Conceptualizing Size and Scale

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> ***"A quarter and a blood cell are the same size because I don't know the size of a blood cell but I have seen pictures in my book of cells and they were a little bit bigger than a quarter."* (Middle School Student)**

This quotation from a middle school student exemplifies the challenges educators have in applying quantitative reasoning to science concepts outside of the human scale that can be directly experienced. New tools and techniques have enabled significant advancements in science and engineering fields from nano to galactic scales. Discoveries at these extremes of scale are increasing at dramatic rates and educators are scrambling to find effective ways to educate the next generation of scientists as well as citizens. Advances in science are accompanied by the need for citizenry that can participate in setting policy and making decisions about funding for research that is tied to scales outside the human experience.

As we struggle with teaching students to use quantitative reasoning, educators are also challenged with how to make use of new forms of instructional technology that allow students to zoom in or out as they visit virtual worlds that model science which exists at scales beyond our human perception. New forms of educational technology allow teachers to provide students with learning experiences at the very large and very small scales.

In the sections that follow, I describe a series of studies that examined concepts and benchmarks of size and scale for people at a variety of ages and levels of expertise. The results of these studies, along with those of others from the fields of mathematics education, geoscience education, and cognitive science provide evidence of the components of quantitative reasoning needed to conceptualize size and scale.

Background

Learning scale begins with learning about quantities and numbers. For young children this translates into learning about and integrating counting, sizes, and amounts, i.e., developing a stable ordering of numbers, mapping numerical quantities to physical representations (Joram, Subrahmanyam, Gelman, 1998; Siegler & Booth, 2004). There is evidence from Piagetian-based research that there are underlying developmental components to understanding objects in space and sizes of objects - part to whole (Piaget & Inhelder, 1971). Understanding measurement and size is also built on students' fundamental understandings of partitioning as well as conservation of length (Piaget & Inhelder, 1971; Piaget, Inhelder, & Szeminska, 1960). Lehrer (2003) suggested that an understanding of measurement and units is built on students' concepts of unit-attribute relations, iteration, tiling, identical units, standardization, proportionality, additivity, and origin of location.

While previous research has looked at components of scale for selected measurement scales at limited age ranges, issues of precision and selecting appropriate units are less common (Lehrer & Schauble, 2004; Petrosino, Lehrer, & Schauble, 2003; Stephan, Bowers, Cobb, & Gravemeijer, 2004). Furthermore, there is limited research that looks comprehensively at individual factors that may contribute to an understanding of scale and scaling (such as spatial visualization, intuitive number-sense, computational speed, quantitative reasoning, estimation accuracy) across school years.

This paper lays a foundation for a potential learning trajectory, with the goal of engaging researchers from different disciplines in delineating and describing how conceptions of size and scale develop over the life span as well as how cultural and educational experiences may influence this development.

Nano to galactic: What do students know?

The United States government has allocated significant amounts of funding into educating the next generation about space science (on the large end of the scale) and nanoscale science (on the small end of the scale) (e.g., Roco, 2002). My own research in this area began with trying to document how nanoscale experiences influenced students' concepts of nanoscale materials (Jones, Andre, Superfine & Taylor, 2003). Early in our research it became apparent that the high school students we worked with had no idea just how small a nanometer was. Our scientist partners wanted the students to understand how very different the nanoscale world is (where intermolecular interactions dominate and forces such as gravity play minor roles) compared to the macroscale. It became apparent in this early research that we had a collision of the worlds of science discovery (atoms, molecules, intermolecular interactions) with the everyday world of schooling where students struggled to understand the metric system (particularly at the small scale).

Concepts of Scale: Elementary to Graduate School

In our early research, we explored fifth grade, seventh grade, ninth grade, twelfth grade, and science doctoral students' accuracy and benchmark concepts for different sizes (Tretter & Jones, 2006a). We began with an examination of students' concepts of metric scale as well as how accurate they were in using a non-standard measure of scale such as body length. Participants were asked to name objects of different sizes (from one nanometer to one billion meters in a mental size line) as well as to name metric and body length sizes for a range of objects given to them (such as viruses or the distance to the moon). Two assessments and interviews were used to document students' accuracy of scale concepts.

