Reverse Engineering a Course in Quantitative Reasoning

Bernard L. Madison
University of Arkansas

Introduction

The process of taking something apart and revealing the way it works is often an effective way to learn how to build a device or make improvements to it; this is an aspect of reverse engineering. The intent of this paper is to describe a version of reverse engineering for a quantitative reasoning (QR) course using as criteria for improvements a half-dozen collections of content and process standards and research findings on how students learn in college classrooms. In brief, these collections are:

- The six core competencies for quantitative literacy (QL) as articulated in the Association of American Colleges and Universities (AAC&U) QL rubric (AAC&U, 2009; Boersma, et al., 2011).
- The five strands of mathematical proficiency from *Adding It Up* (Kilpatrick, Swafford, and Findell, 2001).
- The eight Standards for Mathematical Practice of the Common Core State Standards for Mathematics (CCSSM, 2010).
- The five elements of effective thinking as articulated by Edward Burger and Michael Starbird (2012).
- Three principles from *How People Learn* as applied to successful classroom practice (Bransford, Brown, and Cocking, 2000).
- Ten principles from applying the science of learning to university teaching and beyond (Halpern and Hakel, 2003).

Of course, the real measure of the effectiveness of a course is student learning, especially the learning for long-term retrieval and transfer. Such measures are elusive for single college courses, to say the least, and other reasons why any measures of student learning are both difficult and of limited value will become apparent as we discuss the characteristics of the QR course in question and compare those characteristics to characteristics specified or implied by the six collections of standards and research findings. In the absence of traditional content for a QR course and reliable measures of desired learning outcomes, the six collections of criteria seem a reasonable approach to design specifications for a QR course. Throughout, two rather startling conclusions from a report (Halpern and Hakel, 2003) of the research findings on learning for long-term retrieval and transfer should serve as motivating beacons of a QR course design:

- "But, ironically (and embarrassingly), it would be difficult to design an educational model that is more at odds with the findings of current research about human cognition than the one being used today at most colleges and universities." (p. 38)
- "There is a large amount of well-intentioned feel-good psychobabble about teaching out there that falls apart upon investigation of the validity of the supporting evidence." (p. 41)

Background of Paper

This paper is an expanded and somewhat re-directed version of one of a dozen expert presentations/readings written for An International STEM Research Symposium: Quantitative Reasoning in Mathematics and Science Education held at Savannah, GA, May 31-June 1, 2012. The Symposium had four themes: QR, Technology in Mathematics Teaching, Learning Progressions, and Mathematics as a Lived Experiences. The focus was on QR with discussions in the other three themes framed within QR. This paper reports on the experience in teaching and studying a one-semester college course in QR over a period of eight years. Even though most of the other eleven expert presentations/readings were directly concerned with elementary or middle school QR learning or with science education, issues and discussions in each of the other eleven readings are related to and present, to some extent, in the evolution of the college QR course discussed and evaluated here. Many of these issues shaped the design principles discussed below that now guide the QR course. Some, but not all, connections and relations to the other eleven expert presentations/readings are indicated in the next section.

The Course in the Context of the Symposium

The goal of the QR course discussed here is to build a foundation for successful practice of QR in the everyday contemporary world of our students. For that reason, the course has been referred to as quantitative reasoning in the contemporary world (QRCW), which was the title of the NSF grant (DUE-0715039) that supported its expansion during 2007-2012. The course teaches process and applications, not content, using case studies of public media articles as study materials. Consequently, it is interdisciplinary, even more so that the discussions at the STEM QR Symposium.

The closest connections of QRCW to the Symposium discussions of quantification, proportional reasoning, and co-variational reasoning by Thompson (2012); unitizing by Steffe (2012B), and scale and size by Jones (2012). Understanding of quantitative reasoning within the course is mostly consistent with that put forth by Mayes, Bonilla, and Peterson (2012) with some differences in the concept of quantitative literacy. Most of the case studies of media articles require mathematical modeling, although usually simpler that that discussed by Duschl (2012) in science education. Learning progressions within a one-semester course need extension beyond the course and beyond school, but they have many of the features of and can be informed by the learning progressions discussed by Lehrer (2012); Steffe (2012A); Schwarz (2012); Mayes, Bonilla, and Peterson (2012); and Baroody, Reid, and Purpura (2012). One of the major issues faced in teaching the QR course to college students is overcoming the lack of productive disposition and the lingering effects of previous mathematics experiences (Dingman and Madison, 2010). Although study of mathematics as a lived experience new to this author, these issues as discussed by Luitel (2012) are very much involved in QRCW. Touched on by several of the expert readings but centrally by Olive (2012), the use of technology in QR is very much a part of our course. Because we aim for students to use QR in their worlds beyond the course and beyond school, we utilize the technology of their worlds – mainly computers and calculators, spreadsheets, and smart phones.

The Plight of Education for Quantitative Literacy

Over the past decade or so, education for quantitative literacy (QL) in the US has gained limited recognition as a critical and perhaps distinct component of school and college curricula, but effective educational methods for QL are tentative and unproven. Focused around the publication of *Mathematics and Democracy* in 2001, several authors (see Steen, 1997 and 2001; Madison and Steen, 2003 and 2008) have made the case forcefully for QL education. Various post-secondary professional societies, notably the Mathematical Association of America (MAA, 2004), Association of American Colleges and Universities (AAC&U, 2004), the American Association of Two Year Colleges (AMATYC) (Blair, 2006), and the National Numeracy Network (NNN) (See http://serc.carleton.edu/nnn/index.html), have initiated policies and structures supporting QL education. Courses are being offered or are under development at individual colleges and universities, and consortia of institutions are working in concert to produce effective college level courses in QL, some in conjunction with developmental mathematics and statistics. Two of the efforts by consortia are centered at the Charles A. Dana Center at the University of Texas in Austin (See http://www.utdanacenter.org/amdm/index.php) and at the Carnegie Foundation for the Advancement of Teaching in Palo Alto, California (See http://www.carnegiefoundation.org/quantway).

QL education in post-secondary institutions has two major resource hurdles to overcome. First, it has no academic home in either K-12 or post-secondary education (Madison, 2001; and Steen, 2001). In K-12 it is highly dependent on the mathematics and statistics curricular strand, and less so on the sciences. Most post-secondary courses and quantitative learning centers (Madison and Steen, 2007; Gillman, 2006) have evolved from mathematics or statistics units, but QL units and courses remain largely marginalized in college and university mathematics curricula. In contrast to most mainline collegiate disciplines, collegiate mathematics has long used its standard content-designated courses as general education courses – algebra, geometry, and calculus. Most collegiate mathematics courses have titles derived from the mathematical content of the course – calculus, differential equations, linear algebra, etc. College and university mathematics faculty members, not unlike many of their colleagues in other STEM (science, technology, engineering, and mathematics) disciplines, have limited and varying interests in the role of their courses in the service of general education. Various attempts at general education mathematics courses over the past century have met with limited acceptance, so mathematics faculty are strongly influenced by this in considering and supporting courses such as QL or quantitative reasoning (QR). The titles of such courses do not describe the mathematical content, so faculty are justifiably puzzled by what they are and how effective they would be in promoting learning in mathematics. One of my colleagues characterized the content of our QR course as "fluff." Mathematics in grades 9-16, from high school through the early years of college, is very linear, equaled only by that of a foreign language, and general education courses have no established place in this linearity (Madison, 2003).

