

# **Environmental Bonding for Land Reclamation**

by

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## **1. Introduction**

Over the last decade the country's appetite for energy and other natural resources has grown exponentially, but so has the need for mitigating environmental damage from this resource development. Environmental reclamation and restoration is usually a required part of completing a resource extraction project. Government agencies standards for reclamation vary from agency to agency, but have used performance approach, and design standards as metrics for successful completion. The literature on the economic efficiency of these types of standards is considerable (Hueth and Melkonyan 2009). Coupled with these standards regulators often use some form of bonding scheme to enforce completion of commitments.

Bonding schemes come in several forms as reviewed by Ferreira, et al (2003). Besides regular cash bonds where a firm gives a fixed sum to the agency they can also include letters of

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Credit, Prepaid Collateral Closure Accounts, Leasing Specific Closure Accounts, and Ex-post Insurance Policies. All these pose risks or opportunity costs to the firm, the agency, or both. The risk issue is outside the scope of this article, but is nonetheless important. Environmental bonding related to reclamation has become an increasingly common tool for management of mitigation of natural resource development and impacts. The motivation for environmental bonding as framed in the literature is a mechanism for providing an incentive for a firm to reclaim disturbances on public lands (Perrings, 1989; Constanza and Perrings, 1990; Gerrard 2000). Garrard (2000) argues bonding is a market based approach that imposes a cost on the firm for non-compliance, thereby creating an incentive to follow regulatory commitments. It can also transfer the risk of default to a third party.

The use of a bond to manage commitments has advantages, but also limitations. Shogren (1993) identifies three limitations. A moral hazard exists if firms choose to ignore reclamation commitments because they have already internalized the reclamation cost by posting a bond and plan on forfeiting the bond and shirking on the commitment. Liquidity constraints may occur from setting the bond rate unrealistically high, thereby tying up investment capital with high industry opportunity costs. Finally contract failure can occur because reclamation activities are imperfectly enforced due to monitoring issues and legal restrictions (Shogren et al., 1993), as well as simply the long time period between bonding and

termination.

Limitations aside, there are reasons why firms do complete reclamation commitments. Enterprise level studies that incorporate reclamation decision-making into a complete set of incentives that a company faces are few in the literature. The goal of this analysis.

Sult (2004) developed a model of reclamation decision making for coal mines incorporating fines for non-compliance and reputation costs. The model predicted that reclamation depends upon the entire cost of reclamation, including fines and increased management cost from a change in reputation in the eyes of the regulators. In general reputation effects can insure reclamation activities are completed even in the presence of a small bond that does not fully cover costs (Gerard, 2000). Firms will complete reclamation activities simply to avoid potential business losses because of a damaged reputation, which can affect a company's relationship with regulators, increasing operation costs. A damaged reputation can also affect a company's relationship with consumers, potentially decreasing revenues. In another study of hard rock mining Roan and Martin (1996) develop a similar model that incorporates ecosystem constraints to a mining firm's operation. The authors illustrate two important findings relevant to policy development for hard rock mining. First, the ecosystem constraint (and the regulatory system that follows it) creates an incentive to locate in areas with less environmental restrictions. So firms choose investment locations in part based

upon regulatory cost. Second, regulatory cost associated with ecosystem protection reduces the economic life of mine.

This paper evaluates environmental bonding systems operated by State and Federal Agencies as an enforcement mechanism for future remediation of oil and gas impacts in the Western United States. We look at the Bureau of Land Management (BLM) bonding framework from the perspective of an oil and gas firm. We use a dynamic optimization model to estimate an optimal bond rate for the reclamation of land disturbed by oil and gas development. This study draws from previous work by Andersen and Coupal (2009) that analyzed costs and policies that affect land reclamation decisions by oil and gas firms. We begin by providing a brief description of the current regulatory setting that governs the oil and gas industry in Wyoming and focus our attention on reclamation bonding requirements, which are intended to insure the proper reclamation of disturbed land. The most important issue affecting the decision to reclaim is the cost of reclaiming the disturbed land (although other factors such as clear reclamation guidelines and standards set by land management agencies are important as well). We then empirically test this analysis by constructing a dynamic optimization model in GAMS with empirically derived parameters to calculate an optimal bond per well.

## 2. Model

The theoretical model for a gas development with the cost of the land reclamation is based on a representative oil and gas firm as presented in Pindyck (1978). The formulation represented here as follows:

$$\begin{aligned} \max_{q,w} J &= \int_0^{\infty} [qp - C_1(R)q - C_2(w)] e^{-rt} dt \\ \text{s.t. } \dot{R} &= \dot{x} - q \\ \dot{x} &= f(w, x) \\ R \geq 0, q \geq 0, w \geq 0, x \geq 0. \end{aligned} \tag{1}$$

We assume a competitive market for a non-renewable resource and follow Pindyck's assumptions. Producers take price  $p$  as given and choose a rate of production  $q$ . The average production cost is denoted as,  $C_1(R)$ , where  $R$  denotes the amount of proved oil reserves and the exploration cost is represented as,  $C_2(w)$ , where  $w$  denotes the number of drilled wells.

The average production cost increases as the proved oil reserve decreases. The flow of the oil addition is represented as  $f(w, x)$ . New oil discovery decreases over time because it becomes more difficult to discover new oil reserves when the cumulative oil discovery,  $x$ , increases.

Firms begin drilling in an area which is relatively easy to access first and then move to more costly areas. Firms move to areas of increasing cost over time, thus  $\frac{\partial f(w, x)}{\partial x} < 0$ . Pindyck solved for  $\dot{w}$ , to generate an optimal path of  $\dot{w}$ . In this formulation of Pindyck's model the optimal path for  $\dot{w}$  is as follows:

$$\dot{w} = \frac{C'_2(w) \left[ \frac{f_{wx}}{f_w} f - f_x + \delta \right] + C'_1(R) q f_w}{C''_2(w) - C'_2(w) \frac{f_{ww}}{f_w}} \quad (2)$$

Equation (2) has two state variables and two choice variables. Because (2) has four variables, a graphical phase diagram analysis is not possible, so we introduce a simpler model with one state variable and one choice variable this is suited to the needs of this research.

