Bob (located near Betelgeuse) generates an EPR pair of electrons, $e_1$ and $e_2$, in spin state

$$|\psi_{12}\rangle = \frac{1}{\sqrt{2}}(|++\rangle + |--\rangle).$$

This means that $e_1$ has a 50% chance of being found in a spin ‘up’ state, if measured, and a 50% chance of being found in a spin ‘down’ state. And the same for $e_2$. But regardless of the outcome of measuring either electron, the other electron will always be found to be in the same spin state (up or down); it will never happen that one electron is found to be in a spin up state, and the other in a spin down state. Thus $e_1$ and $e_2$ are maximally entangled. In fact if we measure $e_1$ first, then $e_2$ automatically collapses into a basic state, either $|+\rangle$ or $|--\rangle$, the same as the state in which $e_1$ was observed.

Bob sends the first of his two electrons to Alice (located near Arcturus). Traveling at 90% of the speed of light, it takes about 700 years for $e_1$ to arrive. Alice collects $e_1$ very carefully, being careful not to disturb it. (If she measures the spin state of $e_1$, it collapses into a pure spin up or spin down state, losing its superposition of two states, and teleportation will fail.)

Now Bob has a third electron $e_3$ which he wants to teleport to Alice. This new electron is in some unknown spin state

$$|\psi_3\rangle = \alpha|+\rangle + \beta|--\rangle$$

where $\alpha$ and $\beta$ are unknown complex numbers satisfying $|\alpha|^2 + |\beta|^2 = 1$. Actually Bob doesn’t want to physically send $e_3$ to Alice. Instead he will perform certain operations on electrons $e_2$ and $e_3$ (both of which are located in his lab near Betelgeuse). He will then send two classical bits of information to Alice, who will perform a certain operation on her electron $e_1$, resulting in $e_1$ being transformed into the desired spin state $\alpha|+\rangle + \beta|--\rangle$.

Before performing any operations, $e_1$ and $e_2$ are entangled, but $e_3$ is disentangled from the other two electrons. Therefore the combined spin state of the three electrons is

$$|\psi_{123}\rangle = |\psi_{12}\rangle \otimes |\psi_3\rangle = \frac{1}{\sqrt{2}}(\alpha|++++\rangle + \beta|+--\rangle + \alpha|--\rangle + \beta|--\rangle).$$

It is important to understand that neither Alice nor Bob know, at any time, the actual values of $\alpha$ or $\beta$, i.e. the actual spin state that is being teleported. If for example Bob chooses to measure the spin state of $e_3$, then its state is lost and can never be recovered. In fact such a measurement will result in a single bit of information only, namely the
observation ‘spin up’ (with probability $|\alpha|^2$) or ‘spin down’ (with probability $|\beta|^2$). From this he cannot deduce almost nothing about the actual values of $\alpha$ and $\beta$. (If for example he observes ‘spin up’ then he knows for sure that $\alpha \neq 0$; moreover he can guess that $|\alpha| > |\beta|$, and the likelihood that his guess is correct would be better than 50\%.) However almost all of the information (infinitely many bits of classical information stored in the exact values of $\alpha$ and $\beta$) would be forever lost.

Instead Bob applies a unitary (reversible) transformation on the spin state $|\psi_{23}\rangle$ of his electrons e2 and e3:

$$
|++\rangle \mapsto \frac{1}{\sqrt{2}}(|++\rangle + |--\rangle)
$$

$$
|--\rangle \mapsto \frac{1}{\sqrt{2}}(|++\rangle - |--\rangle)
$$

$$
|+-\rangle \mapsto \frac{1}{\sqrt{2}}(|++\rangle + |--\rangle)
$$

$$
|--\rangle \mapsto \frac{1}{\sqrt{2}}(|++\rangle - |--\rangle)
$$

This linear transformation has unitary matrix

$$
\frac{1}{\sqrt{2}} \begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 \\
0 & 1 & -1 & 0 \\
1 & 0 & 0 & -1
\end{bmatrix}
$$

with respect to the standard basis $|++\rangle$, $|+-\rangle$, $|+--\rangle$, $|--\rangle$ of pure spin states. The state of e1 is unaffected by this transformation; therefore Bob’s action on the combined state of the three electrons is

$$
|+++\rangle \mapsto \frac{1}{\sqrt{2}}(|+++\rangle + |+-+\rangle)
$$

$$
|++-\rangle \mapsto \frac{1}{\sqrt{2}}(|+++\rangle - |+-+\rangle)
$$

$$
|++-\rangle \mapsto \frac{1}{\sqrt{2}}(|+++\rangle + |+-+\rangle)
$$

$$
|++-\rangle \mapsto \frac{1}{\sqrt{2}}(|+++\rangle - |+-+\rangle)
$$

and the unitary matrix of this transformation with respect to the standard basis $|+++\rangle$, $|++-\rangle$, $|+-\rangle$, $|--\rangle$ of pure spin states is

$$
\left[ \begin{array}{c}
1 & 0 \\
0 & 1
\end{array} \right] \otimes \frac{1}{\sqrt{2}} \begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 \\
0 & 1 & -1 & 0 \\
1 & 0 & 0 & -1
\end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix}
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & -1
\end{bmatrix}.
$$
As a result of this transformation, all three electrons are now entangled together; their resulting combined spin state is

$$|\psi_{123}\rangle = \frac{1}{2} [\alpha(|++++\rangle + |+--\rangle) + \beta(|++++\rangle + |+--\rangle)$$

$$\quad + \alpha(|---\rangle - |--\rangle) + \beta(|---\rangle - |--\rangle) ]$$

$$= (\alpha|+\rangle + \beta|--\rangle) \otimes \frac{1}{2}|++\rangle$$

$$\quad + (\alpha|+\rangle - \beta|--\rangle) \otimes \frac{1}{2}|--\rangle$$

$$\quad + (\beta|--\rangle + \alpha|+\rangle) \otimes \frac{1}{2}|+-\rangle$$

$$\quad + (\beta|--\rangle - \alpha|+\rangle) \otimes \frac{1}{2}|-+\rangle.$$

Now Bob measures the electrons e2 and e3. He finds their spins to be up-up, or up-down, or down-up, or down-down, each with probability $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$. Depending on the outcome of this measurement, the state of e1 collapses to

$$\alpha|+\rangle + \beta|--\rangle, \quad \alpha|+\rangle - \beta|--\rangle, \quad \beta|--\rangle + \alpha|+\rangle, \quad \text{or} \quad \beta|--\rangle - \alpha|+\rangle$$

respectively. Bob sends to Alice a message the outcome of his measurement (two bits of classical information) and Alice applies to her electron e1 a unitary operation

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \text{or} \quad \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

respectively. This transforms her electron e1 into the desired spin state $\alpha|+\rangle + \beta|--\rangle$. 