The second major resource hurdle for QL education is connected to the first. There are no clear guidelines for courses and no generally accepted measures of success. Consequently there are no widely accepted curricular materials.

Both of these hurdles were obvious when my mathematical sciences colleagues considered[1] whether or not to establish a course in quantitative reasoning (QR) as an alternative to college algebra for students who would not study further mathematics that needed many of the methods of college algebra, i. e. many of the students who were majoring in non-STEM disciplines. We have been teaching essentially the same QR course as the one proposed for several years, but with college algebra as a pre-requisite. The new version of the course was more visible in that it was being proposed as a course in the (Arkansas) state minimum core as a substitute for college algebra, and, as such, had attracted critiques in the public media (*Arkansas Democrat-Gazette*,

1 The course was recommended narrowly but was questioned as to both its mathematical content and the ways that the success of the course would be measured. To be fair, many of our courses could be questioned on the latter issue, especially college algebra.

2012; Brawner, 2012). The effectiveness of the current mathematics curriculum, including algebra, had been questioned in two highly visible op-ed pieces in the *New York Times*, one by David Mumford and Sol Garfunkel (2011) and one by Andrew Hacker (2012). As was verified by many people who commented on the Hacker article, arguments in favor of QL courses as alternatives to college algebra fall victim to being interpreted as finding an easier route for algebra phobic students. Because QL is neither well established nor well understood and QL courses often do not develop any specific mathematical content, the standards for acceptance within the academic community are higher than those for a course such as statistical methods that indicates some generally acceptable (now, but less so a few decades ago) mathematical content. It was against this backdrop that I re-visited the methods and content of the QR course and analyzed them according to six sets of criteria mentioned above and further outlined below.

I described the current course, called QRCW (Quantitative Reasoning in the Contemporary World) after the title of the NSF grant[2] that supported its expansion, in a pre-conference paper (Madison, 2012) for a QR conference in Savannah, Georgia, in June 2012. In that paper I reviewed the course with respect to four surrounding contexts: traditional high school and college mathematics courses, the six core competencies of the AAC&U QL assessment rubric (#1 below), the five strands of mathematical proficiency from *Adding It Up* (#2 below), and the eight standards for mathematical practice in the Common Core State Standards for Mathematics (CCSSM, #3 below).

Summary Description and Characteristics of Current QR Course

The current QR course, MATH 2183, was first offered experimentally in fall 2004 and has been offered each semester since. At present, the enrollment is approximately 600 students per year, mostly majors in the arts, humanities and social sciences. The course is taught in sections of 25-30 students in interactive classroom environments with tables for four, a document projector, and Internet access. We currently use the third edition of *Case Studies for Quantitative Reasoning* (referred to as the *Casebook*) (Madison, et al., 2012) that evolved from duplicated notes and two earlier editions. The course was expanded and enhanced through the support of the National Science Foundation (DUE-0715039) from 2007 to 2012. Typically, the class meets twice weekly for 80 minutes throughout a semester. The *Casebook* has 30 case studies of media articles, consisting of an article, warm-up exercises, and study questions on the article. The topics of the case studies are sorted into six sections: 1) using numbers and quantities; 2) percent and percent change; 3) measurement and indices; 4) linear and exponential growth; 5) graphical interpretation and production; and 6) counting, probability, odds, and risk. A typical class meeting begins with students presenting media articles (via the document projector) they have found that contain quantitative information. This feature is referred to as News of the Day, and students are awarded credits for presenting articles. There is usually a homework assignment of warm-up exercises, but the core activity is addressing the study questions, which probe the quantitative content of the article being discussed. Often, students address the study questions in groups of 3-4 at a single table. Quizzes and tests consist of exercises similar to the warm-up exercises and study questions on one or two articles new to the students. Mathematics is developed or reviewed as needed, when needed. For example, the sum of a geometric series is developed when needed for compounding interest or exploring installment savings or purchasing.

The success rate (grade of A, B, or C) for the course is over 80%, significantly higher than other introductory mathematics courses. The higher success rate is partly due to the prominent role of daily homework in the course but also due to the students' heightened interest in the

2 Quantitative Reasoning in the Contemporary World, NSF Grant DUE-0715039, 2007-2012.

subject matter. Student evaluations of the course have been favorable, and it has received high marks from faculty advisors in departments whose students enroll in the course. We used pre-post tests in 2007-2008 to compare learning in this course with that in two other similar courses (see Table 1 for some summary results), and we administered a pre and post-attitude survey to the same populations. Although the results were not dramatic, learning gains as measured by the test were greater in the QR course and attitude shifts were all in the desired direction. We administered an email survey to former students after 2-3 years to see if they continued (as we hoped they would) to practice QR in looking at media articles[3]. The response rate was very low (42/300), but about half reported that they continued to practice QR, about 2/3 responded that their confidence in their QR ability had increased, and about ¾ reported that they now believed QR to be more important to them.

Table 1

Fall 2007 – 15 multiple-choice item pre- and post-test to students in three different courses

Course	**Number of Students**	**Number of Items with significant increase in mean scores (p<0.05)**	**Number of items with significant increase in mean scores (p<0.1)**
Survey of Calculus	106	6	9
For All Practical Purposes	77	6	7
QRCW Course	96	9	10

Spring 2008 – 17 multiple-choice item pre- and post-test to students in two different courses

Course	**Number of Students**	**Number of Items with significant increase in mean scores (p<0.05)**	**Number of items with significant increase in mean scores (p<0.1)**
For All Practical Purposes	83	5	6
QRCW Course	95	5	9

Writing and critical reading have been important all along in responding to study questions. In fact, 26 of the 30 case studies have questions that require communication, including writing, and all 30 require interpretation, usually interpreting quantitative information given in words so it can be represented in another form, usually a function or an equation. See Table 2 below for the competency requirements of the case studies in the *Casebook* that are given in full at http://www.cwu.edu/~boersmas/QRCW/mappingtesting/index.html. We have experimented with a few sections of the current course by adding a significant writing component. The results of that and our belief that writing is important to improved QR have convinced us to add a significant writing requirement to a modified course, partially described below.

3 An important outcome of QR courses is development of a QR habit of mind that would be expected to continue beyond the course and beyond school. Assessment of such a habit of mind remains to be developed and demonstrated.

The Modified Course

In this paper I describe a more extensive examination of the QR course as it is proposed as an alternative for college algebra. As noted above, the course, MATH 2183, Mathematical Reasoning in a Quantitative World has been termed QRCW in the three papers we have written about the course and research surrounding it (Dingman and Madison, 2010; Madison and Dingman, 2010; Boersma, et al., 2011). The modified course, which is proposed as a course in the state minimum core for higher education, is MATH 1313, Quantitative Reasoning. We will retain a revised version of MATH 2183 with either college algebra or MATH 1313 as a prerequisite, so we will be able to have a more advanced second QR course. With the change, we expect the enrollment in MATH 1313 (the new QR course) to be approximately 1200 students per year, about double the current yearly enrollment in MATH 2183. Teaching the course in sections of 25-30 will require about 20 sections per term, so we will need 10-12 instructors (mostly graduate assistants or adjunct instructors). Like the current course, we plan to use the *Casebook* of 30 case studies of media articles, and we expect to use at least 20 of them in MATH 1313. But the following features will be in MATH 1313 that were not in MATH 2183:

- MATH 1313 will have a significant formal writing component.
- MATH 1313 will develop some additional algebraic concepts and methods because it does not have college algebra as a prerequisite.