Cumulative oil discovery,  $x$ , is eliminated from the original model for simplicity because it makes the model too complicated to solve analytically. Cumulative discovery is used to represent the depletion of potential oil reserves in one region. Pindyck described  $\frac{\partial f}{\partial x} < 0$  because it gets more difficult to discover new oil reserves. The simpler model for the analytical solutions does not take into account oil depletion in an area. In our model,  $\bar{q}$ , denotes a known rate of production rate per well, and price,  $p$ , is assumed exogenous. In this representation all wells are considered homogeneous and there are no depletion effects. The model has one choice variable and one state variable. Based on the new definitions, our model of profit maximization for a representative firm is represented as:

$$\begin{aligned} \max_w J &= \int_0^T [(\bar{q}w)\bar{p} - C_1(R)(\bar{q}w) - C_2(w)] e^{-rt} dt \\ \text{s.t. } \dot{R} &= f(w) - \bar{q}w \\ R &\geq 0, \bar{q} \geq 0, w \geq 0. \end{aligned} \quad (3)$$

Following Pindyck, the signs of derivatives are as follows:

$$C'_2(w) > 0, C''_2(w) \geq 0, f'(w) > 0. \quad (4)$$

In order to obtain an analytical solution it is necessary to make additional assumptions to the

models. First,  $f(w)$  is a production function representing the oil discovery and we assume that  $f''(w) < 0$ . The marginal productivity would decrease as the number of wells increases.

Second, it is assumed that  $C'_1(R) < 0$  and  $C''_1(R) > 0$ . The function  $C_1(R)$  represents the average cost of production with respect to the amount of oil reserves. The average cost of production  $C_1(R)$  increases as proved reserve base is depleted. This means that  $C_1(R)$  decreases as  $R$  increases, thus  $C'_1(R) < 0$ .

From equation (3) the Hamiltonian is

$$H = [(\bar{q}w)p - C_1(R)(\bar{q}w) - C_2(w)]e^{-rt} + \pi[f'(w) - \bar{q}w]. \quad (5)$$

The maximum principles are given as

$$\frac{\partial H}{\partial w} = [\bar{q}p - C_1(R)\bar{q} - C'_2(w)]e^{-rt} + \pi[f'(w) - \bar{q}] = 0 \quad (6)$$

$$\dot{R} = \frac{\partial H}{\partial \pi} = f(w) - \bar{q}w \quad (7)$$

$$\dot{\pi} = -\frac{\partial H}{\partial R} = C'_1(R)(\bar{q}w)e^{-rt}. \quad (8)$$

Rearranging (6) and solving for  $\pi$ ,

$$\pi = \frac{-[\bar{q}p - C_1(R)\bar{q} - C'_2(w)]e^{-rt}}{[f'(w) - \bar{q}]} \quad (9)$$

Differentiating (9) with respect to time,

$$\dot{\pi} = \frac{[C''_2(w)f' - f''C'_2(w)]\dot{w}e^{-rt} - rC'_2(w)f'e^{-rt}}{[f'(w) - \bar{q}]^2}. \quad (10)$$

Equating (10) with (8), and rearrange it to make the differential equation for  $\dot{w}$ ,

$$\dot{w} = \frac{[C'_1(R)(\bar{q}w)[f'(w) - \bar{q}]^2 - [C'_1(R)\{f(w) - (\bar{q}w)\}\bar{q} + r\{\bar{q}p - C_1(R)\bar{q} - C'_2(w)\}]\{f'(w) - \bar{q}\}]}{[C''_2(w)\{f'(w) - \bar{q}\} + f''(w)\{\bar{q}p - C_1(R)\bar{q} - C'_2(w)\}]} \quad (11)$$

Now,  $\dot{w}$  and  $\dot{R}$  are represented by two equations (7) and (11).

Suppose that a long-run stable equilibrium exists and  $\dot{R} = 0$  and  $\dot{w} = 0$  intersects at one point in the phase diagram between  $w$  and  $R$ . Considering the case when  $\dot{w} = 0$ , it is assumed that the denominator is not equal to zero in (11). Then, the numerator must be equal to zero, and equation (11) becomes

$$C_1'(R)(\bar{q}w)[f'(w) - \bar{q}]^2 - [C_1'(R)\{f(w) - (\bar{q}w)\}\bar{q} + r\{\bar{q}p - C_1(R)\bar{q} - C_2'(w)\}]\{f'(w) - \bar{q}\} = 0. \quad (12)$$

Equation (12) can be rearranged as,

$$[f'(w) - \bar{q}] \left[ \frac{C_1'(R)\bar{q}w[f'(w) - \bar{q}] - [C_1'(R)\bar{q}\{f(w) - (\bar{q}w)\} + r\{\bar{q}p - C_1(R)\bar{q} - C_2'(w)\}]}{[f'(w) - \bar{q}]} \right] = 0. \quad (13)$$

Because  $f'(w) - \bar{q} \neq 0$  from (9), equation (13) can be simplified to:

$$C_1'(R)\bar{q}w[f'(w) - \bar{q}] - [C_1'(R)\bar{q}\{f(w) - (\bar{q}w)\} + r\{\bar{q}p - C_1(R)\bar{q} - C_2'(w)\}] = 0. \quad (14)$$

Assume  $g(w, R)$  has partial continuous derivatives and  $\frac{\partial g}{\partial R} \neq 0$ , then the sign of the slope of

$\dot{w} = 0$  in the phase diagram can be determined from the Implicit Function Theorem,

$$\frac{dR}{dw} = - \frac{\frac{\partial g}{\partial w}}{\frac{\partial g}{\partial R}} = \frac{-\frac{r}{q}C_2''(w) - C_1'(R)[f'(w) + wf''(w) - f'(w)]}{-rC_1'(R) - C_1''(R)[wf'(w) - f(w)]}. \quad (15)$$

Rearranging equation (15),

$$\frac{dR}{dw} = \frac{-\frac{r}{q}C_2''(w) - C_1'(R)wf''(w)}{rC_1'(R) + C_1''(R)f(w) \left[ w \frac{f'(w)}{f(w)} - 1 \right]}. \quad (16)$$

Pindyck set  $f(w, x)$  as  $Aw^\alpha e^{-\beta x}$ . The elimination of  $x$  from the model simplifies the

discovery function to  $f(w) = Aw^\alpha$  The function has regular neoclassical economic properties:

$$f'(w) = A\alpha w^{\alpha-1} > 0, f''(w) = A\alpha(\alpha-1)w^{\alpha-2} < 0 \text{ and } f'''(w) = A\alpha(\alpha-1)(\alpha-2)w^{\alpha-3} > 0 \text{ where}$$

