New Technologies for Developing Quantitative Reasoning: Opportunities and Challenges

John Olive
The University of Georgia

The available technologies for teaching and learning, both in and out of school have expanded tremendously during the first decade of this century. Alongside computers and calculators we have iPods, iPhones, and now iPads; hand-held computing devices such as the TI-nSpire (which operates more like a computer than a calculator); networked calculators; wireless response systems; scientific probes that can be connected to hand-held devices or computers for generating real data in real time (CBL's and CBR's); Interactive SMART Boards and now, interactive SMART Tables for collaborative problem solving activities. Research at the *Kaput Center* is investigating the use of haptic technology, particularly force feedback devices, to enable users of various ages to explore the properties of 2D and 3D computer-generated objects with various senses. The explosion in web-based resources for finding information, for social networking, for entertainment and for collaborative problem solving in on-line communities has changed the way we live our lives – outside of school. Perhaps one powerful reason for why almost a third of the students entering high schools in the USA "drop out" before completing their high school diploma (Gonzalez, 2010) is that education in many schools is presented in the same way as it was in the 19^{th} and 20^{th} centuries. The educational process in school bares little resemblance to how people learn outside of school.

As educators, we need to investigate how children and young adults are making use of the technological environment in which they live and what they are learning from that use. As mathematics educators, we need to understand how we might harness this technological environment to enhance the learning and teaching of mathematics – both in-school and out-of-school.

This paper was commissioned as an expert presentation to the TTAME (Technological Tools for Advancing Mathematics Education) working group at the *International STEM Research Symposium: Quantitative Reasoning in Mathematics and Science Education*, organized jointly by Georgia Southern University Office of Research, the NSF Pathways Project at Colorado State University and the WISDOM[e] center at the University of Wyoming. The conference was held in Savannah, Georgia from May 31 through June 2, 2012. In this paper, I focus on technologies that have the potential to enhance quantitative reasoning in mathematics education, which Pat Thompson has pointed out in his paper (pp xxx) has a different focus from the quantitative reasoning in science:

> In science, once you have a model of a physical system that entails its quantification, you use the model to delve further into the physical system in order to understand it better. In mathematics, once you have a model of a physical system that entails its quantification, you begin to explore properties of the model itself, its structure, ways in which the model can be generalized, and implications for its mathematical properties of loosening or tightening its assumptions. The physical system that gave rise to the model fades over time. (Thompson, this volume, p. xx)

Quantitative reasoning in both realms, however, involves a "dialectic among conceiving an object, conceiving its attribute, and conceiving the attribute's quantification." (Thompson, this volume, p. xx). The technological innovation may enhance (or not) one or more aspects of this dialectic.

In my Plenary talk at the first WISDOM^e conference (Olive, 2010) I posed the following questions with respect to areas of needed research on the use of technology in mathematics teaching and learning:

- *Learning:* How and what do students learn through use of technology?
- *Teaching:* How and what do teachers teach using technology?
- *Curriculum:* What mathematics can and should be accessible through the use of technology?
- *Design of Technology:* How does the specific interface design of a technology impact its use?
- *Use of technology:* Actual use may differ from the designed use – how do the different uses affect learning and teaching outcomes?
- *New Media for learning:* New networking and social interaction technologies offer new media for learning both inside and outside the classroom. How and what kind of learning may take place in these new media?
- *New Media for teaching:* New networking and social interaction technologies offer new media for teaching both inside and outside the classroom. How and what kind of teaching may take place in these new media?

My own focus is shifting from research in the context of the classroom as it exists (based on 19th and 20th century technologies) to the questions relating to *New Media for Learning and Teaching*.

New Media for learning: David Shaffer and colleagues (Shaffer, Squire, Halverson and Glee, 2008) suggested the need for a new model of learning in a world dominated by these new technologies:

> The past century has seen an increasing identification of learning with schooling. But new information technologies challenge this union in fundamental ways. Today's technologies make the world's libraries accessible to anyone with a wireless PDA. A vast social network is literally at the fingertips of anyone with a cell phone. As a result, people have unprecedented freedom to bring resources together to create their own learning trajectories. But classrooms have not adapted. Theories of learning and instruction embodied in school systems designed to teach large numbers of students a standardized curriculum are antiquated in this new world. Good teachers and good school leaders fight for new technologies and new practices. But mavericks grow frustrated at the fundamental mismatch between the social organization of schooling and the realities of life in a post-industrial, global, high-tech society (Sizer, 1984). While the general public and some policy makers may not have recognized this mismatch in the push for standardized instruction, our students have. School is increasingly seen as irrelevant by many students past the primary grades. (p. 17)

Because of this perception by many students that school is irrelevant in this technological milieu, I believe it is critical for us as researchers and practitioners to exam how learning and teaching could be different in these new media. Hoyles et al. (2009) pose the following questions regarding these new interactive media:

- What is the potential for creating virtual communities for mathematics learning and permitting communication between individuals from different educational settings?

- What is the potential contribution to mathematics learning of different levels of interactivity and different modalities of interaction, and how might this potential be realized?
- What is special about the potential of collaborative study of mathematics whilst physically separated, and how might this potential be harnessed so as to support mathematics learning? (p. 440)

The media themselves offer new methodologies for investigating such questions. Because these interactions take place in a digital medium, they can be easily recorded or catalogued for many purposes. Indeed, this is already happening with our use of credit and debit cards and the electronic scanning of all of our purchases. Every single item we purchase from the grocery store adds to our profile in the corporate database, informing the grocery chain of our specific preferences and using this information to focus promotional coupons and email messages specifically for us.