The addition of the writing requirement, to be coordinated with the composition courses in our English department, will add to the rigor of the course and the difficulty of teaching and assessment. Part of the reason for the writing requirement is to maintain the level of rigor and to protect against the course degenerating to the methods contained therein. (Some years ago, we offered a course called mathematics patterns, a rather typical mathematics for the liberal arts that had some very nice mathematical content, but largely due to its being taught by inexperienced instructors and graduate students, it degenerated into a methods course, and there was not much left. Consequently, it was discontinued.) The more important reason for adding writing is that writing strengthens quantitative reasoning. Because the course will require instructors not only familiar with using case studies in a collaborative learning classroom but also with instruction and assessment in writing, we plan to implement a graduate course in the pedagogy of inquiry-based instruction and composition that will be required of all instructors assigned to MATH 1313. We will use a resource book on writing, something akin to *The Chicago Guide to Writing about Numbers* (Miller, 2004), and we will utilize composition instructors from the English department to help (at least initially) with the composition pedagogy of our graduate course. (Our English department has a graduate course in composition pedagogy required of all their entering graduate assistants.)

The additional development of algebraic concepts and methods will not be large, mainly because we had to do much of that in MATH 2183 because most of the students did not recognize the utility of their algebra to answer study questions about media articles. We will face two challenges that we have not solved in teaching MATH 2183:

- What contextual examples should be generalized and abstracted? The power of mathematics is in abstraction and generalization, and students should not only see this power when it is needed but should combine results of contextual examples with abstractions to increase the long-term retrieval and transfer (Halpern and Hakel, 2003; Bransford, Brown and Cocking, 2000).
- One of the research findings (Bransford, Brown, and Cocking, 2000, p. 16) about developing competence in an area of inquiry is to "understand facts and ideas in the

context of a conceptual framework." What are the conceptual frameworks for our QR course, or, more generally, for QR?

This review consists of comparing and contrasting the criteria measures with one another and then with the practices in the QR course as it will be modified for the new offering without college algebra as a prerequisite and as an alternative to college algebra for some students.

Criteria Measures

We analyze below how the QR course fares with respect to the six sets of criteria described briefly in the introduction. These criteria are closely related to a course where students learn for long-term retention and transfer. The first set of criteria, the core proficiencies for QL as developed by AAC&U and adapted by the QRCW course developers (Boersma, et al., 2011), is the only set of the six that was developed with QR or QL in mind, so this set is listed first. The next two sets of criteria (2 and 3) are descriptions of mathematical proficiency (for K-12, but clearly more broadly applicable) that were developed by groups of mathematicians and mathematical educators and have bases in research on teaching and learning mathematics. Criteria set 4 is documented largely with anecdotes from the classrooms of the two authors (Edward Burger and Michael Starbird), both notable award-winning collegiate mathematics faculty members. Criteria sets 5 and 6 are taken from summary reports of extensive research on human cognition.

1. The six core competencies for quantitative literacy as articulated in the VALUE QL rubric developed by the AAC&U and as modified by the QRCW developers, S. Boersma, C. Diefenderfer, S. Dingman, and the author (Boersma, et al., 2011). These QL core competencies are interpretation, representation, calculation, analysis/synthesis, assumption, and communication.
2. The five strands of mathematical proficiency as articulated in the 2001 NRC publication, *Adding It Up* (Kilpatrick, Swafford and Findell, 2001). The five strands are conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition.
3. The eight Standards for Mathematical Practice of the Common Core State Standards for Mathematics (CCSSM, 2010). These eight standards were developed to capture the essence of and articulate the meaning for K-12 mathematics students of the five strands from *Adding It Up* and the five process standards of the 2000 NCTM standards for school mathematics: problem solving, reasoning and proof, communication, connections, and representations. These eight CCSSM standards describe how mathematically proficient students practice mathematics.
4. The five elements of effective thinking as articulated by two award winning college teachers, Edward Burger and Michael Starbird (2012). The five elements are grounding your thinking (understand deeply), igniting insights through mistakes (fail to succeed), creating questions out of thin air (be your own Socrates), seeing the flow of ideas (look back, look forward), and engaging change (transform yourself). (Notice the mnemonic device of earth, fire, air, and water.)
5. What research[4] on learning tells us about how people learn and how that knowledge can be linked to successful classroom practice as articulated in the expanded edition of *How People Learn*, a 2000 publication of the National Research Council.

4 This summary volume has many references to the original research publications.

6. Principles gleaned from applying the science of learning to university teaching and beyond (Halpern and Hakel, 2002). These principles are closely related to some of the findings reported in *How People Learn*, but a summary article (Halpern and Hakel, 2003, p. 38)[5] on teaching for long-term retention and transfer lists ten basic "laboratory-tested principles drawn from what we know about human learning."

Design Principles for the QR Course

As we refined and expanded the QR course over the past eight years, we have generally followed some principles in composing curricular materials and in conducting the QR classes. Some of these are strongly influenced by our circumstances of having a one-semester QR course with no continuing formal education in QR. These ten principles are articulated and discussed with references to the six sets of criteria.

1. *Provide a venue for continued practice beyond the course (and beyond school).* Quantitative reasoning is a habit of mind, and habits are developed by practice. Further, there are several examples of successful application in professional education in the US that use problem-based case studies that prepare one for professional practice; we even use the word, practice. Among these are case studies in education, medicine, law, architecture, social work, and business. Quantitative reasoning is analogous to a lifelong profession, as effective quantitative reasoning will be needed for informed performance as citizens and for personal prosperity. The course utilizes case studies of media articles as the focus of study. We move the students toward developing their own habits of analysis of media articles, taking charge of their learning as promoted by principles from *How People Learn* and by Burger and Starbird (2012). Media articles similar to the ones discussed in the course are now and will continue to be part of the everyday world of the students, which leads to principle #2.

2. *Keep the material relevant to students' everyday contemporary world.* According to John Dewey, "School should be less about preparation for life and more about life itself." Connecting classroom learning to the everyday contemporary world not only can enhance learning at the time of study in the classroom but can lead students to adapt their classroom learning to the changing environment of everyday life. As noted in *How People Learn*, "The ultimate goal of schooling is to help students transfer what they have learned in school to everyday settings of home, community, and workplace" (p. 73). We pay particular attention to adapting thinking in one context to another context because we recognize the changing nature and infinite variety of QR in non-school environments. Again following *How People Learn*, "Since these environments change rapidly, it is also important to explore ways to help students develop the characteristics of adaptive expertise" (p. 73). Adaptive reasoning is one of the five strands of mathematical proficiency from *Adding It Up*.

For various reasons, we strive to keep subject matter fresh and authentic. Even if discussing an old (a few years to an 18-year-old!) article, we relate it to the present. For example, a 2003 article on a political debate about how to measure the budget deficit (nominal dollars, constant dollars or percent of GDP) easily relates to the current continuing discussion of deficits and national debt. Or a 2001 opinion piece about the economics of increasing the fuel efficiency of automobiles is analogous to the economics of choosing between a hybrid version and a gasoline version of a type of automobile.

