Chapter 3
National Income:
Where It Comes From and Where It Goes
Outline of model

A closed economy, market-clearing model

- Supply side
  - factors of production
  - determination of output/income
- Demand side
  - determinants of $C$, $I$, and $G$
- Equilibrium
  - goods market
  - loanable funds market
Factors of production

\[ K = \text{capital:} \]
\[ \quad \text{tools, machines, and structures used in production} \]

\[ L = \text{labor:} \]
\[ \quad \text{the physical and mental efforts of workers} \]
The production function: $Y = F(K,L)$

- shows how much output ($Y$) the economy can produce from $K$ units of capital and $L$ units of labor
- reflects the economy’s level of technology
- exhibits constant returns to scale
Returns to scale: a review

Initially \( Y_1 = F(K_1, L_1) \)

Scale all inputs by the same factor \( z \):
\[
K_2 = zK_1 \quad \text{and} \quad L_2 = zL_1
\]
(e.g., if \( z = 1.2 \), then all inputs are increased by 20%)

What happens to output, \( Y_2 = F(K_2, L_2) \)?

- If constant returns to scale, \( Y_2 = zY_1 \)
- If increasing returns to scale, \( Y_2 > zY_1 \)
- If decreasing returns to scale, \( Y_2 < zY_1 \)
Returns to scale: Example 1

\[ F(K, L) = \sqrt{KL} \]

\[ F(zK, zL) = \sqrt{(zK)(zL)} \]

\[ = \sqrt{z^2KL} \]

\[ = z\sqrt{KL} \]

\[ = zF(K, L) \]

constant returns to scale for any \( z > 0 \)
Determine whether each of these production functions has constant, decreasing, or increasing returns to scale:

(a) \( F(K,L) = \frac{K^2}{L} \)

(b) \( F(K,L) = K + L \)
Assumptions

1. Technology is fixed.
2. The economy’s supplies of capital and labor are fixed at

\[ K = \bar{K} \quad \text{and} \quad L = \bar{L} \]
Determining GDP

Output is determined by the fixed factor supplies and the fixed state of technology:

$$\bar{Y} = F(\bar{K}, \bar{L})$$
The distribution of national income

determined by factor prices, the prices per unit firms pay for the factors of production

- **wage** = price of \( L \)
- **rental rate** = price of \( K \)
Notation

\[ W = \text{nominal wage} \]
\[ R = \text{nominal rental rate} \]
\[ P = \text{price of output} \]
\[ W/P = \text{real wage (measured in units of output)} \]
\[ R/P = \text{real rental rate} \]
How factor prices are determined

- Factor prices determined by supply and demand in factor markets.
- Recall: Supply of each factor is fixed.
- What about demand?
Demand for labor

- Assume markets are competitive: each firm takes $W$, $R$, and $P$ as given.
- Basic idea:
  A firm hires each unit of labor if the benefit exceeds the cost.
  - cost = real wage
  - benefit = marginal product of labor
Marginal product of labor ($\text{MPL}$)

- definition:

  The extra output the firm can produce using an additional unit of labor (holding other inputs fixed):

  \[
  \text{MPL} = F(K, L+1) - F(K, L)
  \]
NOW YOU TRY

Compute & graph $MPL$

a. Determine $MPL$ at each value of $L$.

b. Graph the production function.

c. Graph the $MPL$ curve with $MPL$ on the vertical axis and $L$ on the horizontal axis.

<table>
<thead>
<tr>
<th>$L$</th>
<th>$Y$</th>
<th>$MPL$</th>
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<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>n.a.</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>?</td>
</tr>
<tr>
<td>2</td>
<td>19</td>
<td>?</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>34</td>
<td>?</td>
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<td>5</td>
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<td>?</td>
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<td>9</td>
<td>54</td>
<td>?</td>
</tr>
<tr>
<td>10</td>
<td>55</td>
<td>?</td>
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</tbody>
</table>
Compute & graph $MPL$
As more labor is added, $\text{MPL} \downarrow$

Slope of the production function equals $\text{MPL}$
Diminishing marginal returns

- As an input is increased, its marginal product falls (other things equal).

- Intuition:
  Suppose $\uparrow L$ while holding $K$ fixed
  $\Rightarrow$ fewer machines per worker
  $\Rightarrow$ lower worker productivity
Identifying Diminishing Returns

Which of these production functions have diminishing marginal returns to labor?

a) \( F(K, L) = 2K + 15L \)

b) \( F(K, L) = \sqrt{KL} \)

c) \( F(K, L) = 2\sqrt{K} + 15\sqrt{L} \)
Suppose \( \frac{W}{P} = 6 \).

- If \( L = 3 \), should firm hire more or less labor? Why?
- If \( L = 7 \), should firm hire more or less labor? Why?
Each firm hires labor up to the point where $MPL = W/P$. 

- **Units of output** 
- **Real wage** 
- **Quantity of labor demanded** 
- **Units of labor, $L$** 

$MPL$, Labor demand
The equilibrium real wage

The real wage adjusts to equate labor demand with supply.
The equilibrium real rental rate

The real rental rate adjusts to equate demand for capital with supply.

Units of output

Supply of capital

equilibrium $R/P$

MPK, demand for capital

Units of capital, $K$
The Neoclassical Theory of Distribution

- states that each factor input is paid its marginal product

- a good starting point for thinking about income distribution
How income is distributed to $L$ and $K$

total labor income $= \frac{W}{P}L = MPL \times L$

total capital income $= \frac{R}{P}K = MPK \times K$

If the production function has constant returns to scale, then

$\bar{Y} = MPL \times \bar{L} + MPK \times \bar{K}$

- national income
- labor income
- capital income
The ratio of U.S. labor income to total income

Labor’s share of income is approximately constant over time. (Thus, capital’s share is too.)
The Cobb-Douglas Production Function

- The Cobb-Douglas production function is:

\[ Y = AK^\alpha L^{1-\alpha} \]

where \( A \) represents the level of technology.