$0 < \alpha < 1$  and  $w > 0$ . Considering the term  $\left[ w \frac{f'(w)}{f(w)} - 1 \right]$  in (16) then

$$w \frac{A\alpha w^{\alpha-1}}{Aw^\alpha} - 1 = \alpha - 1 < 0, \text{ where } 0 < \alpha < 1. \quad (17)$$

Thus, the sign of each term in (17) is determined by assumptions.

$$\frac{dR}{dw} = - \frac{\frac{\partial g}{\partial w}}{\frac{\partial g}{\partial R}} = \frac{-\frac{r}{q} C_2''(w) - C_1'(R) w f''(w)}{r C_1'(R) + C_1''(R) f(w) \left[ w \frac{f'(w)}{f(w)} - 1 \right]} > 0. \quad (18)$$

From equation (18) the slope of  $\dot{w} = 0$  is interpreted as negative in the phase space. The

amount of oil reserve decreases when the number of oil wells increases. Then, considering the

shape of the curve,

$$\begin{aligned} \frac{d^2 R}{dw^2} = & \frac{-\left[ \frac{r}{q} C_2'''(w) + C_1''(R) \{ f''(w) + w f'''(w) \} \right] \left[ r C_1'(R) + C_1''(R) \{ w f'(w) + f(w) \} \right]}{\left[ r C_1'(R) + C_1''(R) \{ w f'(w) - f(w) \} \right]^2} \\ & + \frac{\left[ C_1''(R) w f''(w) \right] \left[ \frac{r}{q} C_2''(w) + C_1'(R) w f''(w) \right]}{\left[ r C_1'(R) + C_1''(R) \{ w f'(w) - f(w) \} \right]^2}. \end{aligned} \quad (19)$$

First, we consider the sign of the first term in equation (19). The term  $\{ f''(w) + w f'''(w) \}$  is

modified as:

$$f''(w) \left\{ 1 + w \frac{f'''(w)}{f''(w)} \right\} = f''(w) \{ 1 + (\alpha - 2) \} = f''(w) (-1 + \alpha) > 0. \quad (20)$$

The sign of the first bracket in the numerator of (19) is determined as positive from the

assumptions.

$$-\left[\frac{r}{\bar{q}}C_2'''(w) + C_1'(R)f''(w)\left\{1 + w\frac{f'''(w)}{f''(w)}\right\}\right]^* \quad (21)$$

$$\left[rC_1'(R) + C_1''(R)\{wf'(w) - f(w)\}\right] < 0$$

Likewise, the sign of the second bracket in the numerator of (19) is

$$C_1''(R)wf''(w)\left[\frac{r}{q}C_2''(w) + C_1'(R)wf''(w)\right] < 0. \quad (22)$$

The sign of  $\frac{d^2R}{dw^2}$  is determined as positive from equation (21) and equation (22). Therefore,

$\dot{w} = 0$  is strictly convex because  $\frac{dR}{dw} < 0$  and  $\frac{d^2R}{dw^2} > 0$ .

Next, considering  $\dot{R} = 0$ , equation (3) is modified as:

$$f(w) - \bar{q}w = 0. \quad (23)$$

Incorporating the production functional form in Equation (23) is modified to

$$w(w^{\alpha-1} - \bar{q}) = 0. \quad (24)$$

The optimal number of wells can be zero or a certain real value from. The size of optimal number of wells is determined by the size of the production per well. If the amount of production per well becomes larger, the optimal number of wells decreases corresponding to the size of  $\alpha$  except for  $w^* = 0$ . When the productivity of each well increases, a firm needs fewer wells. On the other hand, a decline of productivity leads to an increase the number of wells to reach the equilibrium. As figure 1 shows,  $\dot{R} = 0$  is a vertical line in  $w$  and  $R$  space. If the optimal number of wells is equal to zero,  $\dot{R} = 0$  locates on the vertical axis, but an optimal number of wells is significantly greater than zero.

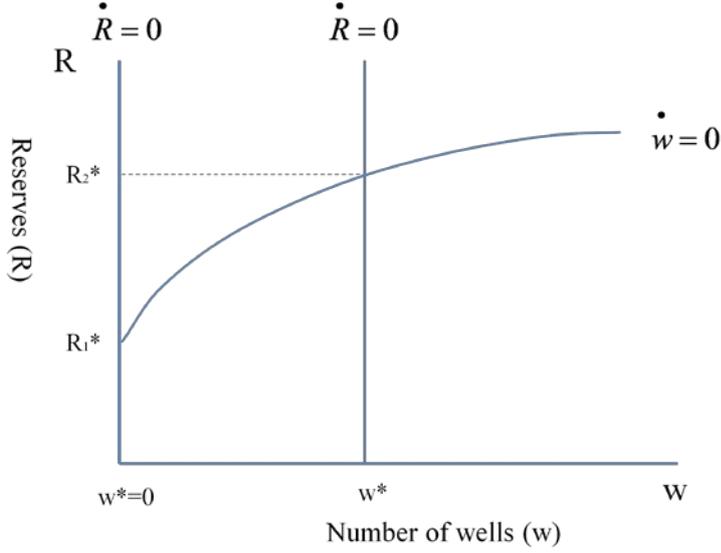


Figure 1. Graph of  $\dot{w} = 0$  and  $\dot{R} = 0$

Producer behavior reflected in the phase diagram is due to the shape of  $\dot{w} = 0$  and

$\dot{R} = 0$  is considered. Because the denominator cannot be zero when  $\dot{w} = 0$ , equation (11) can

be modified as:

$$G(w, R) = r \left\{ p - C_1(R) - \frac{C_2'(w)}{\bar{q}} \right\} - C_1'(R) [f'(w)w - f(w)] = 0. \quad (25)$$

Similarly, when  $\dot{R} = 0$ ,

$$H(w) = f(w) - \bar{q}w = 0. \quad (26)$$

Equation (26) can be linearized around the equilibrium point,  $(w^*, R^*)$ , by a Taylor series

expansion, (27).

$$G(w, R) = \left[ \frac{r}{\bar{q}} C_2''(w^*) + C_1'(R^*) f''(w^*) w^* \right] (w - w^*) + \left[ -r C_1'(R^*) - C_1''(R^*) \{ f'(w^*) w^* - f(w^*) \} \right] (R - R^*). \quad (27)$$

$w^*$  then is the optimal number of wells and  $R^*$  are optimal amount of the proved oil reserves.