Over the decade of developing our QR course, paper copies of newspapers and magazines

5 These summary publications have many references to the original research publications.

have continued to give way to online sources, and online sources are available via numerous personal technologies. This has changed the way students access media articles and has increased the variety (and uncertain reliability) of articles. This has added importance to the question of evaluation of the information reported.

There are potential problems with learning in contexts. As stated in *How People Learn*, “Simply learning to perform procedures and learning in a single context, does not promote flexible transfer” (p. 77). This leads to principle #3.

3. *Use multiple contexts to practice quantitative reasoning.* According to Halpern and Hakel (2002 and 2003), “The purpose of formal education is transfer” (p. 38 in 2003). Halpern and Hakel go on to identify retrieval in multiple contexts as one of the most basic principles to enhancing long-term retention and transfer of learning, and that, spaced, not massed, practice at retrieval is best. In one QR course, significant spacing of retrieval is not possible. Consequently, there is more need for continued practice at retrieval beyond the course. With multiple contexts, students are more likely to abstract the relevant features of concepts and develop a more flexible representation of knowledge, whereas instruction based on single contexts may lead to situated learning. Contextual situations need to be abstracted and generalized, which is closely related to principle #4.

4. *Promote appreciation of arithmetical precision and the power of mathematical concepts and processes.* This is the most difficult principle to apply in a QR course that is based on analyzing contextual situations. Developing mathematical formulas and models when they are needed points to reasons why the work is worthwhile. As stated in *How People Learn*, “An alternative to simply progressing through a series of exercises that derive from a scope and sequence chart is to expose students to the major features of a subject domain as they arise naturally in problem situations. … Ideas are best introduced when students see a need or a reason for their use – this helps them see relevant uses of knowledge to make sense of what they have learned” (p. 139). Much of the power in mathematics is in abstraction and generalization, and this is the motivating principle of the eight CCSSM practice standards. In fact, it is stressed in CCSSM 7, look for and make use of structure, and CCSSM 8, look for and express regularity in repeated reasoning. Abstraction and generalization troubles many students, especially those who are somewhat math-phobic. By seeing uses of and reasons for abstraction and generalization, trouble can be reduced. Multiple uses of similar processes in different contexts give rise to the need for abstraction and generalization.

5. *Help students to structure their quantitative reasoning in resolving problematic situations, including ample doses of critical reading and writing.* One way to do this is by using the core competencies of interpretation, representation, calculation, analysis/synthesis, assumptions, and communication (AAC&U, 2010; Boersma, et al., 2011). If students understand that they need some or all of these six competencies to address a QR situation, then they can organize their responses accordingly and produce a full response. Curricular materials and questioning prompts should be composed in consideration of which competencies are needed for the proper responses. For example, if the student should communicate a response in writing, the prompt should so indicate. Requiring students to write responses promotes clearer thinking and deeper understanding, and writing requirements should progress from sentences to paragraphs to multi-page reports. Students in one of the sections of the QR course in Spring 2012 commented about combining writing and quantitative reasoning (called math by many students). One wrote, “… instead of just working a problem and moving on, I had to evaluate the process and determine how to explain the process in words.” Another was more explicit, showing some negativism

toward mathematics, "Math is virtually useless without proper communication of its meaning." College faculty who were participants at a 2012 Conference on Interdisciplinary Teaching and Learning at Michigan State University discussed why writing was an effective vehicle for assessing interdisciplinary learning. As one participant stated, "writing manifests thinking." Students need to get writing structure down in order to progress intellectually and communicate that progress to others. Reflective writing can reveal how well students are integrating ideas from different sources or disciplines. One participant quoted from Richard Guindon's 1989 *San Francisco Chronicle* cartoon: "Writing is nature's way of showing you how sloppy your thinking is."

6. *Encourage on-the-fly calculations and estimations.* If students are able to quickly assess the validity of a quantitative assertion or mentally compute a numerical result, then they will be more able to practice QR in many aspects of their daily lives. This increased practice will strengthen their analysis and calculation, thereby building formidable QR skills. This is one of the places where one wants to develop automaticity of skills. Facility with mental arithmetic and estimation allows one to "function effectively without being overwhelmed by attentional requirements" (*How People Learn*, p. 139). This is part of the *Adding It Up* strand of procedural fluency, i. e. skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. In the CCSSM practice standards, this is part of the practice standard #6, attend to precision**.**

7. *Increase students' supplies of quantitative benchmarks.* Personal quantitative benchmarks are critical for understanding quantities and being able to determine reasonableness of quantitative assertions or numerical answers to questions. In order to comprehend quantities, especially very large or very small ones, one must express them in personally understandable units. One's personably understandable units depend heavily on one's supply of personal quantitative benchmarks. Joel Best (2008) points to the importance of statistical benchmarks in spotting dubious data. "Having a small store of factual knowledge prepares us to think critically about statistics. Just a little bit of knowledge – a few basic numbers and one important rule of thumb – offers a framework, enough basic information to let us begin to spot questionable figures" (p.7). Best gives four benchmarks that go a long way in understanding US social statistics. These are the US population (approx. 300 million), the annual birth rate (approx. 4 million), the annual death rate (approx. 2.4 million), and the approximate fractions of the population of major ethnic or racial groups.

At the 2012 Quantitative Reasoning Symposium in Mathematics in Savannah, GA, Gail Jones (North Carolina State University) began a presentation by showing a highly magnified image of part of a familiar biological entity and began showing successive images with less magnification. She asked audience members to take note of the point at which they were able to identify the entity. Namely, at what magnification was the entity understandable—i.e. when could you recognize what it was? (In my case, it was at either the penultimate image or the final image that I was able to see that the entity was a common ant.) This is dual to the problem we gave students in a think-aloud session, namely, express \$1.2 trillion in terms that make it understandable to you. One reasonable solution was to note that \$1.2 trillion is enough to purchase every person in the states of Arkansas and Kentucky a house costing approximately \$150,000 each. Note that in the ant visualization example, one understands by seeing the whole, or nearly whole, animal as opposed to small pieces magnified. In the \$1.2 trillion example one understands by breaking the large entity into smaller pieces. Of course, experts on ants might recognize the ant at higher magnifications of its parts, and managers of large money accounts might not need to re-express the \$1.2 trillion.

As students use quantitative benchmarks, their supply grows, as does their understanding of quantities. This is consistent with Burger and Starbird's understanding deeply, clearing the clutter.

8. *Encourage students to use technology to enhance and expedite understanding.* Technology, including personal devices, is omnipresent in the everyday lives of our QR students, so we leverage it in service of understanding. As examples, students are encouraged to use technology for calculations exceeding on-the-fly abilities, to graph functions on graphing calculators, and to use spreadsheets for repetitive calculations. In my QRCW class sections, we sometimes needed a statistic or another piece of information. Often we designated one of the students with a smart phone as our "Googler of the Day." How personal technologies affect learning is not clear; research projects to determine answers will have difficulty keeping pace with the changing technologies. However, since these technologies are certain to be a part of our students' future everyday life, they are a part of how we conduct our QR classes. As stated in the CCSSM practice standards, we encourage the use of appropriate tools strategically.

