1. Determine the character table of the group $S_4$. Show and explain your work.

2. Determine the character table of the group $A_5 \times C_2$. Show and explain your work. Note that this group is the full symmetry group of the dodecahedral graph on 20 vertices, used in class to explicitly construct a degree 3 representation of $A_5$. The group $A_5$ constitutes the rotational symmetry group of the dodecahedron, and the group $A_5 \times C_2$ is the full symmetry group including reflections and other symmetries which reverse orientation. The elements of the group $A_5 \times C_2$ are conveniently represented in the form $\pm g$ for $g \in A_5$. (You may think of these as units in the group ring $\mathbb{Z}A_5$.)

3. Determine the character table of the dihedral group $D_5$ of order 10. Show and explain your work.

4. Consider the Cayley graph $\Gamma = \Gamma(G, S)$ of the group $G = \langle x, y : x^5 = y^2 = (xy)^2 = 1 \rangle$ with respect to the generating set $S = \{x, x^4, y\}$. Use the method described in class, and the character table obtained in answering Question #3, to compute the spectrum of the adjacency matrix of $\Gamma$. 

![Dodecahedral Graph](image.png)