Likewise, equation (27) is linearized as:

$$H(w) = [f'(w^*) - \bar{q}](w - w^*) = 0. \quad (28)$$

The Jacobian matrix is given as:

$$J = \begin{bmatrix} H_R & H_w \\ G_R & G_w \end{bmatrix} = \begin{bmatrix} 0 & f'(w^*) - \bar{q} \\ rC_1'(R^*) + C_1''(R^*)\{f'(w^*)w - f(w^*)\} & \frac{r}{\bar{q}}C_2''(w^*) + C_1'(R^*)f''(w^*)w^* \end{bmatrix} \quad (29)$$

And the eigenvalues are as follows:

$$|J - \lambda I| = \begin{vmatrix} -\lambda & H_w \\ G_R & G_w - \lambda \end{vmatrix} = 0. \quad (30)$$

Solving equation (30),

$$\lambda_{1,2} = \frac{1}{2} \left[ G_w \pm \sqrt{G_w^2 + 4H_w G_R} \right]. \quad (31)$$

The first term in (31),  $G_w$ , is positive from (32).

$$G_w = \frac{r}{\bar{q}}C_2''(w^*) + C_1'(R^*)f''(w^*)w^* > 0. \quad (32)$$

The sign of  $H_w G_R$ , in (31) is positive as represented in (33).

$$H_w G_R = [f'(w^*) - \bar{q}] \left[ rC_1'(R^*) + C_1''(R^*)\{f'(w^*)w^* - f(w^*)\} \right] > 0. \quad (33)$$

From (31), (32) and (33) the phase diagram has a saddle path between  $\lambda_1 < 0 < \lambda_2$ . There are two possible cases to illustrate the phase diagram representation corresponding to the value of optimal number of wells; when  $w^* \neq 0$  and  $w^* = 0$ . Both phase diagrams have saddle paths.

When  $w^* \neq 0$  (26) is converted into (34) because the discovery function is concave and

production is a linear function<sup>2</sup>,

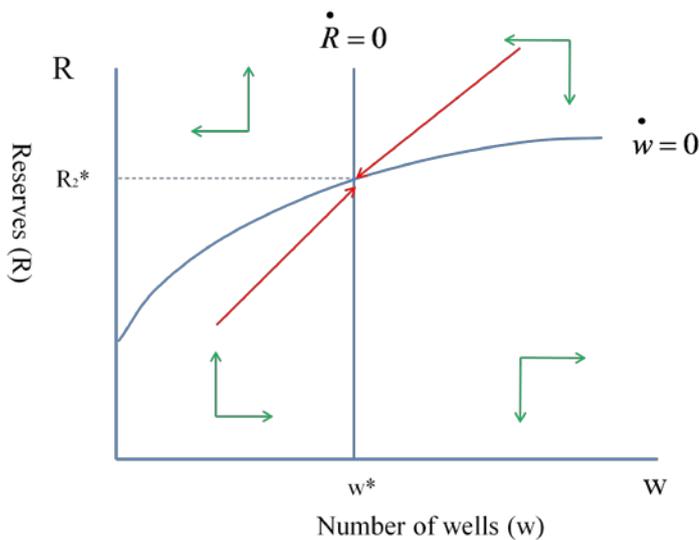
$$H(w^* + \varepsilon) = f(w^* + \varepsilon) - \bar{q}(w^* + \varepsilon) < 0. \quad (34)$$

It means  $\dot{R} = 0$  takes smaller values when the number of wells is larger than the optimal number of wells. Similarly, the change of the number of wells responding to the amount of the proved oil reserve is represented as:

$$G(w^*, R^* + \varepsilon) = r \left\{ p - C_1(R^* + \varepsilon) - \frac{C'_2(w^*)}{\bar{q}} \right\} - C_1'(R^* + \varepsilon) [f'(w^*)w^* - f(w^*)] < 0. \quad (35)$$

Equation (35) implies that as  $\dot{w} = 0$  takes smaller values, the amount of existing oil in reserve is larger than the optimal amount of oil reserve.

Because the behavior of the trajectories around  $\dot{R} = 0$  and  $\dot{w} = 0$  are given, the phase diagram which has a saddle path identifies and optimal path (Figure 2). Because of the saddle path, there is an optimal path.



<sup>2</sup>  $f'(w) > 0$  and  $f''(w) < 0$ .

Figure 2. Saddle path where  $w^* \neq 0$

Second, when the optimal number of wells is equal to zero,  $\dot{R} = 0$  locates on the vertical axis. Equation (26) can be linearized around the equilibrium point,  $(0, R_2^*)$ , by a Taylor series expansion<sup>3</sup>.

$$G(w, R) = -rC_1'(R_2^*)(R - R_2^*). \quad (36)$$

$w^*$  then is the optimal number of drilled wells and  $R^*$  are optimal amount of the proved oil reserves. Likewise, equation (27) is linearized as:

$$H(w) = -\bar{q}w = 0. \quad (37)$$

The Jacobian matrix is given as:

$$J = \begin{bmatrix} H_R & H_w \\ G_R & G_w \end{bmatrix} = \begin{bmatrix} 0 & -\bar{q} \\ rC_1'(R^*) & 0 \end{bmatrix} \quad (38)$$

And the eigenvalues are as follows:

$$|J - \lambda I| = \begin{vmatrix} -\lambda & H_w \\ G_R & G_w - \lambda \end{vmatrix} = 0. \quad (39)$$

Solving equation (39),

$$\lambda_{1,2} = \pm \sqrt{-H_w G_R}. \quad (40)$$

The sign of  $H_w G_R$  is determined from

$$H_w G_R = \bar{q}rC_1'(R^*) < 0. \quad (41)$$

The phase diagram has saddle path from (40) and (41) from  $\lambda_1 < 0 < \lambda_2$ .

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<sup>3</sup>  $C_2''(0) = 0$ ,  $f(0) = 0$  and  $f'(w) = 0$ .