9. *Allow student interests to emerge.* As reported in *How People Learn*, "Students are motivated to spend time needed to learn complex problems that they find interesting. Opportunities to use knowledge to create products and benefits for others are particularly motivating for students" (p. 77). We address student interests in the QR class by way of students finding media articles with quantitative content, bringing them to class and explaining them to the class or formulating questions that they cannot answer. Students who are interested in baseball may bring a comparison of the statistics of Albert Pujols and Henry Aaron. Students who are interested in the military may bring a statistical analysis of military budgets of different countries. This also encourages student generated questioning, one of the four elements of effective thinking.

10. *Provide interactive classroom environment.* Inquiry-based learning is emphasized in the QR classes, and students often work in groups of 3-4. Social interaction is important as a motivation and as a vehicle for developing understanding. The tenth principle articulated by Halpern and Hakel (2003) is, "What learners do determines what and how much is learned, how well it will be remembered, and the conditions under which it will be recalled" (p. 41). Inquiry-based learning and interactive classrooms are fundamental in the elements of effective thinking by Burger and Starbird. Understanding deeply, making mistakes, asking questions, and looking forward and backward are common components of interactive classrooms.

The AAC&U QL Rubric and an Adaptation

In 2009, AAC&U published fifteen rubrics as products of its Valid Assessment of Learning in Undergraduate Education (VALUE) project. One of those fifteen was the Quantitative Literacy rubric**.** According to AAC&U, "the rubrics are intended for institutional-level use in evaluating and discussing student learning, not for grading." My colleagues and I (Boersma, et al. 2011) adapted the AAC&U VALUE QL rubric (http://www.aacu.org/value/rubrics/index_p.cfm?CFID=42815424&CFTOKEN=85621924) to one that we could use to assess individual student work. We modified the VALUE QL rubric in a few ways to make that possible. The result was what we refer to as the Quantitative Literacy Assessment Rubric (QLAR) (See http://www.cwu.edu/~boersmas/QRCW/Casebook/QLAR.pdf). Like the VALUE QL rubric, QLAR has six core competencies that are required for responses to QR prompts: interpretation, representation, calculation, analysis/synthesis[6], assumption, and communication. These are described as follows:

6 This was application/analysis in the QL VALUE rubric.

1. *Interpretation*: Ability to glean and explain mathematical information presented in various forms (e.g. equations, graphs, diagrams, tables, words).

2. *Representation*: Ability to convert information from one mathematical form (e.g. equations, graphs, diagrams, tables, words) into another.

3. *Calculation*: Ability to perform arithmetical and mathematical calculations.

4. *Analysis/Synthesis*: Ability to make and draw conclusions based on quantitative analysis.

5. *Assumptions*: Ability to make and evaluate important assumptions in estimation, modeling, and data analysis.

6. *Communication*: Ability to explain thoughts and processes in terms of what evidence is used, how it is organized, presented, and contextualized.

Two of the six – interpretation and communication – involve critical reading and writing (or speaking). In fact, all but calculation can involve non-quantitative communication. This dependence on writing is noticeably stronger than the two related articulations of mathematical proficiency described below, namely, the five strands from *Adding It Up* and the CCSSM mathematical practice standards.

Discussion of the Core Competencies and the QR Course

The core competencies above serve multiple purposes. They provide the basis for rubrics to assess student work, they offer ways to structure students' understandings, and they are reminders of what we are seeking to develop in curricular materials and assessments. There are 268 study questions in the 30 case studies in the 3rd edition of the *Casebook*. Although most (1st and 2nd editions) of the *Casebook* was written before the QL core competencies were articulated, the changes for the 3rd edition focused on incorporating what we had learned from adapting the AAC&U rubric to assess student work (Boersma, et al., 2011). We clarified what competencies we wanted to assess with study questions, and we included the rubric for scoring student work in the introduction. The fractions of the 30 case studies and the 268 study questions that require each of the six competencies are given in the following Table 2 below.

Table 2

Competency	interpretation	representation	calculation	analysis/ synthesis	assumption	communication
% study questions	67%	30%	48%	35%	7%	38%
% case studies	100%	73%	90%	90%	40%	87%

Five Strands of Mathematical Proficiency from *Adding It Up*

Adding It Up is a 2001 report of the Mathematics Learning Study Committee of the National Research Council that summarizes research results on mathematics learning from pre-kindergarten through grade 8. The model of mathematical proficiency articulated in *Adding It Up* consists of five intertwined strands that are described as follows.

1. *Conceptual understanding*: Comprehension of mathematical concepts, operations and relations.

2. *Procedural fluency*: Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.

3. *Strategic competence*: Ability to formulate, represent, and solve mathematical problems.

4. *Adaptive reasoning*: Capacity for logical thought, reflection, explanation, and justification.

5. *Productive disposition*: Habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence in one's own efficacy.

Discussion of the Strands from *Adding It Up* and the QR Course

Although these five strands were part of the basis for the CCSSM standards for mathematical practice (described below), the articulation of these five as above is more succinct and identifies what appears to be a critical proficiency for many of our students – productive disposition.

The core competencies in QLAR are manifestations of these and related proficiencies. In our work with students, productive disposition seems to be critically important for practicing QR in contemporary society, and all six core competencies seem to depend on productive disposition. As we reported in describing our experience in developing the QRCW course (Dingman and Madison, 2010), the students are initially (on average) negative about their view of and experiences in mathematics, both in its utility to them and their abilities to use it. Improving this productive disposition is paramount in our efforts to help the students toward stronger QR.

Interpretation in QLAR depends more on conceptual understanding; representation depends more on both conceptual understanding and strategic competence; calculation is strongly related to procedural fluency; analysis/synthesis depends on strategic competence and adaptive reasoning as does assumptions; and communication is closest to adaptive reasoning. Reflection, explanation, and justification in adaptive reasoning play major roles in resolving contemporary QR situations.

The CCSSM Mathematical Practice Standards

The CCSSM Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students (CCSSM, 2010). These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics (NCTM, 2000) process standards of problem solving, reasoning and proof, communication, representation, and connections. The second consists of the strands of mathematical proficiency from *Adding It Up* as described above. The eight practice standards are below, each with a one-sentence description. The full descriptions of the standard are at www.corestandards.org/assets/CCSSI_Math%20Standards.pdf.

1. **Make sense of problems and persevere in solving them:** Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution.

2. **Reason abstractly and quantitatively:** Mathematically proficient students make sense of quantities and their relationships in problem situations.

3. **Construct viable arguments and critique the reasoning of others:** Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments.

4. **Model with mathematics:** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.

5. **Use appropriate tools strategically:** Mathematically proficient students consider the available tools when solving a mathematical problem.

6. **Attend to precision:** Mathematically proficient students try to communicate precisely to others.

7. **Look for and make use of structure:** Mathematically proficient students look closely to discern a pattern or structure.

8. **Look for and express regularity in repeated reasoning:** Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts.

Discussion of the CCSSM Practice Standards and the QR Course
Practice standards 1, 2, 3, and 4 are dominant in contemporary QR as addressed in our QR course. Making sense of problems; modeling with mathematics or statistics; reasoning quantitatively; and drawing, supporting and communicating conclusions are integral parts of QR. Critiquing the reasoning of others is often the entry point into a QR situation as they appear in public media articles. Practice standards 5 – 8 are less central to QR. There is attention to precision (standard 6), but most attention focuses on the precision needed or possible in resolving the QR situation. Certainly the use of appropriate units is a big deal in QR and somewhat unusual as noted above in QR course principle 7. Tools (standard 5) for our QR students include calculators (and sometimes, spreadsheets) and quantitative benchmarks for detecting reasonableness of answer. Standards 7 and 8 are less obvious in resolving QR situations.