Students' interactions in a digital learning (or gaming) medium could also be recorded and cataloged in ways that could provide the researcher with data for rich analyses of learning trajectories, modes of communication and collaboration, the emergence of different problem solving strategies, as well as patterns of actual use. Shaffer's (2006) theory of *epistemic frames* that characterize the situated understandings, effective social practices, powerful identities, shared values, and ways of thinking of important communities of practice can be a useful framework for analyzing these digital records. In their paper on video games and the future of learning, Shaffer et al. (2008) make the following point:

To build such games requires understanding how practitioners develop their ways of thinking and acting. Such understanding is uncovered through epistemographies of practice: detailed ethnographic studies of how the epistemic frame of a community of practice is developed by new members. That is more work than is currently invested in most "educational" video games. But the payoff is that such work can become the basis for an alternative educational model. (p. 12)

New Media for teaching: New networking and social interaction technologies offer new media for teaching both inside and outside the classroom. Research on how and what kind of teaching may take place in these new media have focused primarily on distance learning techniques. When distance learning was first introduced, it mimicked the face-to-face lecture modality and was seen as a poor substitute for the real thing. Distance learning is rapidly evolving into a dynamic medium for engaging students at a distance, in both synchronous and asynchronous modes. Social networking platforms, such as "Second Life," are being used by universities to create virtual learning communities in which students and teachers interact via on-screen avatars. Undertaking Shaffer's epistemographies of practice in such virtual teaching and learning communities could help us understand how teaching is transformed in these new media.

My own framework for studying both teaching and learning in these new media takes into account the role of the teacher (or more experienced other) in the didactical situations made possible by the integration of technology. In Chapter 8 (Olive and Makar, 2009) of the 17th ICMI study volume *Mathematics Education and Technology—Rethinking the Terrain,* edited by Celia Hoyles and Jean-Baptiste Lagrange (2009), my co-authors and I focus on the mathematical knowledge and practices that may result from access to digital technologies. We put forward a new tetrahedral model derived from Steinbring's (2005) didactic triangle (see Figure 1) that integrates aspects of instrumentation theory (Verillon & Rabardel, 1995) and the notion of semiotic mediation (Saenz-Ludlow & Presmeg, 2006). This new model illustrates how interactions among the didactical variables: student, teacher, task and technology (that form the vertices of the tetrahedron) create a space within which new mathematical knowledge and practices may emerge. We place the student at the top of this tetrahedron as, from a constructivist point of view, the student is the one who has to construct the new knowledge and develop the new practices, supported by teacher, task and technology. Hollylynne Lee uses our framework in her Plenary Paper to frame her work on probability, data and statistical reasoning (Lee, this volume, pp. xxx). In the following section I will present examples of a variety of these new technologies that are being developed for use both in school and out-of-school. All of these examples share the main theme of this conference: they are designed to promote *quantitative reasoning* in the realm of mathematical learning and teaching.

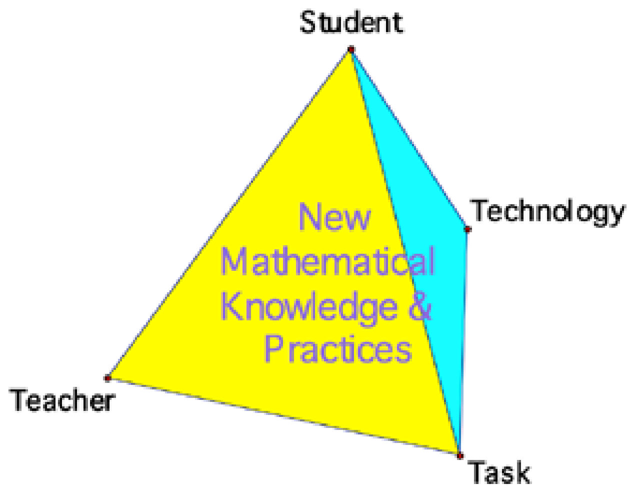

Figure 1: The Didactical Tetrahedron (from Olive & Makar, 2009, p. 169)

Examples of New Media and Technologies for Developing Quantitative Reasoning

Single & Multi-Player web-based gaming environments (New Media for Learning?)

Shaffer et al. (2008) urge the educational research community to look at video and web-based games,

> Not because games that are currently available are going to replace schools as we know them any time soon, but because they give a glimpse of how we might create new and more powerful ways to learn in schools, communities, and workplaces—new ways to learn for a new information age. (p. 2)

Currently there is a large gulf between educators who try to design fun learning games and gamers who try to add learning in fun games. Neither group has met with much success. What is needed is a genuine collaboration between these two groups.

I am currently involved in just such a collaboration (as part of a team of "subject matter experts") with game design teams working for *The Walt Disney Company.* The first result of our collaboration is now available for children online in Disney's *Club Penguin.* It is worth comparing the actual use of this on-line virtual, world-wide community with an on-line gaming environment, *Calculation Nation,* created by The National Council of Teachers of Mathematics (NCTM) in the United States:

Calculation Nation (free): 4 or 5 players on line at any one time.

Club Penguin (subscription required): More than 30 million children have played *Pufflescape* over the past two years.

The contrast is staggering! *Calculation Nation* does have challenging games but their fun appeal is limited. Club Penguin has lots of fun activities but the learning potential of these activities is still to be determined. Disney has developed a *Parent App* that provides parents with data about their child's play in Club Penguin. The data and feedback let parents know which places and which games their children have visited and also what challenges their child has completed in those environments. The portal provides parents with educational suggestions for helping their child get the most (educationally) from their time online.