When  $w^* \neq 0$ , because the discovery function is concave and production is a linear function<sup>4</sup>, Equation (26) is represented as

$$H(0 + \varepsilon) = f(\varepsilon) - \bar{q}(\varepsilon) < 0. \quad (42)$$

It means  $\dot{R} = 0$  takes smaller values when the number of wells is larger than the optimal number of wells. Similarly, the change of the number of wells responding to the amount of the proved oil reserve is represented as:

$$G(0, R_2^* + \varepsilon) = -\bar{q}rC_1'(R_2^* + \varepsilon) < 0. \quad (43)$$

From (42) and (43), the trajectories around the equilibrium  $(0, R_2^*)$  are determined.

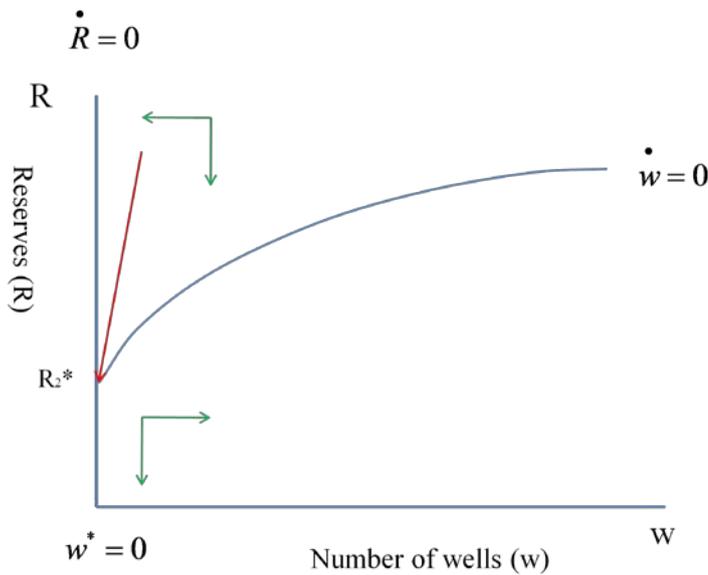


Figure 3. Saddle path where  $w^* = 0$

From  $w \geq 0$ , there is a saddle path only above  $\dot{w} = 0$ . Thus, the initial amount of proved reserve and initial number of wells must be greater than the combinations on  $\dot{w} = 0$ . If an initial point starts above  $\dot{w} = 0$ , it converges to the equilibrium  $(0, R_2^*)$ .

<sup>4</sup>  $f'(w) > 0$  and  $f''(w) < 0$ .

The behaviors of the trajectories around two equilibria,  $(w_1^*, R_2^*)$  and  $(0, R_2^*)$ , are shown on figure 2 and figure 3. A combination of an initial number of well and an initial amount of oil reserve converges to either of the two equilibria. When  $\bar{q}$  is extremely large and does not have intersection except for  $w^* = 0$ , the phase diagram has only one equilibrium on the vertical axis. As is shown on figure 3, the initial combinations above  $\dot{w} = 0$  are able to reach the equilibrium  $(0, R_2^*)$ .

### *The Simplified Pindyck's Model with Reclamation*

We now incorporate a fixed initial lump sum bond and reclamation costs to the simplified model described above. Since reclamation cost is highly correlated with total well depth  $w$  is an appropriate choice variable for both production and reclamation (Andersen and Coupal, 2009).<sup>5</sup> The final choice variable included is exploration costs (which can be as simple as adding another gas field to the operation). Exploration effort is measured in terms of cumulative depth they drill in one region at time  $t$ . Additionally, the firm must pay the reclamation cost,  $RC(w)$ , which covers the restoration costs for the land disturbance occurred by the exploration effort. Assuming the cost of reclamation for each well is same, the function of reclamation cost is linear. Thus,  $RC'(w) > 0$  and  $RC''(w) = 0$ . Following Deacon (1993) we limited the number of control variables to facilitate the likelihood of a solution.<sup>6</sup> The total cost of drilling wells is

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<sup>5</sup>  $\text{Corr}[\text{depth of wells, reclamation cost}] = 0.994$ .

<sup>6</sup> Deacon (1993) combined development and exploration into one variable.

$$C_3(w) = C_2(w) + RC(w). \quad (44)$$

Exploration efforts produce two kinds of costs, although the exploration efforts facilitate the oil or gas discovery. The derivatives of the total cost of drilling wells are  $C_3'(w) > C_2'(w) > 0$  from  $RC'(w) > 0$  and  $C_3''(w) = C_2''(w) > 0$  from  $RC''(w) = 0$ .

Analytical solutions of the model for the optimal bond are considered with reclamation costs.

The model has one choice variable and one state variable:

$$\begin{aligned} \max_w J &= \int_0^T [(\bar{q}w)p - C_1(R)(\bar{q}w) - C_3(w)] e^{-rt} dt - (1 - e^{-rT})B_0 \\ \text{s.t. } \dot{R} &= f(w) - \bar{q}w \\ R &\geq 0, \bar{q} \geq 0, p \geq 0, w \geq 0, r \geq 0, B_0 \geq 0. \end{aligned} \quad (45)$$

$w$  denotes the depth they drill in region  $i$ . The initial bond posted by a firm can be retrieved at the terminal state. The bond retrieved is smaller than the posted bond due to the opportunity cost of time. The retrieved bond becomes smaller when the planned termination date is longer.

The Hamiltonian from (45) is as follows:

$$H = [(\bar{q}w)p - C_1(R)(\bar{q}w) - C_3(w)] e^{-rt} + \pi [f(w) - \bar{q}w]. \quad (46)$$

The initial posted bond does not affect the optimal solutions because it is a sunk cost from the perspective of the firm. The maximum principles are given as

$$\frac{\partial H}{\partial w} = [\bar{q}p - C_1(R)\bar{q} - C_3'(w)] e^{-rt} + \pi [f'(w) - \bar{q}] = 0 \quad (47)$$

$$\dot{R} = \frac{\partial H}{\partial \pi} = f(w) - \bar{q}w \quad (48)$$

$$\dot{\pi} = -\frac{\partial H}{\partial R} = C_1'(R)(\bar{q}w) e^{-rt}. \quad (49)$$