The Five Elements of Effective Thinking

Although the Burger and Starbird (2012) book is entitled "Five Elements," there are four building blocks of effective thinking with the fifth one, change, being an expected outcome of applying the first four. They have used the five classical elements that were once believed to be the essential parts of nature and matter – earth, fire, air, and water, plus the quintessential heavenly element aether. Contrary to what was believed about aether (that it was incapable of change), Burger and Starbird have change as their fifth and quintessential element. Briefly, these four building blocks of effective thinking are (p. 6):

- Earth – Understand deeply. Don't face complex issues head-on; first understand simple ideas deeply. Clear the clutter and expose what is really important.
- Fire – Ignite insights by making mistakes. Fail to succeed. Intentionally get it wrong

to inevitably get it more right. Mistakes are great teachers – they highlight unforeseen opportunities and holes in your thinking.

- Air – Raise questions. Constantly create questions to clarify and extend your understanding. What's the real question? Working on the wrong question can waste a lifetime. Be your own Socrates.
- Water – Follow the flow of ideas. Look back to see where ideas came from and then look ahead to see where the ideas may lead. A new idea is a beginning, not an end.

Discussion of *Five Elements* and the QR Course

Earth. Deep understanding at first blush seems like something that one cannot achieve in a one-semester QR course. In fact, as mathematics faculty tend to judge mathematics courses, they are likely to consider a QR course such as the one discussed here as not promoting or requiring deep understanding. They likely are judging on the depth of understanding of the mathematical concepts and not on the sophisticated and habitual use of rather elementary mathematical concepts to understand quantitative situations. Deep understanding of ratios, proportions, rates of change, and graphical representations are not the aim of most college mathematics courses, but they are among the aims of our QR course. Clearing the clutter in analyzing a quantitative argument in a media article and getting to the gist is a critical first step in understanding. This requirement of depth in understanding contextual situations is one of the major distinctions of a quality QR course.

Fire. Mistakes can be great teachers, but our QR students initially are not inclined to venture opinions or propose solutions. In my QR class I treat every mistake as a learning opportunity. This is a major issue in the student presentations of News of the Day articles. Many students are reluctant to stand up in front of a class (and the teacher) and demonstrate their quantitative reasoning, which often contains errors. I try to defuse the reluctance by handling mistakes carefully and straightforward as if everyone makes mistakes and we all can learn from them. One of the most common mistakes occurs in backing up a percentage change. Canonically, one knows the value of a quantity now and a percent change from some point in the past and wants to find the value at the point in the past. About ¾ of the students entering our QR course answer this incorrectly, and these same mistakes persist throughout the semester. When a student makes the canonical mistake in a News of the Day presentation I take the opportunity to point out how common this is and urge that we remember the correct way. By semester's end we have about half the students still making this mistake.

Air. Raising questions by our QR students is initially stymied by the same attitudes that keep them from venturing solutions or opinions. Their experiences in traditional mathematics and statistics courses point them toward responding to questions that have definite and often unique answers. The core material in the *Casebook* we use consists of study questions on media articles that serve as examples of questioning that they should employ in QR in everyday life beyond the course. Many of these questions do not have clearly defined answers, which can be frustrating to students not accustomed to such situations. However, the vague nature of some situations invites student questioning, and QR instructors model such questioning, especially in regard to News of the Day articles being presented by students.

Water. News media articles invite looking backward at the origins of the information and forward to where it might lead. Further, the ideas developed in exploring and understanding one media article are often applicable to other articles. So the flow of ideas has two channels, one regarding a particular context of one article and one that takes the understanding of one article and utilizes it in understanding other articles, perhaps even in very different contexts. As an example, one of the QR case studies aims at understanding inflation by way of looking at the cost of a product (in this case, the Chuck Taylor All Star canvas shoe) that has remained essentially the same over the past half century. This is a very real situation as it is often the case that some student in a QR class may be wearing the All Star shoe. One has the chance to think backward to the 1950s and forward to see what the shoe might cost in 20 years. And the ideas here easily extend to more complex situations, say, considering arguments about how to measure federal revenues, spending, and deficits or surpluses.

Learning Research Findings from *How People Learn*

Quantitative reasoning has become an indispensible skill for 21st century US residents. In *How People Learn* (2000), the situation is summarized as follows.

> In the early part of the twentieth century education focused on the acquisition of literacy skills: simple reading, writing, and calculating. It was not the general rule for educational system to train people to think and read critically, to express themselves clearly and persuasively, to solve complex problems in science and mathematics. Now, at the end of the century, these aspects of high literacy are required of almost everyone in order to successfully negotiate the complexities of contemporary life. The skill demands for work have increased dramatically, as has the need for organization and workers to change in response to competitive workplace pressures. Thoughtful participation in the democratic process has also become increasingly complicated, as the focus of attention has shifted from local to national and global concerns. (p. 4-5)

The expanded edition of *How People Learn* represents reports on the work of two National Research Council committees, both published in 1999, one that summarized research developments in the science of learning, and one that summarized research findings on linking learning research to classroom practices. The expanded volume, published in 2000, begins with three key findings on how students learn. These have strong implications for teaching and are connected to our practices in the QR course as discussed above.

1. Students come to the classroom with preconceptions about how the world works. If their initial understanding is not engaged, they may fail to grasp the new concepts and information that they are taught, or they may learn them for purposes of a test but revert to their preconceptions outside the classroom.

Therefore:

1T. Teachers must draw out and work with preexisting understandings that their students bring to them.

2. To develop competence in an area of inquiry, students must: (a) have a deep foundation of factual knowledge, (b) understand facts and ideas in the context of a conceptual framework, and (c) organize knowledge in ways that facilitate retrieval and application.

Therefore:

2T. Teachers must teach some subject matter in depth, providing many examples in which the same concept is at work and providing a firm foundation of factual knowledge. Burger and Starbird (2012) get at this in several ways. While giving advice on how to understand deeply, they say, "Sweat the small stuff." (p. 25). They note that when studying some complex issue, instead of attacking it in its entirety, find one small element of it and solve that part completely.

3. A "metacognitive" approach to instruction can help students learn to take control of their own learning by defining learning goals and monitoring their progress in achieving them. Burger and Starbird's five elements are aimed at students (and others) taking control of their own learning. Although there are anecdotes from their classrooms that illustrate the five elements in action, the real message is to the learner-thinker.

Discussion of *How People Learn* and the QR Course

How does the QR course respond to these principles?

Principle #1 above. Some of the preconceptions that students bring to the QR course are molded by their experiences in previous mathematics classes (Dingman and Madison, 2010). They are accustomed to courses with structured lectures, template problems, textbooks with numerous example exercises, and homework that utilizes the method of the day to solve problems that have one and only one solution. Because this is very different from the everyday QR challenges these students will face, the QR course and "textbook" are different. The absence of multiple template problems frustrates some students, illustrating that varying conditions of learning makes it more difficult for students but results in more learning (number II below). Students are also not accustomed to seeing mathematics, especially algebra, as a tool for understanding media articles, and this is the central purpose of the QR course.