The Parent App is a product of *Disney Connected Learning (DCL)*, a collaboration among game designers and educators that attempts to integrate learning trajectories based on an inter-connected knowledge map that links over 5000 concepts in mathematics, science, language arts, social studies, music and art. DCL has recently released five new games specifically designed for mobile devices such as iPads and iPhones that cover about 5% of these concepts. Their goal is to develop games that will encompass at least 50% of the knowledge map. Information about DCL can be found at http://connectedlearning.disney.com/. An article on DCL can be found here: https://www.edsurge.com/n/2013-02-06-disney-connected-learning-aims-to-infuse-games-with-learning

GapMinder: Dynamic Data Visualization of Multi-variable data sets

In her Plenary and her paper in this volume, Hollylynne Lee uses *GapMinder* as an example of a dynamic visualization tool for complex data. This free internet-based tool is available from http://www.gapminder.org/. There is also a free laptop application version that can be used without an internet connection. However, the databases for the application are not as up to date as the internet databases. Some schools are beginning to make use of this dynamic data visualization tool for students to explore the time-sequenced data bases to answer their own data driven questions. One such school in the USA is the New York City iSchool (NYC iSchool). The following quote is taken from their website -- https://sites.google.com/a/nycischool.org/gapminder/about-gapminder-at-the-ischool:

> Gapminder at the iSchool is an experimental course that challenges 10th and 11th grade students to use a quantitative lens to analyze the last 200 years of global history. Students look for trends and meaningful changes with the data visualization tools on Gapminder.org. They create research questions, work through the research process, draft papers, and present their findings at the end of the course. The purpose of the course is for students to develop the following skills:
>
> 1. Data analysis
> 2. Quantitative reasoning
> 3. Research process
> 4. Writing and argument
> 5. Public speaking
>
> The course is designed and taught by Jesse Spevack, (jspevack@mail.nycischool.org) the Assistant Principal of the NYC iSchool.

In the TTAME working group sessions at the Savannah Conference we explored some questions related to education and technology using GapMinder. The following screen shot shows the rapid rise of internet use from 1990-2008. I have tracked the rise in use for both Turkey and the USA. You might ask, which country is at the top of the graph in 2008 (the tiny orange dot at 90% of the population)? You will probably be very surprised!

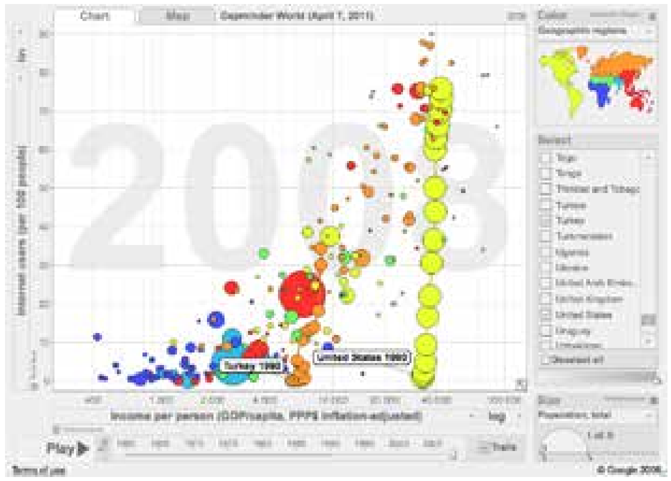

Figure 2: Internet use across the world from 1990-2008

The vertical axis on the GapMinder chart in Figure 2 shows the number of internet users per 100 people. The horizontal axis shows the income per person (adjusted for inflation). It is important to realize that this scale is a logarithmic scale where each linear unit on the scale indicates a doubling of income. The size of each bubble is proportional to the country's population. The time-sequence data is revealed as an animation by clicking the *Play* button at the bottom left of the chart. All of the bubbles move from their positions in 1990 through each year in a continuous motion through 2008. I tracked the motion for both Turkey and the USA in this screen shot (follow the connected orange and yellow bubbles as they shoot up almost vertically over the time span). Not surprisingly, we see an almost exponential trend by the year 2008 between internet use and income/person. That is, the higher the income/person in a country, the higher the number of people using the internet per 100 people in that country.

As education researchers, we should be asking questions about the potential for what and how students might learn when given access to such a powerful visualization tool. The work that students are producing at NYC iSchool provides us with some initial answers to such questions. According to the NYC iSchool GapMinder web site:

> The purpose of the course is not to teach content, but rather develop a life-long skill set in students. Students learned to analyze data using Gapminder.org data visualization. They developed a framework of questions that can be asked when examining any graphic representation of data. Students then learned how to seek out answers to their questions and publish their findings in the form of a research paper. (https://sites.google.com/a/nycischool.org/gapminder/student-work)

These are, indeed, necessary 21st Century Skills in the area of quantitative reasoning that we all need to develop in this era of information overload.

