (Eds.) *Quantitative reasoning and mathematical modeling: A driver for STEM integrated education and teaching in context* (pp.149-167). Laramie WY: University of Wyoming.

Mayes, R., Hatfield, L. & Belbase, S. (Eds.) (2012). *Quantitative reasoning and mathematical modeling: A driver for STEM integrated education and teaching in context.* Laramie WY: University of Wyoming.

Steffe, L. & Olive, J. (2010). *Children's fractional knowledge*. New York: Springer.

Thompson, P. (2011). Quantitative reasoning and mathematical modeling. In Chamberlin, S., Hatfield, L. & Belbase, S. (Eds.) *New perspectives and directions for collaborative research in mathematics education: Papers from a planning conference for WISDOM^e* (pp. 33-57). Laramie WY: University of Wyoming.

Thompson, P. (2012). Advances in research on quantitative reasoning. In Mayes, R., Hatfield, L. & Belbase, S. (Eds.) *Quantitative reasoning and mathematical modeling: A driver for STEM integrated education and teaching in context* (pp.143-8). Laramie WY: University of Wyoming.