A short construction of highly chromatic digraphs without short cycles

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A natural digraph analogue of the graph-theoretic concept of an ‘independent set’ is that of an ‘acyclic set’, namely a set of vertices not spanning a directed cycle. Hence a digraph analogue of a graph coloring is a decomposition of the vertex set into acyclic sets. In the spirit of a famous theorem of P. Erdős [Graph theory and probability, Canad. J. Math., 11:34–38, (1959)], it was shown probabilistically in [D. Bokal et al., The circular chromatic number of a digraph, J. Graph Theory, 46(3): 227–240, 2004] that there exist digraphs with arbitrarily large girth and chromatic number. Here I give a construction of such digraphs using acyclic homomorphisms to show that they have the desired properties.