Dynamic Number Tools for Elementary Students

KCP Technologies (developers of Dynamic Geometry and Dynamic Statistics software) has been awarded a research and development grant from the National Science Foundation (NSF) to develop *Dynamic Number* (DN) tools for students and teachers in elementary and middle school. In the NSF proposal for the project, Scher and Rasmussen (2009) make the following points:

> Currently, Dynamic Number ideas only exist in highly controlled, narrowly content-focused "applet"- like incarnations. Alas, these interactive models are useful only at the rarest triple concurrence of technological availability, curricular relevance, and student need. Furthermore, they are capable of being built only by those equally rare individuals who combine curriculum development expertise with sufficient technological prowess and appropriate professional contexts to pursue such work. If, instead, these ideas became infrastructure in a general-purpose mathematical tool accessible not only to curriculum developers but to students and teachers, **the educational, technological, and social school conditions are ripe for Dynamic Number technology to have broad and transformative impact, at a national scale, on students' mathematical understanding and performance relating to core number constructs, elementary number theory, and early algebra ideas across the grades 2–8 curriculum.** (p. 2 emphasis in the original)

Actualizing this potential is the goal of the Dynamic Number Project. During the first year of the project, 19 teachers in grades 2 through 8 were recruited across the USA and were provided with six weeks of on-line training in the use of *The Geometer's Sketchpad* (GSP 5) and initial prototypes of Dynamic Number tools constructed in GSP. In the summer of 2010, four more teachers in Croatia joined the project. My role in the project is as director of the evaluation component.

The project is now in its third year and most of the initial classroom implementation of the prototype tools designed using GSP has been completed. The project development team is now focusing on creating a more child-friendly and teacher-friendly user interface for the tools and adapting them for web-based and iPad implementation. I would like to share a couple of the GSP prototypes in this paper that we explored in the TTAME working group. The first one is the Color Calculator, and the second is Zooming Decimals. Both of these tools were designed to help students construct a deep understanding of the relation between rational numbers and decimals, and the denseness of the rational numbers as expressed in decimal notation.

I have chosen the Color Calculator because of its possible link to research concerning connections among number development, color and spatial visualization (Seron, Pesenti, Noel, Deloche, and Cornet, 1992) and more recently neuroimaging research that supports those connections (Terao, Koedinger, Sohn, Qin, Anderson, & Carter, 2004) and the role that color and shape plays in the amazing numerical abilities of an autistic savant (see Born on a Blue Day, by Daniel Tammett, 2007).

The Color Calculator assigns a different color for each of the digits 0-9 and displays the result of a fraction less than one to as many decimal places as can fit up to a 20x20 grid (i.e. 400 places). The number of rows and the number of columns in the grid can be changed by dragging key points in the sketch, thus enabling the user to detect color patterns in the display. Figures 3 and 4 show displays of the decimal representation of 1/7, first with ten rows and ten columns, and then with ten rows and six columns. In Figure 3 it is hard to discern any kind of pattern but in Figure 4 the six repeating decimal digits line up to show six columns, each a different color.

The color sequence (using the key) shows that the repeating decimal for 1/7 is .142857, but what is even more amazing to discover is that as the numerator is increased by 1, all the way to 6/7, these digits cycle in order, with the leading digit changing as follows: 1, 2, 4, 5, 7 and 8. So every proper fraction of sevenths has exactly the same six repeating digits in the same order, but starting with a different leading decimal. Thus the repeating decimal for 6/7 is .857142.

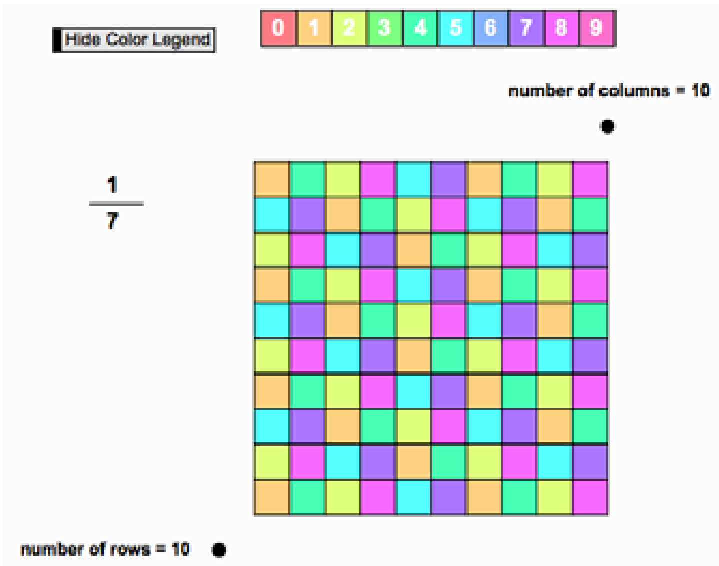

Figure 3: Color Calculator showing 1/7

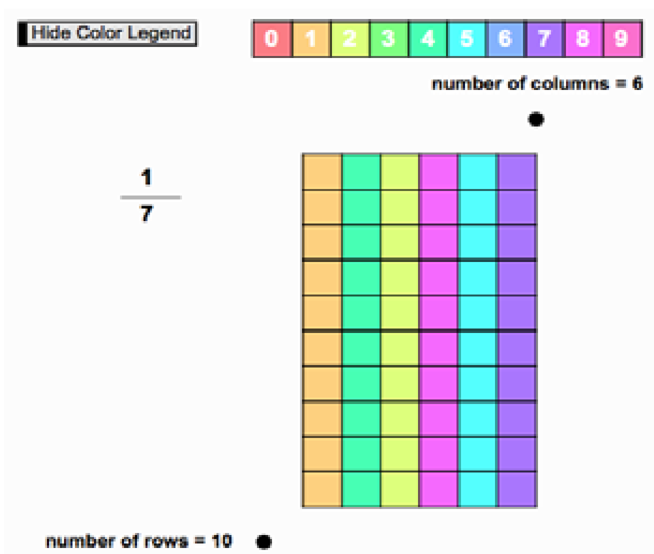

Figure 4: Color Calculator showing 1/7 with six repeating decimals

Why do I see such pattern discovery in the decimal representation of a fraction as being important for quantitative reasoning? Decimals are used as the representation of a numerical quantity in most technological devices used by STEM professionals but the use of fractions is the more natural way for children to express a relationship between two quantities. Finding ways to relate these two representations of the same quantitative relationship could help students make sense of the data provided by the tools that professionals use.

