References

PARTICIPANT RESEARCH ESSAY
FOR QRaMM RESEARCH TEAM

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My current research focus stems from my dissertation study and involves investigating the role of quantitative reasoning and covariational reasoning that students use in learning precalculus and calculus ideas. In my dissertation study, I focused on precalculus students learning various topics of trigonometry, such as angle measure and the sine and cosine functions. The results of my dissertation study have also led to an interest in the relationship between quantitative reasoning and students’ approaches to problem solving.

With the results of my dissertation study, I intend to add to the limited research in the area of students and teachers’ understandings of trigonometric functions and topics foundational to these functions (e.g., angle measure). The design of my dissertation study consisted of a teaching experiment (Steffe & Thompson, 2000) with three undergraduate precalculus students. The design of the instructional sequence used during the teaching experiment was informed by research literature on the foundational reasoning abilities needed for success in calculus. For instance, research on quantitative reasoning (Smith III & Thompson, 2008), covariational reasoning (Carlson et al., 2002), and function (Harel & Dubinsky, 1992) informed the design of the study and instructional sequence. Also, the instructional sequence was designed to promote the students constructing conceptions of angle measure that supported constructing the sine and cosine functions.

Previous research on students and teachers’ conceptions of trigonometric functions reports that they hold very shallow and limited understandings of these functions (Akkoc, 2008; Brown,
2006; Fi, 2006; Thompson, Carlson, & Silverman, 2007; Weber, 2005). Thompson et al. (2007) observed teachers having difficulty conceptualizing teaching trigonometric functions in a context other than right triangles, and Weber (2005) revealed that students were often unable to leverage various geometric objects as tools to solve problems involving trigonometric functions. Recently, Thompson (2008) drew attention to the incoherence of trigonometry curriculum, and pointedly identified the importance of students developing meaningful understandings of topics foundational to trigonometry. These findings and suggestions informed the design of my dissertation study and the instructional sequence implemented during the teaching experiment.

Consistent with previous research findings, the students involved in my study held limited and fragmented understandings of angle measure upon entering the teaching experiment. The students referred to various aspects of a circle and distances when reasoning about angle measure, but they were unable to systematically coordinate various quantities to solve the presented problems. As an example, the students described areas and arcs when attempting to reason about angle measures, but they did not reason about measurements of these attributes in relation to angle measure. Furthermore, the students frequently treated various angle measures as predefined labels of geometric objects, as opposed to results of a measurement process. This included referring to 180 degrees as “a straight line,” 90 degrees as a “right angle” and 360 degrees as “a full circle.” As the students were prompted to justify their statements, many of the students added that they had not thought deeply about angle measure in their past courses.

After two days of instruction designed to promote the students conceptualizing processes for measuring angles in degrees and radians, a majority of the students developed meaningful and flexible understandings of angle measure. As an example of a task, the students were asked to use the supplies of a ruler, a string, a calculator, and a compass to create protractors to measure in various units of angle measure. After encountering difficulty in creating the protractor when attempting to create sections of equal area (e.g., pie slices), multiple students identified that they could measure the circumference of the protractor and partition this length into equal arc lengths, where the number of arc lengths was equivalent to the total number of units corresponding to a full rotation. As the students reflected on this solution and the use of circles of various radii, they conceptualized an angle’s measure as conveying the fractional amount of any circle’s circumference subtended by the angle, given that the circle is centered at the vertex of the angle.

This image of measuring along a subtended arc supported the students’ reasoning about the radius as a unit of angle measure. Initially, the students determined the number of radius lengths in the circumference of any circle by using a piece of string the length of the radius to measure along circles. These actions enabled the students to conceptualize the circumference of any circle as \(2\pi\) radius lengths \( (C = 2\pi r) \). Additionally, the image of the radius forming a unit for measuring an arc enabled multiple students to reason that a number of radians implied the subtended arc was that many times as large as the corresponding radius.

The students’ ability to reason about measuring along an arc, as well as measuring a length in a number of radii, further formed a foundation for their constructing the sine (and cosine) function. The sine function was introduced in the context of circular motion, which supported the students in covarying the angle measure swept out by an object and its vertical position. In order to create a graph of the relationship between these two quantities, the students used a GSP applet of the situation to compare changes of vertical distance for successive equal changes of arc length. This step led to them constructing a correct graphical representation of the relationship. Furthermore, their reasoning that led to creating the graph consisted of comparing
magnitudes rather than numerical values, which supported a process view of function. This step then led to a discussion centered on the possible units for measuring the vertical position of the object, with the students deciding that measuring both quantities relative to the radius would result in a graphical representation with values that would correspond to a circle of any size, given that the radius remained the unit of measure. In turn, this also enabled the instructor to introduce the notation \( y = \sin(q) \) as formalizing the quantitative relationship constructed by the students. As the study progressed, this understanding of the sine function enabled them to produce symbolic and graphical representations that reflected various quantitative relationships within the context of circular motion.