When we asked students to name something the size of a nanometer we found students were often off by multiple powers of ten. Elementary students reported that eyelashes, hairs, and rocks were nanometer sized. Middle school students reported that objects such as crayons, sand, notebooks, and coins were nanometer sized. High school students were no more accurate and named objects such as germs, length of a door, salt, and an ant leg as being nanometer in size.

Accuracy of Scale Concepts

This study across grade levels also examined accuracy of scale concepts (Tretter & Jones, 2006a). Results showed that students were more accurate at large scales (declining in accuracy as the scale increased) than small scales (see Figure 1). In both large and small scales, accuracy decreased with the ability to perceive or experience objects or distances. At small scales, there was a sharp decrease in accuracy below a millimeter. Although many students (at all levels) had likely learned about microscopic objects, their accuracy for objects this size was low. When doctoral students were interviewed about their knowledge of size and scale they noted that they tended to jump from one scale to the next with automatic shifts in the powers of ten.

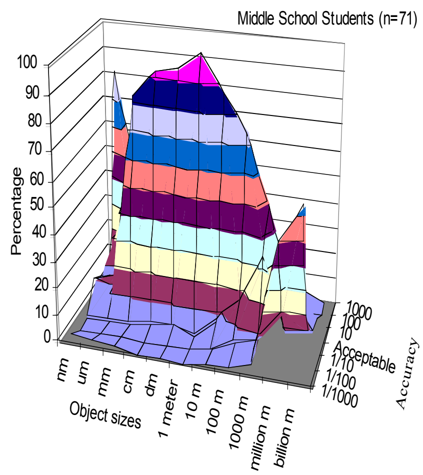

Figure 1. Middle school students' accuracy of metric size. (Reprinted from *Conceptual boundaries and distances: Students' and adults' concepts of the scale of scientific phenomena*, by T. Tretter, M. G. Jones, T. Andre, A. Negishi & J. Minogue, 2006. *Journal of Research in Science Teaching, 83,* 282-319. Copyright 2006 by The Journal of Research in Science Teaching.

Conceptual Boundaries

When concepts of relative size are compared to concepts of absolute size, we find that students have conceptual boundaries for different size ranges (Tretter & Jones, 2006b). Relative size appears to be more easily understood than absolute size (Tretter & Jones, 2006b). Furthermore, we found that students have benchmarks (landmarks) that they use to anchor their web of scales. These mental reference points are useful in estimating lengths (Bright, 1976). In this study, we used a card sorting task with different sizes of objects and distances to examine how students conceptually grouped objects by size. We found students' concepts appeared to be influenced by experiences that range from visual to kinesthetic as well as the degree to which objects can be experienced holistically to sequentially. The results of this study revealed that elementary students tended to have scale benchmarks that included: small, person-sized, room-sized, field sized, and big. Middle school students tended to think of scales as: small, person-sized, room-sized, football field-sized, practical sized, and big. High school students' benchmarks were like middle school students but with the addition of a shopping mall as a large unit of size. Experts included more differentiations in benchmarks of scale and included: atomic-sized, many atom sized, barely visible, very small, small, person-sized, room-sized, field-sized, travel-sized, and big.

Estimation is Key

While students may hold benchmark concepts of size and scale, the degree to which they can use and manipulate this information is important in tasks of estimation (particularly for length). Estimation is an essential concept that has been emphasized in the science standards as well as the mathematics standards (National Science Education Standards, 1996; National Council of Teachers of Mathematics, 2000). Estimation includes numerosity estimation (estimation of the number of objects), computational estimation, and measurement estimation (Hogan & Brezinski, 2003; Siegler & Booth, 2004; Siegler & Opfer, 2003). Estimating linear size and scale specifically involves measurement estimation (e.g. Joram, Subrahmanyam, & Gelman, 1998; Sowder, 1992). Furthermore, estimating measurements appears to be tied to measurement experiences and may be context bound (Harel & Sowder, 2005). Contextual elements that may influence the estimation of length include cognitive distractors, dimensionality and orientation of the object in space (Heller et al., 2003; Leek, Reppa, Rodriguez, & Arguin, 2005; Zacks, Mires, Tversky, & Hazeltine, 2000).

