PARTICIPANT ESSAY


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FOR TTAME RESEARCH TEAM

Sergei Abramovich
SUNY Potsdam, USA
abramovs@potsdam.edu

*Perhaps the greatest of all pedagogical fallacies is the notion that a person learns only the particular thing he is studying at the time.*

*John Dewey*

Introduction

This essay is based on the author’s experience in teaching a computer-enhanced capstone course for prospective secondary mathematics teachers. It discusses how the use of technology can support the notion of a computational experiment in mathematics education. An experimental approach in mathematics draws on computers to perform extensive numerical computations and graphical constructions in the case when ideas and objects under study are too complex to be approached by using mathematical machinery and mental computation alone. The approach includes one’s engagement in recognizing and analyzing numerical patterns formed by modeling data as well as formulating properties of the studied mathematical models through interpreting behavior of their geometric/graphic representations. A computer-enabled mathematical experiment makes it possible to balance formal and informal approaches to the subject matter allowing one
to learn how the two approaches complement each other and support the Conference Board of the Mathematical Sciences (2001) recommendation that mathematics has to be approached “at least initially ... from an experientially based direction, rather than an abstract/deductive one” (p. 96).

By using computing technology as a medium for experimentation with a traditional curricular content, mathematics educators (perhaps in cooperation with mathematicians involved in the preparation of teachers) can significantly enrich mathematics curricula of a capstone course sometimes reaching a mathematical frontier. Even a modest exposure to complicated ideas of modern mathematics can significantly help prospective teachers of mathematics to learn how to teach in accord with the current principles and standards for teaching and recommendations for teacher preparation. Indeed, technology “influences the mathematics that is taught and enhances students’ learning” (National Council of Teachers of Mathematics, 2000, p. 24) therefore enabling many opportunities for “a quality encounter with the situations and ideas that stimulate student activity” (Hatfield, 1991, p. 241, italics in the original). One of the characteristics of such an encounter is the notion that the appropriate use of technology “for complicated computation does not eliminate the need for mathematical thinking but rather raises a different set of mathematical problems” (Conference Board of the Mathematical Sciences, 2001, p. 48). In other words, when used appropriately, technology stimulates formal learning of mathematics through the need to make sense of a computational experiment. This implies that a worthy direction of mathematics education research is the development of technology-rich curriculum materials for teacher education programs that take advantage of the experimental approach to mathematics. Materials of that kind should provide teacher candidates with “deep experiences in understanding of what it feels like or means to learn with the computer as a tool” (Hatfield, 1982, p. 43). The treatment of many curricular topics included in the author’s recently published book (Abramovich, 2011) may be recommended as a capstone experience for prospective teachers of secondary mathematics.

**Reflective Inquiry and Mathematical Thinking**

One of the core recommendations of the Conference Board of the Mathematical Sciences (2001) regarding teacher preparation in mathematics includes the need for courses that help teachers “develop the habits of mind of a mathematical thinker” (p. 8). One aspect of mathematical thinking is using experience for furthering knowledge. In education, experience is viewed in part as a vehicle that promotes one’s intellectual growth (Dewey, 1938). Such growth typically requires support on the part of a teacher—“the more knowledgeable other”—who encourages students to reflect on their actions, to inquire about the meaning of their experience, to develop alternative representations of the meaning found, and to uncover hidden connections among the alternatives.

Dewey (1933) also promoted the pedagogy of reflective inquiry—a problem-solving method that integrates knowing with doing and knowledge with experience. In a computerized teacher education classroom, reflective inquiry pedagogy becomes a useful approach to the development of mathematical knowledge. From examining a problem, to recognizing patterns, to generalizing from observations and experience—by going along this route, teacher candidates can come to recognize how seemingly disconnected ideas and concepts in mathematics become connected through a common conceptual structure.

**Computer Experimentation Motivates Collateral Learning**

A possibility of learning by reflecting on a computational experiment enables the emergence of what Dewey (1938) called “collateral learning” (p. 49), something that does not result from the immediate objective of the experiment but rather emerges as a powerful by-product from its hidden domain. A more recent educational construct closely related to collateral learning is known as hidden curriculum—“those nonacademic but educationally significant consequences of schooling that occur systematically” (Martin, 1983, p. 124). This nonacademic learning expe-
experience is taking place within a context that is much broader than a topic of any given lesson and, through reflection, enables one to become aware of rules and guidelines typically associated with social relations and control of individual actions.

The notion of hidden curriculum, in turn, can be extended to include collateral learning that may take place within a pure academic domain when one is expected and even encouraged to make connections among seemingly disconnected ideas and concepts related to a particular subject matter. Thus, one can talk about hidden mathematics curriculum (Abramovich & Brouwer, 2006)—a didactic approach to the teaching of mathematics that motivates and encourages collateral learning to occur in a broader context that one has been exploring. In the mathematics classroom, such extended explorations into a hidden conceptual domain require a certain level of mathematical competence and intellectual courage on the part of the teacher. Once the hidden mathematics curriculum approach becomes an accepted pedagogical tool in a technology-rich classroom, teachers can investigate different ideas under the guidance of their mathematics education instructor.