In summary, a majority of the students in the study constructed understandings of angle measure (e.g., measuring along a subtended arc and partitioning circumferences into equal parts) that created coherence between various units of angle measure, while also supporting their construction of the sine function. As they encountered novel tasks, the students were able to draw on these understandings to reason about quantities and relationships between quantities. In these instances, it was important that the students identified and conceptualized the quantities when confronting a novel problem. If the students did not conceptualize the appropriate quantities and their relationships, they were unable to define the functional relationship between quantities and describe how the quantities changed in tandem.

Over the course of the instructional sequence, the students’ responses to the problems also offered insights into the role of quantitative reasoning in their problem solving behaviors. Specifically, the students’ attempts to solve the problems revealed various mental actions, e.g., orienting, planning, executing, and checking, at play during the problem solving phases identified by Carlson and Bloom (2005). Students who focused on executing calculations and past procedures rarely engaged in reasoning about a problem’s context. Their approach to solving problems did not involve their conceptualizing distinct quantities, which led them to perform calculations that were not rooted in quantitative relationships. Additionally, these students were rarely observed planning or checking their solutions independent from calculating numerical values. To say more, the students expressed a need to make calculations before discussing a meaning of their calculations and possible future calculations. Students who exhibited such behaviors had difficulty solving novel problems and rarely provided correct solutions without aid from the instructor or other students.

On the other hand, the students that spent a significant time creating diagrams and focusing on the problem contexts provided meaningful and correct solutions rooted in their understanding of the situation. For instance, these students often constructed quantities and relationships between quantities prior to executing calculations and procedures. As a result, when executing calculations between numerical values, their actions were grounded in a robust mental structure of the problem’s quantities. This mental structure also provided a foundation for planning and checking their solutions. These students were frequently observed anticipating a series of calculations by reasoning about relationships between quantities previous to performing calculations between numerical values. That is, the students reasoned about quantities with undetermined values to plan their solution steps. They also frequently reflected on and refined their image of the problem situation as they progressed through a problem. When obtaining an unexpected result, this led to the students further orienting to the problem’s context and constructing an alternative solution grounded in their refined image of the situation.

Moving forward, I plan to continue investigating the mental actions at play when students are solving novel applied problems. The students’ propensity to construct and leverage their
image of a problem’s context significantly influenced the nature of their problem solving behaviors. The differing approaches to problem solving exhibited by the students shed light on the role of quantitative reasoning in solving novel problems and factors that contribute or hinder a student’s ability to solve these problems. Future investigations into students’ reasoning will provide a fine-grained analysis of students’ mental actions during problem solving. These insights will inform both curriculum design and instructional activities relative to supporting students constructing mathematical understandings through problem solving. This design research will result in research-based course materials that can subsequently be utilized to investigate student thinking, resulting in further refinement of my theories.

I also plan on continuing to investigate the role of quantitative reasoning in students learning precalculus ideas, and particularly exploring other topics related to trigonometric functions and angle measure (e.g., polar coordinates, periodicity, and modeling non-circular periodic motion). Such investigations will offer insights into the conceptions of angle measure, trigonometric functions, and other mathematical ideas necessary to construct meaningful and coherent understandings of these topics that are frequently used in calculus, the sciences, and engineering.

Lastly, I am interested in pre-service and in-service teachers’ interpretation and implementation of research-based curriculum. The trigonometry sequence designed during my dissertation study was based on the findings from various research studies, but a teacher trying to implement these materials might not have knowledge of such research literature. Also, a teacher’s understandings of the topics that are explored by the tasks will vary from teacher to teacher. Thus, I am interested in determining the supports necessary for a teacher to implement such research-based curriculum in a way that is consistent with the design intentions. Such investigations will include determining the teachers’ conceptions of various topics previous to implementing the curriculum, monitoring their shifts in content knowledge as they make sense of and implement the curriculum, and documenting any shifts that happen in their pedagogical practices. Further, such information will also reveal the causal conditions for these shifts and the role of research-based curriculum in producing these shifts.

References


