Answer two of the following problems.

1. Denote by $S_3$ the nonabelian group of order 6. How many homomorphisms are there from $S_3$ to $S_3$? List them all.

2. Let $G$ be a group, and let $A, B$ be normal subgroups of $G$ such that $G = AB$. Let $N = A \cap B$. Show that $G/N \cong A/N \times B/N$.

3. Let $G$ be a group generated by two distinct elements $x, y$ of order 2.
   (a) Show that $G$ has an abelian normal subgroup $H$ of index 2.
   (b) What are the possibilities for the order $|G|$? In each case indicate also the order $|Z(G)|$ of the centre. Justify your answers.

4. Let $G = GL_n(\mathbb{C})$, the group of invertible complex $n \times n$ matrices, and let $T \leq G$ be the subgroup consisting of all diagonal matrices with nonzero diagonal entries. Show that $T$ is a maximal abelian subgroup of $G$, i.e. the only abelian subgroup of $G$ containing $T$, is $T$ itself.