Zooming Decimals

The Zooming Decimals tool is designed to enable the user to zoom in on a section of a number line between two integers in order to specify the decimal representation of a target point between those two integers. At each level of zoom, the interval is expanded to the length of the original segment of the number line (which has ten integer intervals). Thus each zoom magnifies the interval by a factor of ten (see Figure 5).

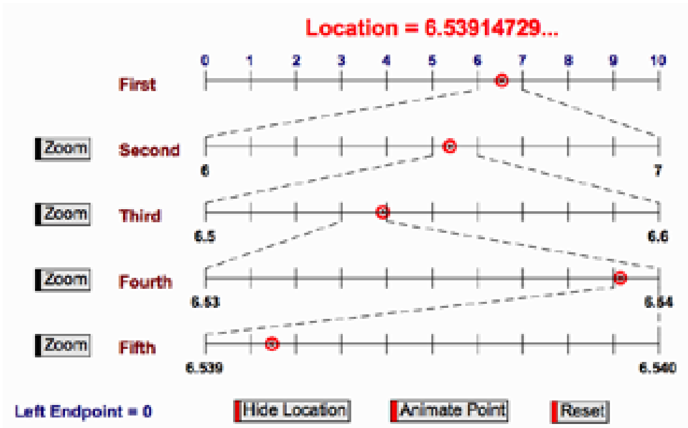

Figure 5: Zooming Decimals after the fifth zoom

Before each zoom, the students should make an estimate for the next decimal place in the number (the location is hidden to begin with). The first estimate would probably be around 6.5. The second estimate might be a little more difficult as the target ends up very close to 6.54. However, the really exciting part comes when the student clicks (or taps on the iPad) the *Animate Point* button. This causes the point on the last zoom (bottom line segment) to move across its line segment from left to right. When it reaches the right end point, that becomes the value of the new left end point (in Figure 5 the left end-point of the bottom line changes to 6.540 and the right end-point to 6.541). The bottom target point continues to moves in this way until the animation is stopped. All of the other points are also changing their positions relative to the bottom point. The point on the fourth zoom actually moves at one tenth the speed of the bottom point, the point on the third zoom at one hundredth the speed of the bottom point. I challenge the reader to figure out how long it would take the top target point to reach the number 7 if it takes the bottom point one second to go from the left to the right end!

Zooming Decimals was designed to give the user a sense of the *density* of the real number system. This notion of density is critical in all measurements of continuous quantities. Students need to realize that every measurement made by any instrument has a margin of error. Zooming decimals uses a point on a number line to represent the target of a measurement and illustrates very powerfully how one can get closer and closer to the *measure* of this point (its distance from zero) by repeatedly zooming in on the location of the point as represented by its decimal notation. The final animation (and the challenge I posed with respect to that animation) illustrates the power structure of each place in a decimal representation and how this power structure can be illustrated by the differing speeds of each point.

Multi-touch Apps for the iPad

The iPad and other multi-touch electronic tablets are becoming more and more accessible both in and out of school. Several elementary and secondary schools in the USA have classroom sets of iPads for students to use. Many middle-class families in the USA own an iPad. Gaming and Learning companies are rapidly producing *Apps* that are free or very inexpensive to download for individual use. The potential for learning for even very young children using this multi-touch technology is yet to be determined but I will share some potentially powerful positive examples and some potentially very damaging examples! I'll begin with what I consider to be some potentially positive uses of this technology.

SketchExplorer from KCP Technologies

SketchExplorer® is a free App from KCP Technologies that allows the user to explore any *Geometer's Sketchpad*® (GSP) sketch on the iPad. KCP Technologies and a large community of GSP users are currently developing GSP sketches tailored to the iPad. Both of the GSP sketches illustrated above in the *Dynamic Number* Project can be used successfully on the iPad. Following are two more examples that were specifically designed for iPad use:

Ex: *Bunny Times* (KCP Technologies)

- Bunny Times was designed to help young children (ages 7 – 9) develop their multiplication facts and strategies for solving multiplicative situations using the distributive property of multiplication over addition. The context for the problems is a farmer's field of carrots and a bunny (or team of bunnies in higher levels) who wants to eat all the carrots. The carrots are arranged in rows, with the same number of carrots in each row. In the basic levels there are a maximum of 6 rows with a maximum of 6 carrots per row. In the advanced levels there are a maximum of 12 rows with a maximum of 12 carrots per row (see Figure 6).