Principle #2 above. Presentation of an organized set of facts is not specified in the QR course. The knowledge that students are to apply consists of mathematics and statistics learned in school or early college. Beyond that, they need to understand or learn the basics of various contexts – political, social, economic, etc. – of the media articles in the case studies and articles brought to class by students. One of the weaknesses (noted above) of the QR course is in developing conceptual frameworks for QR, and the absence of conceptual frameworks takes away a powerful retrieval and transfer mechanism.

Principle #3 above. Having students take charge of their learning is a major goal of the QR course. Much of what we do is aimed at that: creating a venue for continued practice, contexts from contemporary student life, increasing the supply of personal quantitative benchmarks, etc.

Principles from Applying the Science of Learning to the University and Beyond

What can research on human learning tell us about how to best conduct classes in college (or in any adult education setting) to teach for long-term retention and transfer? About ten years ago 30 experts from different areas of the learning sciences met to answer this question. As reported by Halpern and Hakel (2003), these experts identified ten "basic laboratory-tested" principles drawn from what we know about human learning. They follow below, and after each principle, the connection to the QR course is given.

1. *The single most important variable in promoting long-term retention and transfer is "practice at retrieval."* Practice at retrieval within the QR course can take place with questioning in class, collaborative learning situations where one student explains to another, and responding to assessment items or homework assignments. Spaced practice is better than massed practice, so spreading concepts such as relative change versus absolute change over an entire course, in different contexts, will facilitate learning for long term transfer.

2. *Varying the conditions under which learning takes place makes learning harder for learners but results in better learning.* The absence of template problems, as noted above, is the main adherence of our QR course to this principle. Each case study is different, but there are conceptual strands that run through multiple cases. Identifying and emphasizing these strands remains one of the challenges of the course.

3. *Learning is generally enhanced when learners are required to take information that is presented in one format and "re-represent" it in an alternate format.* As noted earlier, all of the case studies and 2/3 of the study questions require interpretation (i.e. gleaning and explaining information presented in various forms) and ¾ of the cases require representation, i.e. converting information from one mathematical form to another.

4. *What and how much is learned in any situation depends heavily on prior knowledge and experience*. We know very little about prior knowledge and experience of our students. We have surveyed them about attitudes and administered pre-tests, but this information has not been systematically used in instruction.

5. *Learning is influenced by both our students' and our own epistemologies.* One of our findings about the QR students is that they are weak on productive disposition, i.e. the habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence in one's own efficacy. We work hard to convince students that the mathematics and statistics in the QR course is important to them and that they are capable of understanding it and making use of it in their daily lives.

6. *Experience alone is a poor teacher. Too few examples can situate learning. Many learners don't know the quality of their comprehension and need systematic and corrective feedback.* The use of authentic cases can point out to students the consequences of various conclusions in real-life situations. The feedback can convince students that their experiences are not conclusive and push them to consider other alternatives.

7. *Lectures work well for learning assessed with recognition tests, but work badly for understanding.* We discourage extensive lecturing, relying more on just-in-time mini-lectures to address a needed concept.

8. *The act of remembering itself influences what learners will and will not remember in the future. Asking learners to recall particular pieces of information (as on a test) that have been taught often leads to "selective forgetting" of related information that they were not asked to recall.* We try to avoid prescribing certain facts and processes that students need to remember at the expense of others. Identifying a few conceptual frameworks that have broad application would alleviate the possibility of promoting "selective forgetting."

9. *Less is more, especially when we think about long-term retention and transfer. Restricted content is better.* The mathematical and statistical methods in the QR course are quite restricted but broadly applicable. Mathematical formulas or concepts are developed only if there is an immediate reason, and most of those developed have broad applications to QR.

10. *What learners do determines what and how much is learned, how well it will be remembered, and the conditions under which it will be recalled.* We keep the admonition that the mind remembers what it does in front of all our instruction. Collaborative inquiry-based learning is a major theme of the course.

Final Thoughts

Our QR course was not designed with the principles listed above explicitly stated. Nor was it designed in consideration of any of the six sets of criteria, except perhaps the research results on human cognition, which were reasonably well known to me as I designed the course (with colleagues Dingman, Boersma, and Diefenderfer) over the past eight years. And looking at the result in light of the six sets of criteria has no doubt influenced forming the ten design principles that we now recognize. The qualitative evidence that the design principles of the course align reasonably well with most of the principles in the six sets of criteria is a good starting point for a more rigorous evaluation of the course. The alignment is far from perfect. As noted earlier we have two unresolved alignment issues:

1. What contextual examples should be generalized and abstracted to take advantage of the power of mathematics?
2. What are the conceptual frameworks for QR?

The alignment with the QL core competencies is understandably strong since these are competencies for QR. The alignment with the five strands of mathematical proficiency is stronger than that with the practice standards of CCSSM, which are attuned more to traditional mathematics proficiency. Alignment with the *Five Elements* of Burger and Starbird (2012) seems reasonably strong, but the explication of these in their book by the authors points clearly to the personal pedagogies of the authors, so alignment here is likely to depend more on the implemented course. Alignment with the principles from *How People Learn* (Bransford, Brown, and Cocking, 2000) and those articulated by Halpern and Hakel (2003) is probably the strongest of all, and this might be surprising except for the fact noted above that I knew of these principles before I began designing and teaching the QR course. We have sprinkles of other evaluative evidence, some of it quantitative – surveys of faculty advisors, student evaluations, some pre- and post-test data, and some follow up survey data of former students. Most of the evidence appears to support the conclusion that the design of the course supports strong learning by QR students. However there are uncertainties. One is that we cannot be sure how well aligned the implemented course is with the designed course. With most of our instructors inexperienced in leading this kind of course, implementation can vary from design. We are currently reviewing the design rather thoroughly and plan to implement professional development programs for QR instructors. Until there are assessment instruments that are reliable measures of long term retention and transfer or QR habits of mind, we will continue to rely on qualitative evidence of alignment with accepted principles that apply to QR learning. Accepted principles is a fairly high standard as indicated by the six sets of criteria here, and not the "well-intentioned feel-good psychobabble about teaching out there that falls apart upon investigation of the validity of the supporting evidence," as we quoted from Halpern and Hakel (2003) in the introduction. Such alignment with accepted principles adds some concurrent validity to the face validity of the course we teach.

The validity of the QR course as now taught and possible future innovations were both enhanced by the discussions at the STEM QR Symposium discussed in the introduction. Most of the issues faced in the evolution of the course involve the major themes of the Symposium: size and scaling, unitizing, quantification, proportional reasoning, learning progressions, use of technology, modeling, and mathematics as a lived experience. In spite of its dispersion across most of education and all of contemporary society, there seems to be a coalescing of understanding of the elements of effective QR.

References

Association of American Colleges and Universities (AAC&U). (2004). Issue of *Peer Review* dedicated to quantitative literacy. *Peer Review*, *6*(4).

Association of American Colleges and Universities (AAC&U). (2009). Assessing learning outcomes: Lessons from AAC&U's VALUE project. *Peer Review*, Winter 2009, *11*(1). Washington, DC: Association of American Colleges & Universities.

Baroody, A. J., Reid E. E., & Purpura, D. P. (2012). An example of a hypothetical learning progression: The successor principle and emergence of informal mathematical induction. Expert Reading, An International STEM Research Symposium: Quantitative Reasoning & Science Education, Savannah, GA, May 31- June 2. http://coe.georgiasouthern.edu/QR/Learning%20Progression%20-%20Baroody.pdf

Best, J. (2008). *Stat-spotting: A field guide to identifying dubious data*. Berkeley and Los Angeles, CA: University of California Press.