- The Cobb-Douglas production function has constant factor shares:

\( \alpha = (MPK \times K)/Y = \text{capital’s share of income} \)
\( (1 - \alpha) = (MPL \times L)/Y = \text{labor’s share of income} \)
Labor productivity and wages

- Theory: wages depend on labor productivity
- U.S. data:

<table>
<thead>
<tr>
<th>period</th>
<th>productivity growth</th>
<th>real wage growth</th>
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<tbody>
<tr>
<td>1960–2010</td>
<td>2.2%</td>
<td>1.9%</td>
</tr>
<tr>
<td>1960–1973</td>
<td>2.9%</td>
<td>2.8%</td>
</tr>
<tr>
<td>1973–1995</td>
<td>1.4%</td>
<td>1.2%</td>
</tr>
<tr>
<td>1995–2010</td>
<td>2.7%</td>
<td>2.2%</td>
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Outline of model

A closed economy, market-clearing model

Supply side

- factor markets (supply, demand, price)
- determination of output/income

Demand side

- determinants of $C$, $I$, and $G$

Equilibrium

- goods market
- loanable funds market
Demand for goods and services

Components of aggregate demand:

\[ C = \text{consumer demand for goods \& services} \]

\[ I = \text{demand for investment goods} \]

\[ G = \text{government demand for goods \& services} \]

(closed economy: no \( NX \))
Consumption, $C$

- **def:** Disposable income is total income minus total taxes: $Y - T$.

- Consumption function: $C = C(Y - T)$
  Shows that $(Y - T) \Rightarrow C$

- **def:** Marginal propensity to consume (MPC) is the change in $C$ when disposable income increases by one dollar.
The consumption function

\[ C = C(Y - T) \]

The slope of the consumption function is the \textit{MPC}. 

\[ \text{MPC} \]
Investment, $I$

- The investment function is $I = I(r)$ where $r$ denotes the **real interest rate**, the nominal interest rate corrected for inflation.

- The real interest rate is
  - the cost of borrowing
  - the opportunity cost of using one’s own funds to finance investment spending

  So, $\uparrow r \implies \downarrow I$
The investment function

Spending on investment goods depends negatively on the real interest rate.
Government spending, $G$

- $G = \text{govt spending on goods and services}$
- $G$ excludes transfer payments (e.g., Social Security benefits, unemployment insurance benefits)

Assume government spending and total taxes are exogenous:

\[ G = \bar{G} \quad \text{and} \quad T = \bar{T} \]
The market for goods & services

- Aggregate demand: $C(\bar{Y} - \bar{T}) + I(r) + \bar{G}$

- Aggregate supply: $\bar{Y} = F(\bar{K}, \bar{L})$

- Equilibrium: $\bar{Y} = C(\bar{Y} - \bar{T}) + I(r) + \bar{G}$

The real interest rate adjusts to equate demand with supply.
The loanable funds market

- A simple supply–demand model of the financial system.

- One asset: “loanable funds”
  - demand for funds: investment
  - supply of funds: saving
  - “price” of funds: real interest rate
The investment curve is also the demand curve for loanable funds.
Types of saving

private saving  = (Y – T) – C

public saving = T – G

national saving, \( S \)

= private saving + public saving

= (Y – T) – C + T – G

= Y – C – G
Budget surpluses and deficits

- If $T > G$, **budget surplus** $= (T - G)$ and public saving is positive.

- If $T < G$, **budget deficit** $= (G - T)$ and public saving is negative.

- If $T = G$, **balanced budget** and public saving is zero.

- The U.S. government finances its deficit by issuing Treasury bonds—i.e., borrowing.
Loanable funds supply curve

National saving does not depend on \( r \), so the supply curve is vertical.

\[
S = Y - C(Y - T) - G
\]
Loanable funds market equilibrium

\[ S = Y - C(Y - T) - G \]

Equilibrium real interest rate

Equilibrium level of investment
The special role of $r$

$r$ adjusts to equilibrate the goods market and the loanable funds market simultaneously:

If L.F. market in equilibrium, then

$$Y - C - G = I$$

Add $(C + G)$ to both sides to get

$$Y = C + I + G \quad (goods \ market \ eq'm)$$

Thus,

$$\text{Eq'm in L.F. market} \iff \text{Eq'm in goods market}$$
CASE STUDY: The Reagan deficits

1. The increase in the deficit reduces saving…

2. …which causes the real interest rate to rise…

3. …which reduces the level of investment.
Are the data consistent with these results?

<table>
<thead>
<tr>
<th></th>
<th>1970s</th>
<th>1980s</th>
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<tbody>
<tr>
<td>$T - G$</td>
<td>-2.2</td>
<td>-3.9</td>
</tr>
<tr>
<td>$S$</td>
<td>19.6</td>
<td>17.4</td>
</tr>
<tr>
<td>$r$</td>
<td>1.1</td>
<td>6.3</td>
</tr>
<tr>
<td>$I$</td>
<td>19.9</td>
<td>19.4</td>
</tr>
</tbody>
</table>

$T - G$, $S$, and $I$ are expressed as a percent of GDP. All figures are averages over the decade shown.
An increase in investment demand raises the interest rate. But the equilibrium level of investment cannot increase because the supply of loanable funds is fixed.