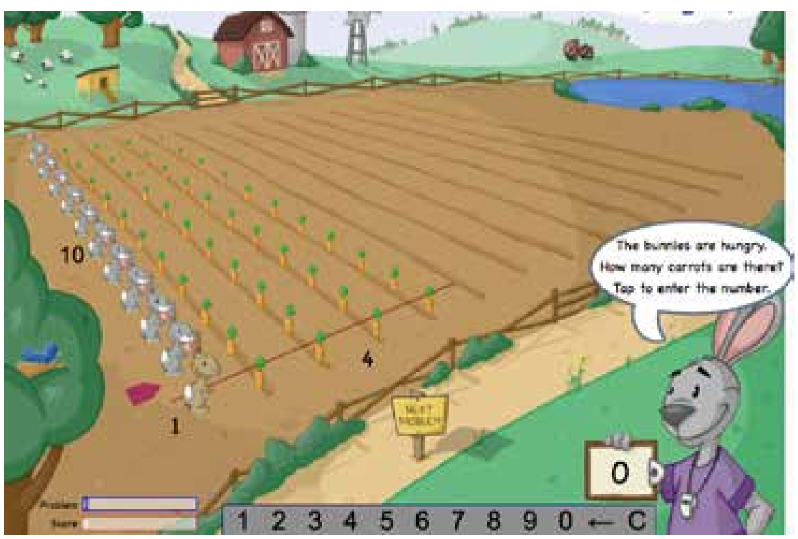

Figure 6: Screen shot from *Bunny Times*

The App presents problems in a logical sequence to enable children to use what they have just solved to answer the next problem. The levels also advance from a basic level where children can solve the problem by counting individual carrots to levels where some of the rows of carrots are hidden by a mist or cloud and onto higher levels where a team of bunnies eat the carrots simultaneously (encouraging the use of "counting by" strategies). These higher levels include the use of a moveable border to divide the bunnies into two teams. Each team of bunnies then eat the carrots in two sequential acts, thus leading to solving the problem through use of the distributive property. In the screen shot shown in Figure 6, the problem presents 4 rows of carrots with 11 carrots in each row. There are a total of 11 bunnies that have been split into two teams using the moveable border: a team of 10 and a single bunny. The child could solve this problem by counting by tens first to get 40 and then by ones to get 4 more carrots.

I have used this iPad App with below-level 10-year old children in a small rural elementary school in Georgia. The two children were completely engaged in solving the problems and "fought" over who gets to go next!

Ex: *Area of a Circle* (my own contribution)

- This GSP sketch, designed to be explored on the iPad using *SketchExplorer*, links a representation of a circle divided into a certain even number of sectors with a rearrangement of those sectors to create a "curvy" parallelogram. Half of the sectors are in one color and half in another (see Figure 7).

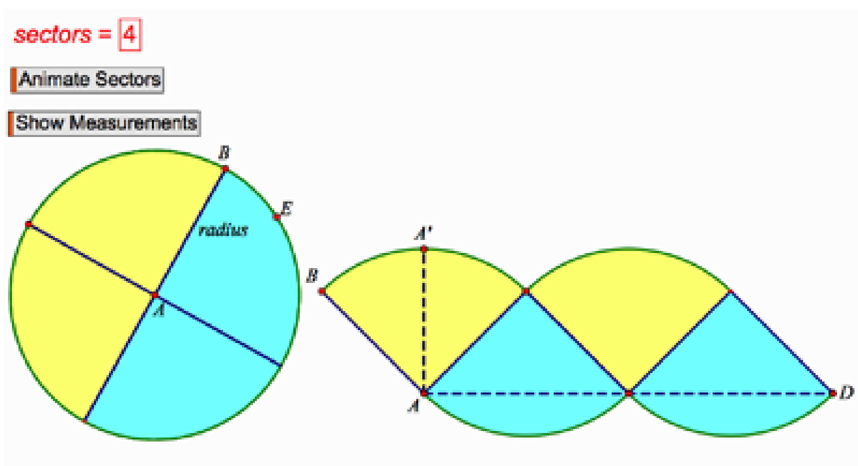

Figure 7: iPad GSP sketch to investigate the area of a circle.

- By tapping the *Animate Sectors* button, the number of sectors increases discretely by two up to 100 and then starts descending back down to four. The animation can be stopped at any time by tapping the button a second time.
- Tapping the *Show Measurements* button will display the length of the radius of the circle (AB), the height of the parallelogram (AA'), the width of the parallelogram (AD) and calculations involving the product of these last two measures as well as the value for $\pi \cdot radius^2$ (see Figure 8).\

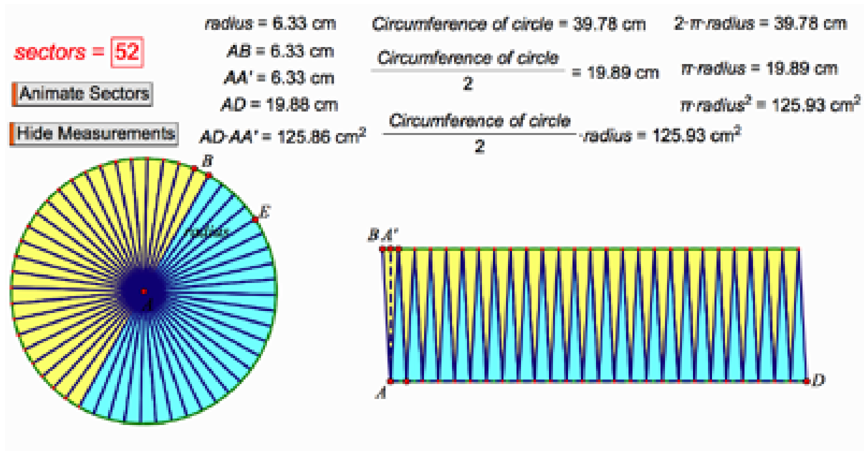

Figure 8: After animating sectors to 52 and showing measurements

- As you can see in Figure 8, the "curvy" parallelogram is approaching the shape of a rectangle with height the radius of the circle and width half the circumference. The value for width•height (AD•AA') is approaching the value (Circumference of circle/2)•radius, which is the same as π•radius^2.
- The red points on the circle are moveable points (tap and drag). These points can be used for changing the size and position of the circle, thus enabling students to discover that the relationships are independent of the size of the circle.

