INVITED COMMENTARY FOR OLIVE’S PAPER AND PRESENTATION

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I would first like to thank John for his paper, which provides an excellent framing for the current needs in research related to the use of digital technologies. In my response, I will first make comments on the theoretical framework section of John’s paper, pointing to aspects of the technology-based mathematical experience that I see as needing theorizing of a slightly different kind than offered either by the instrumental theory or the theory of semiotic mediation. I will then comment on John’s Example Implementations section to draw specific attention to dynamic representation technologies and their links to both theories of embodied cognition and to mathematics.

Comments on Theoretical Frameworks
After outlining the context of current technology use, the central questions for research, and the sense of urgency they carry, Olive discusses the two major theoretical frameworks currently being used to study these questions. Both the instrumental approach and the theory of semiotic mediation take the role of the tool/artifact/instrument in mathematical activity as central. They both offer frameworks for thinking about the use of digital technologies that render the initial question driving research—Is the use of digital technology X better than the use of pencil-and-paper?—seem naive and irrelevant. As Artigue (2002) notes, such a question assumes that what is needed are “pedagogical instruments for the learning of mathematical knowledge and values which were defined in the past, mostly before these tools existed” (p. 246). Instead, the frameworks Olive describes take as a fundamental assumption that the use of digital tools changes the very conceptions that students will develop—these tools are not mere “optional avatars of independently existing mathematical ideas” (Sfard & McCain, 2002, p. 155). Papert (1980), though not seeped in the same theoretical traditions as those described by Olive, knew this well, asking not “Can Logo help kids learn mathematics better?” but, rather, “What mathematics is there for kids to learn with Logo?”

At the risk of sounding nostalgic, I’d like to draw attention back to Papert’s work because I think he was concerned about an aspect of learning with digital technologies that seems to me missing from these frameworks, and that is the emotional dimension of human experience. Those of us who have worked with expressive technologies such as Logo are familiar with the sense of pride, accomplishment, joy, curiosity, excitement, autonomy, motivation and pleasure that learners have shown. These dimensions of experience may seem epiphenomenal to the learning of mathematics but they are central to our educational goals and values. The fact that so many papers on the use of technology in mathematics education are compelled to close, sometimes sheepishly, with a comment on how excited and engaged the students and teachers were, but insist on including it as an add-on to the real study being reported, strikes me as very problematic—particularly in the context of Olive’s comments on the alarming rate of high school drop-outs and the growing disconnect between in- and out-of-school learning.

But what is it about the use of expressive technologies (here I want to distinguish certain digital tools that promote student mathematical creation and construction from those that require more passive interactions) that encourages such aesthetic experiences, in the sense of
Dewey? For Papert (1980), they upheld the constructionist view that students need and want to make things and that their ability to do so in Logo was mediated by the close feedback loop that led to tinkering and debugging as well as the simple yet powerful means by which the turtle geometry language could be used to create ideas. For Jackiw (2006), the designer of The Geometer’s Sketchpad, the particular pleasure of using his dynamic geometry software can be better understood in the historical and cultural context of mechanical devices (rods and pulleys, pistons and axels), and relates to the constant interplay of—and assertion of the ego in—making (constructing) and performing (dragging, animating) mathematics. Sketchpad’s particular mechanisms offer a thrilling alliance between the visceral, temporal human with the immutable, detached mathematics: “the individual ‘touches’ raw mathematical ideas, where personal volition and psychical exertion can have seismic impact on disembodied abstractions (p. 155).

Whatever their origins, the emphasis that both Papert and Jackiw place on the de-segregation of touch and feeling is consistent with Vygotsky’s (1934/1986) insistence on the major weakness of traditional psychology: “since it [made] the thought process appear as an autonomous flow of “thoughts thinking themselves,” segregated from the fullness of life, from the personal need and interests, the inclinations and impulses of the thinker” (p. 10). According to Roth and Lee (2007), it is precisely this “neglected” aspect of Vygotsky’s work (in favor of semiotic mediation) that forms the basis of the Cultural Historical Activity Theory (CHAT) developed by his student Leont’ev. CHAT construes learning as occurring “during the expansion of the subject’s action possibilities in the pursuit of meaningful objects in activity” (p. 199). While CHAT contains elements of the theoretical framework of semiotic mediation described in Olive’s paper, its unit of analysis is not the individual learner, but meaningful activity, with an emphasis on the cultural-historical context of that activity and on the motives of activity. In the context of using expressive technologies, CHAT would interpret aesthetic experiences to the degree to which a learner has expanding control and action possibilities. While Roth and Lee (2007) offer examples in which the learners find motivation through social good (learning about environmentalism), educators such as Papert (1980), diSessa (2000), Shaffer (1997) and myself (Sinclair, 2001) have described the aesthetic goals that can motivate students’ activities with digital technologies.

CHAT may not turn out to be the most fruitful framework, but it seems to be the only one currently being used (though in a very limited way) that offers a central role for affect in the student technology experience.

Examples: Three of the Four Examples Involved Dynamic Representations
I found it interesting to note that all three of John’s examples (of current technologies) involve dynamic mathematical representations. While the Dynamic Number Project has yet to yield systematic data, we now have abundant research on the use of DGEs (dynamic geometry environments) as well as SimCalc indicating that something about dynamic representations is extremely powerful for learners. I don’t think that we have sufficiently explored the reasons behind this, though we can hypothesize that dynamism occupies a unique position in being central to both mathematics and to learning.