Comparing the maximum principles (47), (48) and (49) with (6), (7) and (8); the maximum

principle for the optimal bonding system uses  $C_3(w)$  instead of  $C_2(w)$ .<sup>7</sup> Thus,  $\dot{w}$  is represented

as

$$\dot{w} = \frac{C_1'(R)(\bar{q}w)[f'(w) - \bar{q}]^2 - [C_1'(R)\{f(w) - (\bar{q}w)\}\bar{q} + r\{\bar{q}p - C_1(R)\bar{q} - C_3'(w)\}]\{f'(w) - \bar{q}\}}{[C_3''(w)\{f'(w) - \bar{q}\} + f''(w)\{\bar{q}p - C_1(R)\bar{q} - C_3'(w)\}]} \quad (50)$$

Because  $C_3(w)$  is represented as a quadratic function, the signs of the derivatives are determined as  $C_3'(w) > 0$  and  $C_3''(w) > 0$ . Because the derivatives of  $C_3(w)$  are

interchangeable with those of  $C_2(w)$  due to the signs of themselves<sup>8</sup>. Thus,  $\dot{w} = 0$  is

convex to the origin because  $\frac{dR}{dw} < 0$  and  $\frac{d^2R}{dw^2} < 0$ . The curve  $\dot{R} = 0$  is a vertical line in the

phase space. The phase diagram is represented in figure 2. Because the signs of the

derivatives of  $C_3(w)$  are same with the derivatives of  $C_2(w)$ , the phase diagram

representation is similar with the Pindyck's model. When  $\dot{w} = 0$ , equation (50) is simplified

to:

$$r \left\{ p - C_1(R) - \frac{C_3'(w)}{\bar{q}} \right\} - C_1'(R)[f'(w)w - f(w)] = 0. \quad (51)$$

Because  $\dot{R} = 0$  does not include  $C_3(w)$ , the optimal amount of oil reserve is not different from

the model without reclamation cost. Thus, the optimal well depth must differ from the

previous model<sup>9</sup>.  $C_3'(w)$  makes the first term in (7) smaller. When the optimal amount of

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<sup>7</sup>  $C_3(w) = C_2(w) + R(w)$ .

<sup>8</sup>  $C_2'(w) > 0$  and  $C_2''(w) \geq 0$ .

<sup>9</sup> The exploration effort was represented by the number of wells instead of the well depth for the model without reclamation. But, the concept of exploration effort is same between the number of wells and the well depth.

oil reserves of the model without reclamation is represented as  $R_{w/out}^*$ , the optimal amount of oil reserve of the model with reclamation is denoted as  $R^*$ , where  $R^* = R_{w/out}^* + \mu$ . The new equilibrium must move up or down from the previous equilibrium depending on  $\mu$ . Here, some terms of (7) are used to determine the sign of  $\mu$ . The following relation must be satisfied to hold a new equilibrium from  $C'_3(w) > C'_2(w)$ .

$$\begin{aligned}
 & -rC_1(R_{w/out}^*) - C_1'(R_{w/out}^*) [f'(w^*)w^* - f(w^*)] \\
 & < -rC_1(R^*) - C_1'(R^*) [f'(w^*)w^* - f(w^*)].
 \end{aligned} \tag{52}$$

From  $C'(R) < 0$  and  $C''(R) > 0$ ,  $\mu$  must be a positive value to cancel out the effect of  $C'_3(w)$ . Figure 4 shows the equilibrium with and without reclamation. The optimal number of wells does not change and the optimal amount of reserve increases because  $\dot{w} = 0$  shifts up. The firm makes the production cost lower by increasing the amount of reserve at the equilibrium because the cost from drilling increases by internalizing the cost of reclamation.

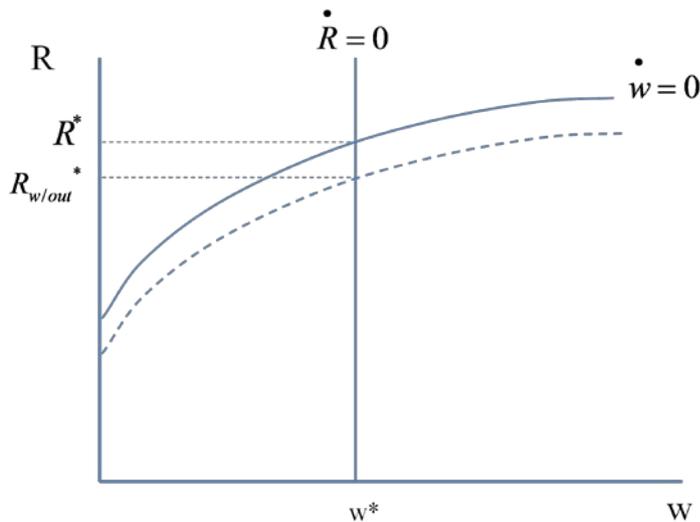


Figure 4. Graph of and  $\dot{R} = 0$  with reclamation

The new phase space has a saddle path because of the consistency of the signs of derivatives.

### *The Optimal Bond*

Next we consider the optimal amount of the bond from (45). The ideal amount of a reclamation bond covers the total reclamation costs. The total reclamation cost is determined by solving (45). Then, the simulation with the regression models determines the optimal values of the choice variable for each time period. Because the total cost of reclamation is equal to the initial bond, the equation is represented as

$$\int_0^T RC(w_t^*)e^{-rt} dt = B_0. \quad (53)$$

$w_t^*$  denotes the current optimal depth of wells, and they are given by the simulation result.

The real value of the posted bond at the terminal period covers the reclamation costs with time preferences. Rearranging (53), the amount of optimal bond is represented as

$$B_0^* = \int_0^T RC(w_t^*)e^{-rt} dt. \quad (54)$$

$B_0^*$  is the exact amount which covers the expected cost of reclamation. Thus, the agency can reclaim the disturbed land with the initially posted bond even if the firm fails to reclaim. The amount of initial bond a firm has to pay changes by the size of the discount rate.

### **3. Case Study**

We apply the above model to oil and gas development in the Rocky Mountain West. Natural resource booms and busts in the energy industry are a common and almost expected in resource dependent regions. From 1988 to 1998 well counts grew at an annual average rate of

15 percent per year compared to 41 percent per year in the period 1998 to 2008.<sup>10</sup> As of 2009, there were more than 68,000 active oil and gas wells in the state operated by approximately 900 separate firms. This level of activity suggests that reclamation issues will become more important in the future as production in these wells ends and are plugged and released or abandoned. Factors that become important in successful reclamation include the regulatory environment, industry structure, and environmental factors associated with the specific location of the field or well. Given the sheer number of wells and their distribution across varying ecological and precipitation regimes, as well as the sharp increase in development over the past decade, the structure and expectations of reclamation regulations becomes an important policy issue for State and Federal Agencies.