When using this App with students, I first have them draw sectors on cardboard circles and cut them out and rearrange the sectors to actually create a "curvy" parallelogram from their own circles. I have them measure the width and height of their curvy parallelograms and compare the product of these two measures to the product of the radius and half their circle's circumference (determined either by rolling their cardboard circle across a sheet of paper or using string to surround the circle). What their exploration with the App adds to this initial exploration, is the ability to see a limiting process as the number of sectors increases. This limiting process is impossible to replicate with physical circles but very easy to see and understand using the iPad app (or computer sketch).

Some Warning Signs

With the almost universal access to un-moderated teaching and learning activities via the World-Wide-Web and the ease with which children and adults can engage in these un-moderated and un-evaluated activities using smart phones or iPads there is as much potential for learning mathematics that is either wrong (or detrimental to further mathematical development) as there is for generative learning. I provide one example that I find very disturbing.

Fraction App by Tap To Learn. This iPad app-based learning system is available for download from the *Apple App Store* for 99 cents. It boasts that it is "The Top Selling App to Practice Fractions in a Fun Visual Way" and is "In Use by over 200,000 Students worldwide." (from the *About this app* window). The app also provides "links to the best content available directly via YouTube that is being constantly updated." The first YouTube video that students are linked to is from *Math Made Easy:* http://www.youtube.com/watch?v=7e_JGthEE-Q which provides the first part of an "introduction to fractions." At 4 minutes into this introduction, the narrator makes the following rule concerning numerator and denominator: “the numerator is always the smaller number in fractions that are correct. The denominator is always the larger number in fractions that are correct.” In referring to the fraction ½, the narrator goes on to indicate that if you ever come across a fraction that looks like this: 2/1 “it is incorrect – it is not O.K. It is breaking the rules. It has to turn around so that the larger number is on the bottom and the smaller number is on the top.” (While rewriting 2/1 as ½ -- see Figure 9).

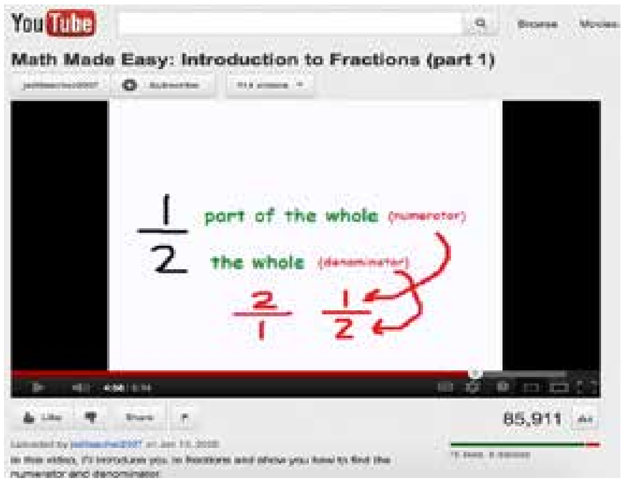

Figure 9: First screen shot from Introduction to Fractions (Part 1)

Some of you may feel that the video is not that bad at this point (even though the narrator just told the viewer to rewrite 2/1 as ½). What comes next, however, is an attempt to brainwash the young viewer so that they would NEVER regard an “improper fraction” as O.K. The narrator introduces “Arnold Schwarzenegger” holding a board over his head with a little baby sitting on top (see Figure 10). The narrator emphasizes that the big guy (the denominator) has to be on the bottom in order to hold up the little baby on top (the numerator).

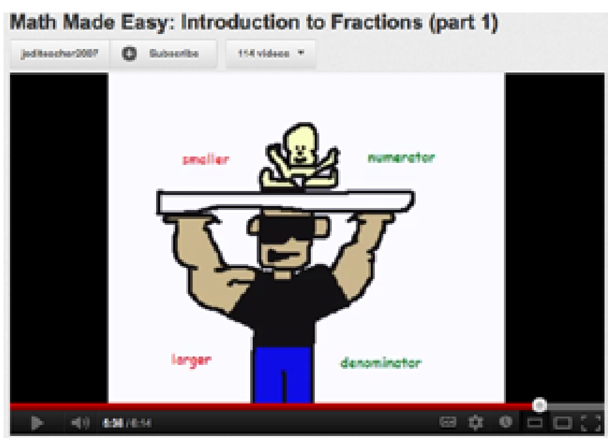

Figure 10: Second screen-shot from Introduction to Fractions (part 1)

At this point in the video, the narrator says that if the larger number "is ever on the top, you've got to switch things around so it is always on the bottom." The narrator continues:

> If the larger number is on the bottom then the baby is happy. If the larger number is on the top, then -- *Hasta la vista, baby!* Yeah, the baby will be crushed – it won't be able to hold this guy up! So, smaller number on the top, larger number on the bottom.

End of video!

As teachers, mathematics educators and as researchers, I believe we have a duty to monitor what is available in these new media and let the unsuspecting public know that some of what is out there can be dangerous to their children's development!

Final Example: A Research-Based Dynamic Manipulative for Developing Children's Fractional Knowledge

Dr. Leslie P. Steffe and I conducted a Teaching Experiment with 12 children (6 pairs) for three years (from the children's third to fifth grade) during the early 1990's. The project, *Children's Construction of the Rational Numbers of Arithmetic* (Steffe and Olive, 1990) was funded by the NSF for 5 years, with a further 3-year grant to continue the data analysis. The culmination of this research was recently published in the book *Children's Fractional Knowledge* (Steffe and Olive, 2010). An integral part of the project was the development of computer-based Tools for Interactive Mathematical Activity (TIMA). These tools were developed using *Object Logo* an object-oriented programing language derived from LISP. Three tools were created: *TIMA: Toys* (which used discrete shapes, called toys, for investigating children's multiplicative structures), *TIMA: Sticks* (which used partitionable line segments to transition from operations with discrete quantities to operations on continuous quantities) and *TIMA: Bars*, which used rectangular regions to further develop children's operations with continuous quantities in two dimensions (see Olive, 2000 for a complete description of the TIMA tools). Unfortunately, Object Logo died with the advent of the Power Mac computer (Mac OS system 8 and above). We did, however, create an amalgam of TIMA: Sticks and TIMA: Bars using the platform independent programming language, Java. The current versions of JavaBars are freely available from my web site: http://math.coe.uga.edu/olive/welcome.html.

