1 Multiple Regression Analysis: Inference

Normality Assumption

- $t$ statistic & distribution
- $F$ statistic & distribution
- Large $n$ & the central limit theorem
- Jarque-Bera test

Hypothesis Testing for Individual Coefficients

- Regression model: $Y_i = \beta_1 + \beta_2X_{2i} + \beta_3X_{3i} + u_i$
- Hypothesis test on $\beta_2$
- One tailed: $H_0: \beta_2 \leq 0$ vs. $H_1: \beta_2 > 0$
- Two tailed: $H_0: \beta_2 = 0$ vs. $H_1: \beta_2 \neq 0$

Hypothesis Testing for Overall Significance

- **Question.** Does the regression model explain any significant variation in $Y$?
- Economic significance?
- Two tailed: $H_0: \beta_2 = \beta_3 = 0$ vs. $H_1$: $H_0$ false
- Analysis of variance: $TSS = ESS + RSS$
- $F = \frac{ESS/(k-1)}{RSS/(n-k)} = \frac{R^2/(k-1)}{(1-R^2)/(n-k)} \sim F(k - 1, n - k)$
- Relationship between individual and overall tests of significance

General (Linear) Hypothesis Testing

- For example, how do you test $H_0: \beta_2 = \beta_3$ vs. $H_1: \beta_2 \neq \beta_3$?
- You can use a general $F$ test
- $F = \frac{(R^2_{U}-R^2_{N})/m}{(1-R^2_{U,NI})/(n-k)} \sim F(m, n - k)$ where $m$ is the number of restrictions
- In this case, you can also perform a $t$ test
• Rewrite hypotheses as $H_0$: $\beta_2 - \beta_3 = 0$ vs. $H_1$: $\beta_2 - \beta_3 \neq 0$

• $t = \frac{\hat{\beta}_2 - \hat{\beta}_3}{s_e(\hat{\beta}_2 - \hat{\beta}_3)} = \frac{\hat{\beta}_2 - \hat{\beta}_3}{\sqrt{\text{var}(\beta_2) + \text{var}(\beta_3) - 2\text{cov}(\beta_2, \beta_3)}}$

• Unrestricted vs. restricted approach