The ability to estimate length involves understanding how continuous measurement can be broken down into segments (Jorman et al., 1998; Sowder, 2005). It has been suggested that estimation ability is not directly related to number calculation (Dehaene, 1997) or general mathematical ability (Hogan & Brezinski, 2003; Sowder, 1992). However, perceptual abilities such as spatial visualization may play important roles in estimating length (Hogan & Brezinski, 2003).

We recently investigated estimation accuracy for linear distances and the relationship of linear estimation accuracy to logical thinking (Jones, Forrester, Taylor, & Gardner, 2012) for middle school students. Results showed that students had difficulty estimating known objects with accuracy and that the context of the task influenced students' accuracy. When students were asked to estimate with non-standard units, accuracy was correlated with logical thinking.

These studies provided evidence that students are not able to conceptualize size and scale of common objects with accuracy. These studies led us to examine how adults who use scale in their jobs frequently learned to use scale. The goal was to understand how we might structure instruction to enable students to apply scale flexibly in real world contexts.

Scientists' Concepts of Size and Scale

As noted earlier, many of the significant advancements that are being made in science are taking place at the extremes of scale. Furthermore, we know students have difficulty estimating sizes and distances (e.g., Tretter, Jones, & Minogue, 2006a), but what is less well understood is how individuals move from having limited skills in conceptualizing scale (novices) to having and using the sophisticated skills that are necessary to do cutting edge science (expert).

Scientists and others in the community that frequently use scale in their work were interviewed to document perceptions of the importance of scale and perceptions of the ways scale expertise developed (Jones, & Taylor, 2009). Fifty individuals with a variety of training and educational backgrounds were asked to describe how they learned about scale from childhood to adulthood. Participants were specifically asked to recall experiences in and out-of-school that helped them understand size and scale.

School Experiences. Using tools of measurement in laboratories was cited by 14% of the participants as having had a significant influence on their knowledge of size and scale (Jones & Taylor, 2009). Twenty percent also recalled creating scaled maps and models as having had a significant influence on their knowledge of scale. Eighteen percent of the participants also recalled learning to convert measurement units as having helped them learn to use scale.

Out-of-School Experiences. When asked about out-of school experiences that were recalled as helping participants learn scale, movement was reported by 76% of the interviewees as having a significant influence on their understandings of distances (Jones & Taylor, 2009). These experiences include activities such as walking, biking, or traveling in a car.

The Process of Conceptualizing Scale. Many of the participants used conceptual benchmarks like the size of red blood cells or a virus to help them conceptualize sizes outside of the human experience. They also reported that they used "body benchmarks" as conceptual rulers to quickly make estimations by measuring with their fingers, arms, or by pacing off distances (e.g., Steffe, 1991).

Participants noted that they often change scales without conscious thought and developed automaticity in working with different scales. This was true for occupations like a forester estimating the amount of timber or for a microbiologist working with cell organelles. Visualization was also reported as a strategy that participants used when working at other scales. As one person noted, *"I have a pretty good mental picture of what I think those cells are doing. And then I can scale that picture down and you can zoom in"* (Jones & Taylor 2009, p. 470). Some participants reported that they moved from one scale to another through mentally visualizing benchmarks and transitioning through images of scale.

Importance of Scale. Finally, we asked participants about the role of scale in their work. Repeatedly, participants described how scale was essential to their work. For example, one scientist noted, "Scale is critically important… it is fundamentally determinative of what you can and can't do."

Future Research: Documenting a Learning Trajectory

These studies provided preliminary evidence that estimating size and conceptualizing different scales (particularly for length) is difficult for most students (elementary through high school). There is also evidence that adults learn to use scale flexibly in work-related contexts. Figure 2 shows a trajectory framework that shows components of learning concepts of size and scale (Jones & Taylor 2009). Although the exact sequence of conceptual components and developmental skills is not fully known, this framework can serve as a starting place for further research that can more accurately pinpoint the development of knowledge and skills needed to be scale literate.