The following screen shot from JavaBars illustrates a solution to the following typical problem that we posed to children in the teaching experiment: This bar (red) is 3/5 of my bar. Create my bar:

Figure 11: Making a unit bar from 3/5 of a bar

In order to solve the problem using the actions that are possible with JavaBars, the child could copy the red bar, use the *Parts* buttons to create three equal parts in the copy (the middle blue bar in Figure 11) then *Pullout* two of those three parts and add them to the 3/5 bar (the two purple parts in the middle bar). It is important to note that this *Pullout* action is not possible with physical bars but is only made possible in this digital medium. It is an action that allows students to actualize their mental disembedding operations (taking a part out of a whole while also leaving it in the whole).

The blue bar in the third row of Figure 11 illustrates the result of an iterative approach to solving the problem. Having partitioned the original bar into three equal parts and having pulled out just one of those parts (1/5 of the unknown bar), the child then *Repeats* (or iterates) that pulled-out part to make a bar that is five of those 1/5-parts – i.e. a 5/5-bar, or "my bar" that the child was challenged to create. This second method is a more generalizable procedure that lays the foundation for the child to construct what we call an *Iterative Fraction* scheme (Steffe and Olive, 2010). An iterative fraction scheme enables the child to construct and makes sense of fractions greater than one (those pesky *Improper Fractions*!) by simply iterating a unit fraction any number of times. Thus 7/5 is just seven times 1/5 of a unit or, in the context of JavaBars, a bar that is seven times as large as 1/5 of a unit bar – a bar that *Arnold* should enjoy lifting above his head!

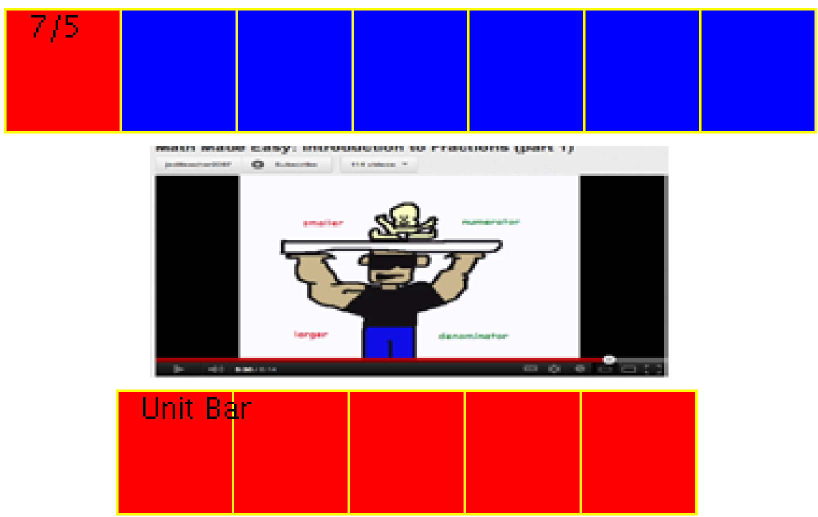

Figure 12: It's easy to lift an improper fraction bar when you know how!

Summary

In this paper I have focused primarily on new media for developing quantitative reasoning from a mathematical point of view. The examples I have provided (with the exception of the *Math Made Easy* web tutorial) all share what Hollylynne Lee cited in her presentation as "Mathematical Fidelity" (Zbeik, Heid, Blume and Dick, 2007). That is, the "technology-generated external representation must be faithful to the underlying mathematical properties of that object." (p. 1174). Certainly, the web-based games in *Calculation Nation* exhibit mathematical fidelity. The *Pufflescape* game in *Club Penguin* is actually built on a model of Newtonian mechanics, thus this may be said to exhibit "scientific fidelity." The *GapMinder* dynamic data visualization tool uses both logarithmic and linear scales, as well as proportional areas to faithfully represent the quantitative relations among the data being represented. The examples of *Dynamic Number* tools for elementary students provide accurate representations of numerical quantities using various models (color patterns and zooming number lines). The Multi-touch iPad apps (*Bunny Times* and *Area of a Circle*) provide students with ways to enact strategies for making sense of multiplicative problems and area measurements.

The *Math Made Easy* example of a widely used web-based tutorial program illustrates the danger for children and their parents who may be attracted to the simplistic approach and the promise to make learning mathematics easy. This tutorial is not only mathematically flawed, but also cognitively dangerous. It aims to imprint on the young child's mind a conceptually disastrous, incorrect relation concerning the numerator and denominator of a fraction.

My final example (the TIMA software) was described by Zbiek et al (2007) as having *both* Mathematical Fidelity and Cognitive Fidelity. Cognitive Fidelity is the "degree to which the (cognitive) tool actions explicitly reflect the user's cognitive actions." (p. 1176). The authors go on to say that "a tool having a high degree of cognitive fidelity holds particular promise to researchers in making visible the thinking of the user." (p. 1176) For the role of technology in research on quantitative reasoning, I can think of no more important challenge and opportunity than to make visible the quantitative thinking of the student.

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