Blair, R. (Ed.). (2006). *Beyond crossroads: Implementing mathematics standards in the first two years of college.* Memphis, TN: American Mathematics Association of Two-Year Colleges. Available at beyondcrossroads.amatyc.org/doc/PDFs/BCAll.pdf

Boersma, S., Diefenderfer, C. L., Dingman, S. W., & Madison, B. L. (2011). Quantitative reasoning in the contemporary world, 3: Assessing student learning. *Numeracy 4*(2), Article 8.

Bransford, J. D., Brown, A. L., & Cocking, R. R. (Eds.) (2000) *How people learn: Brain, mind, experience, and school*. Washington, DC: National Academies Press.

Burger, E., & Starbird, M. (2012). *The 5 elements of effective thinking*. Princeton, NJ: Princeton University Press.

Common Core State Standards for Mathematics (CCSSM). (2010). Washington, DC: National Governors Association and The Council of Chief State School Officers. Available at: http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Dingman, S. W., & Madison, B. L. (2010). Quantitative reasoning in the contemporary world, 1: The course and its challenges. *Numeracy 3*(2): Article 4. http://services.bepress.com/numeracy/vol3/iss2/art4/

Duschl, R. A. (2012). Quantitative reasoning & mathematical modeling in STEM: A commentary. Expert Reading, An International STEM Research Symposium: Quantitative Reasoning & Science Education, Savannah, GA, May 31- June 2. http://coe.georgiasouthern.edu/QR/Quantitative%20Reasoning%20-%20Duschl.pdf

Gillman, R. (2006). *Current practices in quantitative literacy*. Washington, DC: Mathematical Association of America

Hacker, A. (2012). Is algebra necessary? *New York Times*, 29 July.

Halpern, D. F., & Hakel, M. D. (Eds.) (2002). *Applying the science of learning to the university and beyond.* New York, NY: Jossey-Bass.

Halpern, D. F., & Hakel, M. D. (2003). Applying the science of learning to the university and beyond." *Change*, July/August issue.

Jones, M. G. (2012). Conceptualizing size and scale. Expert Reading, An International STEM Research Symposium: Quantitative Reasoning & Science Education, Savannah, GA, May 31- June 2. http://coe.georgiasouthern.edu/QR/Concepts%20of%20Size%20and%20scale%20-%20Jones.pdf

Lehrer, R. (2012) A learning progression emerges in a trading zone of professional community and identity. Expert Reading, An International STEM Research Symposium: Quantitative Reasoning & Science Education, Savannah, GA, May 31- June 2. http://coe.georgiasouthern.edu/QR/Trading_Zone_LP-Lehrer.pdf

Luitel, B. C. (2012) A multi-paradigmatic approach to researching lived mathematical experiences. Expert Reading, An International STEM Research Symposium: Quantitative Reasoning & Science Education, Savannah, GA, May 31- June 2. http://coe.georgiasouthern.edu/QR/Math%20Lived%20Experience%20-%20Luitel.pdf

Madison, B. L. (2001). Quantitative literacy: Everybody's orphan. *MAA FOCUS* (6), 10-11.

Madison, B. L., & Steen, L. A. (Eds.) (2003). *Quantitative literacy: Why numeracy matters for schools and colleges.* Princeton, NJ: National Council on Education and the Disciplines. http://www.maa.org/ql/qltoc.html

Madison, B. L., & Dingman, S. W. (2010). Quantitative reasoning in the contemporary world, 2: Focus questions for the numeracy community. *Numeracy*, 3(2). Article 5. http://services.bepress.com/numeracy/vol3/iss2/art5/

Madison, B. L., Boersma, S., Diefenderfer, C. L., & Dingman, S. W. (2012). *Case studies for quantitative reasoning: A casebook of media articles*, 3rd ed. New York, NY: Pearson.

Madison, B. L., & Steen, L. A. (Eds.) (2008). *Calculation vs. context: Quantitative literacy and its implications for teacher education*. Washington, DC: Mathematical Association of America.

Madison, B. L., & Steen, L. A. (2008). Evolution of numeracy and the national numeracy network, *Numeracy*: Vol.1: Iss.1, Article 2. Available at: http://scholarcommons.usf.edu/numeracy/vol1/iss1/art2

Mathematical Association of America (MAA). (2004). *Undergraduate programs and courses in the mathematical sciences: CUPM curriculum guide 2004.* Washington D.C.: author. Available at www.maa.org/cupm/cupm2004.pdf.

Mayes, R., Bonilla, R., & Peterson, F. (2012). Quantitative reasoning: Current state of understanding. An International STEM Research Symposium: Quantitative Reasoning & Science Education, Savannah, GA, May 31- June 2. http://coe.georgiasouthern.edu/QR/QR%20Overview%20-%20Mayes,%20Peterson,%20Bonilla.pdf

Miller, J. E. (2004). *The Chicago guide to writing about numbers*. Chicago, IL: The University of Chicago Press.

Mumford, D., & Garfunkel, S. (2011). How to fix our math education. *New York Times,* 24 Aug. www.nytimes.com/2011/08/25/opinion/how-to-fix-our-math-education.html

National Council of Teachers of Mathematics (NCTM). (2000). Principles and standards for school mathematics. http://www.nctm.org/standards/content.aspx?id=16909

Olive, J. (2012). Critical themes in the teaching and learning of mathematics with technology. Expert Reading, An International STEM Research Symposium: Quantitative Reasoning & Science Education, Savannah, GA, May 31- June 2. http://coe.georgiasouthern.edu/QR/Technology%20Critical%20Issues%20-%20Olive.pdf

Schwarz, C. (2012). Supporting scientific practices in elementary and middle school classrooms: The role of quantitative reasoning. Expert Reading, An International STEM Research Symposium: Quantitative Reasoning & Science Education, Savannah, GA, May 31- June 2. http://coe.georgiasouthern.edu/QR/Schwarz_Quantitative_Reasoning_Paper_Final.pdf

Steen, L. A. (Ed.) (1997). *Why numbers count: Quantitative literacy for tomorrow's America.* New York, NY: The College Board.

Steen, L. A. (Ed.) (2001). *Mathematics and democracy*. Princeton, NJ: National Council on Education and the Disciplines.

Steffe, L. P. (2012A). Delphi study. Expert Reading, An International STEM Research Symposium: Quantitative Reasoning & Science Education, Savannah, GA, May 31- June 2. http://coe.georgiasouthern.edu/QR/Delphi_Study_KU-Steffe.pdf

Steffe, L. P. (2012B). On the construction of units and their systems in quantitative reasoning. Expert Reading, An International STEM Research Symposium: Quantitative Reasoning & Science Education, Savannah, GA, May 31- June 2. http://coe.georgiasouthern.edu/QR/Construction%20of%20Units%20-%20Steffe.pdf

Thompson, P. W. (2012). Advances in research on quantitative reasoning. Expert Reading, An International STEM Research Symposium: Quantitative Reasoning & Science Education, Savannah, GA, May 31- June 2. http://coe.georgiasouthern.edu/QR/Advances%20in%20Research%20on%20Quantitative%20Reasoning%20-%20Thompson.pdf