1. Solve

\[ y'' + 2y' - 3y = 0. \]

2. Solve

\[ y'' + 3y' = 0, \ y(0) = -2, \ y'(0) = 3. \]

3. Solve

\[ 2y'' + y' - 4y = 0, \ y(0) = 0, \ y'(0) = 1. \]

4. Find a differential equation whose general solution is

\[ y = c_1 e^{2t} + c_2 e^{-3t}. \]

5. Find a differential equation whose general solution is

\[ y = c_1 e^t + c_2. \]

6. Solve

\[ y'' - y' - 2y = 0, \ y(0) = \alpha, \ y'(0) = 2. \]

Then find \( \alpha \) so that the solution approaches zero as \( t \to \infty \).

7. Solve

\[ y'' + 4y = 0, \ y(0) = 0, \ y'(0) = 1. \]

8. Solve

\[ y'' + y = 0, \ y(\pi/3) = 2, \ y'(\pi/3) = -4. \]

9. Find the Wronskian of the given pair of functions
   (i) \( e^{2t}, e^{-3t}/2 \)
   (ii) \( \cos t, \sin t \)
   (iii) \( \cos^2 \theta, 1 + \cos 2\theta \)
   (iv) \( x, xe^x \)

10. If the Wronskian \( W \) of \( f \) and \( g \) is \( t^2 e^t \) and if \( f(t) = t \) find \( g(t) \).
11. Show that 
\[ W(e^{\lambda t} \cos \mu t, e^{\lambda t} \sin \mu t) = \mu e^{2\lambda t} \]

12. Consider the initial value problem

\[ y'' + 2y' + 6y = 0, \ y(0) = 2, \ y'(0) = \alpha \geq 0. \]

(i) Find the solution \( y(t) \) of this problem.
(ii) Find \( \alpha \) so that \( y = 0 \) when \( t = 1 \).

13. Solve

\[ 4y'' + 4y' + y = 0, \ y(0) = 1, \ y'(0) = 2. \]

14. Solve

\[ 25y'' - 20y' + 4y = 0. \]

15. Consider the initial value problem

\[ 9y'' + 12y' + 4y = 0, \ y(0) = a > 0, \ y'(0) = -1. \]

(i) Find the solution \( y(t) \) of this problem.
(ii) Find the critical value \( a \) that separates solutions that becomes negative from those that are always positive.

16. Solve

\[ 4y'' + 4y = 3 \sin t, \ y(0) = 2, \ y'(0) = -1. \]

17. Solve

\[ y'' + y = 3 \sin t + t \cos 2t. \]

18. Solve

\[ y'' - 3y' - 4y = 3e^{2t}. \]

19. Solve

\[ y'' + y = \tan t \quad 0 < t < \pi/2. \]

20. Solve

\[ y'' - 2y' + y = \frac{e^t}{1 + t^2}. \]
21. Determine $\omega_0$, $R$, and $\delta$ so as to write the given expression in the form

$$u = R \cos(\omega_0 t - \delta)$$

(i) $u = 3 \cos 2t + 4 \sin 2t$
(ii) $u = 4 \cos 3t - 2 \sin 3t$
(iii) $u = -\cos t + \sqrt{3} \sin t$
(iv) $u = -2 \cos \pi t - 3 \sin \pi t$

22. Consider the initial value problem

$$mu'' + \gamma u' + ku = 0, \ u(0) = u_0, \ u'(0) = v_0.$$

Assume

$$\gamma^2 < 4km.$$

(i) Solve the initial value problem.
(ii) Write the solution in the form

$$R = Re^{-\gamma t/2m} \cos(\mu t - \delta)$$

. Determine $R$ in terms of $m$, $\gamma$, $k$, $u_0$ and $v_0$.
(iii) Investigate the dependence of $R$ on the damping coefficient $\gamma$ for fixed values of the other parameters.