The two primary regulations that provide the regulatory basis for reclamation in oil and gas development are the Stock Raising Homestead Act of 1916 (SRHA) and Mineral Leasing Act (MLA). SHRA allowed non-surface owners access to subsurface mineral rights because the rights were held by the Federal government. The Act required that companies compensate surface owners for loss of use at a fair market agricultural level. Amendments to the Act in the latter half of the 20<sup>th</sup> century required these compensations to surface owners be in the form of a bond. MLA introduced the bond to ensure compliance with all the lease terms, which includes protection of environment. The Bureau of Land Management's (BLM) policy

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<sup>10</sup> Author's calculations based on Wyoming Oil and Gas Conservation Commission (WOGCC) data, available online at <http://wogcc.state.wy.us>.

on bonding allows firms to choose from three options for the right to extract: 1) pay a single-site lease fee of \$10,000; 2) pay a blanket bond of \$25,000 for all the wells they drill in the state; or 3) post a blanket bond of \$150,000 that covers all wells in the entire nation.<sup>11</sup>

The Agency's current environmental bonding system suffers from two design flaws: the failure to properly account for time value of money, and that the possibility that a bond level is not tied to production (Andersen and Coupal, 2009). Operators do not receive interest on cash bonds and given the long life of a typical oil and gas well, operators incur a substantial opportunity cost of capital as the initial value is largely forfeited over the life of the operating well.

The use of blanket bonds is also inefficient because it is not linked to production. Webber (1985) argues that the amount of fee, deposit, tax, or bond should vary due to several factors, such as a well location, production technique employed, and the amount of oil or gas produced. The blanket bonds and other current bonding requirements result in bonding amounts that are insufficient to cover the full cost of reclamation as a producer increases the number of operating wells. For example, data on the cost of reclaiming 255 orphaned wells in the state of Wyoming from 1997-2007 showed that bond levels were as low as 20 of the actual cost (Table 1).<sup>12</sup> These cost figures represent the actual costs incurred by contractors for the

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<sup>11</sup> On a per well basis firms need to pay \$10,000 for one lease by the federal surface lease requirement (WORC, 2004).

<sup>12</sup> The data in this analysis were provided by Don Likwartz, *State Oil and Gas Supervisor*, WOGCC (Fall 2008).

WOGCC in the process of fully reclaiming a total of 48 separate locations on state fee lands.

Table 1 shows the actual cost, bond amount, and variance (difference between cost and bond) for the full set of 255 wells: 1) per foot of drilling depth; and 2) per well.

Table 1: Orphaned Oil & Gas Wells in Wyoming (1997-2007)

	Actual Cost	Bond	Variance
per foot of drilling depth	\$10.81	\$1.79	\$9.02
per well	\$29,136	\$5,989	\$23,147

Note: All values in constant 2007 dollars.

While the actual cost of the full reclamation of the 255 wells averaged \$10.81 per foot of well depth, and approximately \$29,136 per well, the bond per foot of well depth was \$1.79, and per well was \$5,589, respectively. Part of the reason why the bond amount per foot of well depth and per well seems low is because the full sample includes some wells that had no bond posted, as their development likely pre-dated the bonding regulations. However, this gives a good indication of the variance that likely currently exists in Wyoming because there is a mix of older wells with no bond posted, and newer wells that are fully bonded. The existence of the older un-reclaimed wells with no bond places an added financial burden on the state, above and beyond insuring that funds are available in the future to reclaim current development.

As shown in Table 1, the posted bond is considerably smaller than the actual costs. This discrepancy between actual costs and posted bond suggests that the current bonding system is

not a viable deterrence to walking away from reclamation obligations. If the bond is set too low and the company reneges on its commitment, the state could get stuck with the bill for the reclamation. On the other hand, if the bond rate is set too high it can have an adverse effect on the industry and a sub-optimal outcome. The optimal bond rate will be determined by a proper accounting of the time dimension of this problem, and incorporation of planned reclamation costs. In order to estimate the optimal bond rate we construct a dynamic optimization model of an oil and gas firm that incorporates the bond and opportunity cost of capital.

This section describes the empirical model we use to estimate optimal bond under different bonding structures. We use GAMS to build a dynamic optimization model based upon the theoretical model in the previous section. The empirical model requires estimation of three functions – average total cost, drilling cost, and reserve addition. We estimate these functions econometrically using data from the Wyoming Oil and Gas Conservation Commission (WOGCC) and the U.S. Energy Information Administration (EIA). All prices and costs used for the estimation are in real 2005 dollars adjusted using the GDP implicit price deflator.

#### *Average production costs*

The average production cost function is assumed to increase as the proved reserves

decrease. The regression estimated by Pindyck was:

$$C_1(R) = \frac{m}{R}. \quad (55)$$

It is an inverse function of proved reserves with a constant,  $m$ . In reality when there are huge gas reserves firms can choose the reserves with lowest cost to produce, although in this paper we consider each well are identical. We modified equation (55) to:

$$C_1(R) = \frac{m}{\text{average gas reserves}} = \frac{m}{R/n}. \quad (55)$$

Where  $R$  represents the total amount of the gas reserve, the average gas reserves are represented as the total reserves divided by the number of wells,  $n$ . The number of wells is given by:

$$n = \frac{w}{k} \quad (56)$$

where  $w$  represents the depth of wells in thousand feet and  $k$  is constant which represents the average depth of wells in thousand feet per well. Equation (55) is then:

$$C_1(R, w) = m \frac{w}{kR}. \quad (57)$$

The data from the Energy Information Agency (EIA) includes annual data on average production costs in the Rocky mountain region from 1989 to 2006. The OLS regression results for the average cost function for the sample of 18 observations is:

$$C_1(R, w) = 998.008 \frac{w}{kR}.$$

(t-statistic) (21.61) (58)

$$R^2 = 0.9699$$

*Drilling costs*

The drilling cost per foot is represented by the function of the depth of wells:

$$\frac{C_2(w)}{w} = 49.924w + 0.0015117w^2. \quad (59)$$

(t-statistics)(4.016)      (15.06)

The quadratic form was chosen because the costs rise exponentially with drill depth.

