Embedding Factorizations in Hypergraphs

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A hypergraph $\mathcal{G}$ is a pair $(V, E)$ where $V$ is a finite set called the vertex set, and $E$ is the edge set, where every edge is a subset of $V$. The degree of a vertex is the number of edges containing that vertex. A $k$-edge-coloring of $\mathcal{G}$ is a mapping $f : E \rightarrow C$, where $C$ is a set of $k$ colors, and the edges of one color form a color class. An $r$-factor $\mathcal{H}$ of $\mathcal{G}$ is composed of all the vertices and some (or perhaps all) of the edges of $\mathcal{G}$ so that each vertex in $\mathcal{H}$ is of degree $r$.

Let $[m] = \{1, \ldots, m\}$, and let $K_m^h = ([m], \binom{[m]}{h})$ ($\binom{[m]}{h}$ is the collection of all $h$-subsets of $[m]$). Given a $k$-edge-coloring of $K_m^h$, we discuss the conditions under which this coloring can be extended to a coloring of $K_n^h$ (for $n \geq m$) so that each color class in $K_n^h$ is an $r$-factor.