In terms of the mathematics, we have ample evidence to suggest that the great mathematicians of history have always thought dynamically. Archimedes provides the quintessential example of this in his solution of the parabolic section, which he conceives of in terms of a balancing of areas. Moreover, the central constructs of mathematics have emerged from the mathematizing of dynamic phenomena: take, for example, the whole of calculus, often described as the mathematics of change, or, more specifically, the concept of eigenvector emerging from the pendulum. More closely related to the Dynamic Number Project that John describes, it is worth recalling Simon Stevin, whose own dynamic conception of number (and creation of the
geometric numberline model of the reals) made possible and compelling the idea of 0 as a number, not to mention the integers and the rationals (Klein, 1968). That these concepts eventually become communicated in a formal language that insists on detemporalization and reification does not mean that they shed their dynamic origins, particularly in mathematician’s personal conceptions of them (see Sinclair & Gol Tabaghi, 2010). Thinking dynamically in mathematics requires an ability to imagine sequences of images changing over time, something that the digital tools John describes offers so readily to learners.

In terms of learning, the various theories of embodied cognition that have emerged in the past two decades have drawn attention to the way in which our lived experiences in the world, our sensorimotor interactions with the environment, make learning possible. Temporality and motion are central aspects of this lived experience. Indeed, the cognitive psychologist Seitz has offered the following telling aphorism ‘I think therefore I move.’ Similarly, the cognitive linguist Talmy (1996) has argued that even in the absence of it, people impose motion on their environment, talking about things as if they were changing, moving, transforming. From a more philosophical point of view, Châtelet (1993) has articulated the way in which the diagram, which “capture gestures mid-flight” (p. 10), transduces the mobility of the body. For Châtelet, the diagram does not simply illustrate or translate an already available content; it come from the gesture, and it carries the meaning of the gesture—the motion, the visceral, the corporeal. Again, in the context of the Dynamic Number Project, it is interesting to note Châtelet’s criticism of the static conceptions of number and operations mediated by the elementary school curriculum.

While dynamic representations are widely used and studied in mathematics education, research often focuses on the secondary effects of dynamism, such as how DGEs promote conjecturing or motivate proof. There is much less attention given to how learners perceive dynamic representations, and how they use these in developing concepts. For example, when a learner drags a triangle on the screen, is she seeing many different examples of three-sided polygons or is she seeing one figure morphing in time? The difference might be crucial to her way of thinking and talking about the concept ‘triangle.’ In their longitudinal study of children’s (aged 6-8) thinking about geometry, Lehrer et al. (1998) found that 40% of the students mentioned “pulling on the corners” to morph the one figure into a second one in order to reason about whether the two were the same. However, they also note that this thinking strategy is not well-supported or developed through the static representations children experience at school. But if such representations were available, how might students strategies develop, and be used in context other than naming and classifying shapes? This is a developmental question in great need to study.

And here’s another question: the three dynamic representations that John describes are visual in nature, but can dynamism extend beyond vision? Jackiw and Sinclair (2009) have suggested ways in which sound can also be used as a dynamic representation of mathematics, even though its historical roots are much less developed—examples include using rhythm to learn about multiples and ratios or hearing fractions and functions. In the context of working with blind students, Lulu Healy has adapted the color calculator into a sound one, using it to help learners make sense of rational numbers. In addition to sound, we are now also seeing examples of haptic environments (for example, in a project led by Stephen Hegedus) in which touch can be used as the dynamic mediator. A predecessor to this work was carried out at the Centre for Experimental and Constructive Mathematics at Simon Fraser University, where mathematicians could enter “the cave” where they could touch—and move around in—3-dimensional mathematical objects such as knots and surfaces. In thinking about the new media for learning, it seems worth considering how the dynamic representations that have been so successful thus far might be further extended to new modalities and forms of interaction.

Another question that deserves attention, in the context of dynamic representations, relates
to the changing discourse that these representations might occasion and, in particular, the way in which teachers might navigate these changes. Olive cites the Panel of Educational Technology as saying that “The more ambitious and promising pedagogic applications of computers call for considerably more skill from the teacher, who must select appropriate software, effectively integrate technology into the curriculum, and devise ways of assessing student work based on potentially complex individual and group projects.” But this is not enough. Consider Dave, a mathematically sophisticated, skillful user of Sketchpad, experienced integrator of technology in the classroom. After designing a lesson for his high school students on the topic of integration, he realized that he had never thought of Reimann sums as dynamic before, and further realized that he didn’t know how to talk about them with students, and even further realized that he no longer was certain what were the most important ideas for him to emphasize for his students. The case isn’t always so extreme, as evidenced by the number of users of DGEs in the world, but teachers are in a somewhat delicate role of mediating between two very different mathematical discourses, one well-established and explicit in schools and textbooks and the other usually more implicit or unknown. Sinclair and Yurita (2008) provide an example of these discourses in a grade 10 geometry course where the teacher has great difficulty in mediating between the traditional mathematical discourse of shape identification with the emerging discourse of dynamic geometry.

The question of what a new mathematical discourse might look like, in the context of new digital technologies, is an open one. Consonant with CHAT, the philosopher of mathematics Rotman (2008) sees mathematics as having been “engaged in a two-way co-evolutionary traffic with machines since its inception” (p. 58), citing, for example, the way in which cyclicity and angular motion are concepts abstracted from the wheel-and-axle, concepts that are then used to create new machines from which new mathematical concepts are abstracted. Rotman also sees the advent of digital technologies as leading to a radically new co-evolutionary dynamic. He acknowledges the way in which these technologies will lead to new concepts, reconfiguring the mathematical terrain “not only by affecting changes in content and method over the wide mathematical terrain, but more importantly, altering the practice and perhaps the very conception we have of mathematics” (p. 59). In thinking about research in the use of technology in mathematics education, I agree with John that it is very important to consider out-of-school learning contexts that effectively involve the use of technology, but it is also very important to continuously re-evaluate the educational opportunities available within the changing mathematical discourse.

References