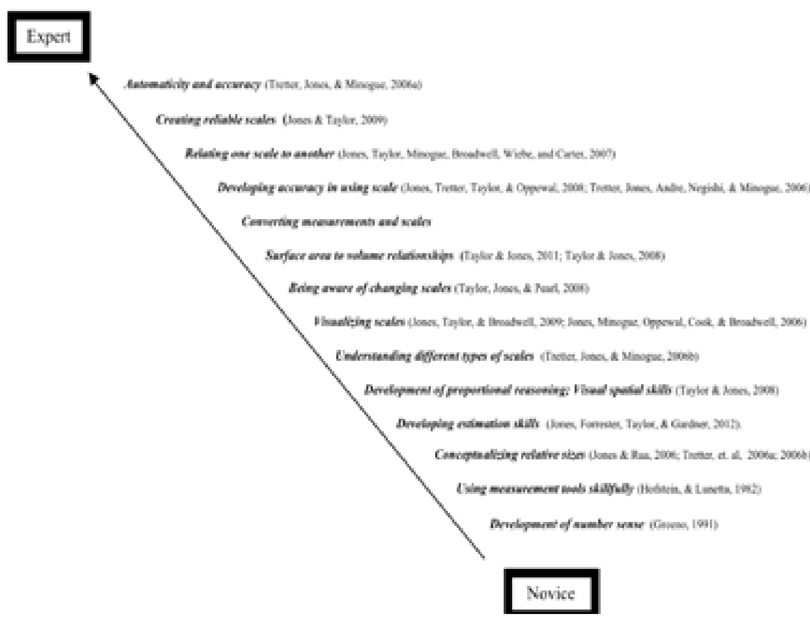

Figure 2. Trajectory Framework For Learning Size and Scale

The trajectory framework begins with the development of number sense, using measurement tools, and conceptualizing relative size. These fundamental concepts underlie the development of estimation skills, understandings of different types of scales, and changes that take place as we move from one scale to another. There is initial evidence that these intermediate concepts and skills are components of development that include proportional reasoning and spatial visualization. At the upper end of the trajectory, individuals are able to move from one scale to another with fluidity and can create and use new scales. Furthermore, as seen in the study of adults that use scale in their work, there is the ability to use scale with automaticity and accuracy.

Although this trajectory provides skeletal evidence for the development of concepts, there is little known about how individuals learn scale and how scale concepts change over a lifetime. In these studies we have focused on linear scales but little is known about how people develop understandings of other types of scales such as volume or mass. Furthermore, it is not clear whether different types of scale concepts tap into different cognitive skill sets or abilities (Hogan & Brezinski, 2003; Sowder, 1992).

There is some evidence that visual-spatial skills may contribute to linear estimation skills, but the precise contribution of visual-spatial development to concepts of scale is not fully known. Our recent studies have found that there is a correlation between visual spatial skills and students' understandings of surface area to volume understandings as well as conceptualizing magnified images (Taylor & Jones, 2011; Jones, Gardner, Taylor, Wiebe, & Forrester, 2011). However, we need to know more about how we can utilize visualization as a tool (particularly with new virtual reality technologies) to teach size and scale.

To explore development and individual difference factors more closely, we are currently examining whether there is a relationship between the non-verbal number acuity, the approximate number system (ANS), and students' abilities to accurately identify size and scale as well as reason about scale and scaling effects. The ANS has been found across development and across animal species (Feigenson, et al., 2004; Barth, et al., 2003; McCrink & Wynn, 2004). ANS resolution as specified by the Weber fraction has been shown to increase with age (Halberda & Feigenson, 2008). A link between the ANS and quantitative reasoning has been verified, but the relationship of the ANS to estimation of continuous quantities is not known. The ANS may be tied to quantitative reasoning through a link with numerical facility (Hogan & Briezinki, 2003; Royer, Tronsky, Chan, Jackson, Marchant, 1999). Of interest to us is the degree to which the ANS is related to students' abilities to accurately identify size and scale as well as reason about scale and scaling effects.

Although many researchers have been investigating components of estimation and concepts of size (particularly in mathematics education), there is promise in an interdisciplinary approach based on quantitative reasoning that can weave what we know about learning mathematics with research on learning size and scale from the perspectives of cognitive science, geospatial studies, and science education. Furthermore, researchers of educational technology are beginning to investigate how virtual worlds may contribute further to an individual's understandings of those worlds that exist at the very large and the very small scales that until recent history have existed outside of our everyday perception. A comprehensive effort to situate quantitative reasoning at the basis of STEM instruction has the potential to help students develop size and scale literacy that is needed to learn science both now and in the future.

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