From a Computational Experiment to a Mathematical Frontier

As an illustration, consider one of the most common uses of one of the most popular technology tool—the computing of Fibonacci numbers $\ldots$, $F_n = F_{n-1} + F_{n-2}$, subject to initial conditions $F_0 = F_1 = 1$. The next step is to use this recursive definition in defining the contents of three consecutive cells and replicate the third cell to generate a sufficient number of terms. In that way, an electronic spreadsheet turns into an agent of a mathematical activity which results in the conceptualization of Fibonacci numbers as a solution to a linear difference equation of the second order. By generating the ratios of two consecutive Fibonacci numbers for a sufficient number of terms, one can see how the spreadsheet consumes the mathematical activity. This stage continues as one alters the contents of the cells assigned for the first two terms $F_0$ and $F_1$ to observe the invariance in the limiting behavior of the ratios.

Going beyond the use of a spreadsheet as an agent and a consumer of the activity of creating a computational model for Fibonacci numbers and the Golden Ratio, one can use the tool to amplify the results obtained so far. Toward this end, the following task can be formulated.

Considering the equation $F_{n+1} = aF_n + bF_{n-1}$, $F_0 = x, F_1 = y$, reorganize the spreadsheet to enable each cell beginning from the third to be a linear combination of the previous two cells, and numerically model the so generalized Fibonacci numbers along with the ratios of their two consecutive terms. Through exploring this task, several interesting phenomena can be discovered. For example, when $\{a, b, x, y\} = \{3, -1, 1, 2\}$ (these values of parameters were determined experimentally) the spreadsheet generates the sequence $1, 2, 5, 13, 34, 89, \ldots$. Surprise! The spreadsheet eliminates every second Fibonacci number from the original sequence while keeping the limiting value of the ratios of two consecutive terms of the so modified sequence without change. Now, one can consider the sequence $1, 5, 34, 233, \ldots$ and find a recursive rule through which this sequence develops. In that way, a spreadsheet-based experimentation with a two-parametric linear difference equation of the second order leads to the emergence of collateral learning about subsequences of Fibonacci numbers and their recursive definitions.
Furthermore, when \( \{a, b, x, y\} = \{6, -12, 1, 1\} \), the sequence of numbers 1, 6, -48, -216, -720, -1728, 10368, ... results, the ratios of which 1, -6, 8, 9/2, 10/3, 12/5, 1, -6, 8, ... form a cycle of period six. \( \{1, -6, 8, 9/2, 10/3, 12/5\} \). This unexpected discovery motivates collateral learning for it raises many questions that can be appreciated by a curious mind. How can one find other values of \( a \) and \( b \) that cause the ratios to oscillate with period six (rather to converge to a single value called the Golden Ratio)? How can one find all such values of parameters? How can one find loci in the plane of parameters \( a \) and \( b \) where cycles of other periods realize? These and other questions about the above two-parametric difference equation of the second order can be addressed by using, jointly with a spreadsheet, Wolfram Alpha and Maple—powerful computational tools for mathematical modeling activities. This time, through the spreadsheet-based amplification of a rather mundane activity of generating a string of consecutive Fibonacci numbers one can come to discover intriguing phenomena of modern mathematics, like the existence of cycles of any integer period formed by the orbits of a two-parametric difference equation of the second order, and further explore the behavior of the orbits with additional computer tools. In doing so, one can come across new polynomials the roots of which are responsible for the emergence of the cycles. It can be shown that if \( X \) is a root of such a polynomial and \( a^2 = x^*b \), then the ratios of two consecutive terms generated by the above difference equation always form a cycle the period of which is defined by the degree of the polynomial. The discovery of such polynomials occurred in a collateral way as it emerged quite unexpectedly from a spreadsheet-based experimentation with Fibonacci numbers. In other words, through the study of a familiar mathematical concept in a computational environment the multitude of new concepts became available for learning. More details on this topic can be found in (Abramovich & Leonov, 2009).

**Concluding Remarks**

The development of experimental approaches to mathematics based on the appropriate use of computational tools is one of the directions of mathematics education research where collaboration between mathematicians and mathematics educators can be especially fruitful. The exploratory power of technology tools and the variety of their educational applications can bridge research efforts of the two groups. The unity of content and pedagogy of a capstone course in a technological paradigm provides many opportunities for collateral learning. Using the agent-consumer-amplifier framework as a theoretical underpinning of a computational experiment can lead to the discovery of new mathematical knowledge in the context of education, thus enabling prospective secondary mathematics teachers’ true experience with a mathematical frontier.

**References**