### *Reserve additions*

The function for the reserve addition is assumed to be strictly increasing and concave.

As the cumulative discovery increases, the reserve addition gets smaller. The amount of gas discovery increases in response to the exploration effort. The discovery function is

represented by the following regression model without a constant:

$$\ln f(w, x) = 1.4106 \ln w - 0.00000006054x \quad (60)$$

(t-statistics) (49.22)      (-3.535)

### *Reclamation costs*

The reclamation cost has a high correlation with depth of wells. The reclamation costs are estimated with the data from WOGCC. The data includes the 255 orphaned wells in Wyoming from 1997 to 2007. The regression model for reclamation cost is represented as:

$$RC(w) = 5800.4w \quad (61)$$

(t-statistic) (63.88)

$$R^2 = 0.9880.$$

The reclamation cost function is a linear function from the origin because the cost of plugging and abandoning does not increase exponentially like the exploration cost function.

#### 4. Simulation results

Simulations are carried out under the following conditions.<sup>13</sup> The simulations are run in GAMS. The interest rate used to a discount rate of 5% annually. The length of the simulation is 20 periods. The well head price of natural gas in Wyoming per million cubic feet (Mcf) at the initial year starts at \$3.50. The price is assumed to increase 1.25% per year as Deacon used it for his simulation. The average depth of well among the orphaned well is 3,500 feet per well. The dynamic model used for the simulation is represented as:

$$\begin{aligned}
 \max_{q,w} J &= \int_0^T [qp - C_1(R,W)q - C_2(w) - RC(w)] e^{-rt} dt - (1 - e^{-rT}) B_0 \\
 \text{s.t. } \dot{R} &= \dot{x} - q \\
 \dot{x} &= f(W, x) \\
 \dot{W} &= w \\
 R \geq 0, q \geq 0, w \geq 0, x \geq 0.
 \end{aligned} \tag{62}$$

The amount of production,  $q$ , and the depth of wells,  $w$ , are control variables. The amount of proved reserve,  $R$ , cumulative gas discovery,  $x$ , and cumulative depth of wells,  $W$ , are state variables. Deacon separated the depth of wells and the cumulative depth of wells (1993). The average production cost denotes  $C_1(R,W)$  from the regression model. The reclamation cost is determined with respect to the depth of wells because gas development causes land disturbance. The developed wells are not reclaimed until the terminal state because the depth of wells assumed to be positive. Thus, all of the developed wells continue their productions. Although the reclamation costs are charged every year, the final reclamation for all wells is

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<sup>13</sup> The dynamic programming model is attached as the appendix.

completed at the terminal state.

The initial well depth is 10 thousand feet and the initial amount of proved reserve is 20 Mcf. The rate of the additional well depth is highest for the first year. Afterwards, the rate of addition continues to decrease, and it hits to zero at the terminal state. Due to the change of the rate of depth additions, the curve of cumulative depth has concave shape. Because Cumulative depth is a variable of the discovery function, the firm does not need higher rate of depth additions after it reached certain level of Cumulative depth.

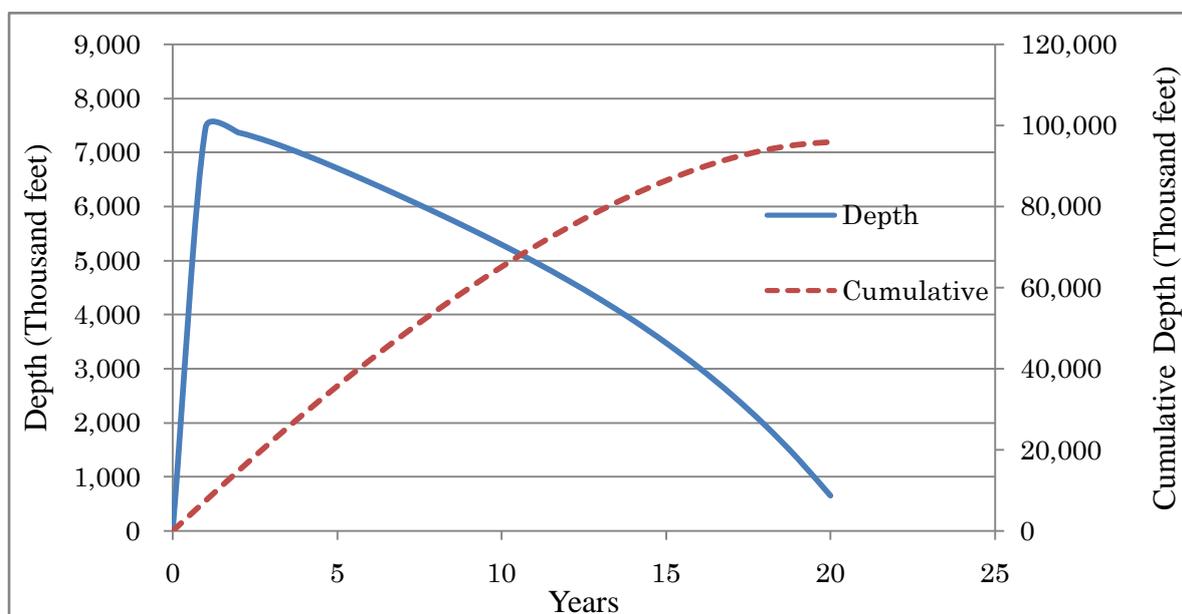


Figure 5. Time paths for drilling (feet  $\times 10^3$ ) and cumulative depth (cf  $\times 10^3$ ).

Although cumulative well depth increases over time, the amount of discovery decreases after the 9<sup>th</sup> year. The decline of the amount of discovery is explained by depletion of natural gas. When cumulative discovery increases, the rate of discovery decreases. The abundant proved reserve reduces the production cost. The amount of gas reserve holds the almost same level from the 10<sup>th</sup> year to the 15<sup>th</sup> year. It indicates that  $\dot{R} = f(w, x) - q \approx 0$ .

Even if simulation results which have different terminal period, they still have stable state in the figures. If the terminal period is extended to 40 years, the stable term is a figure becomes longer.

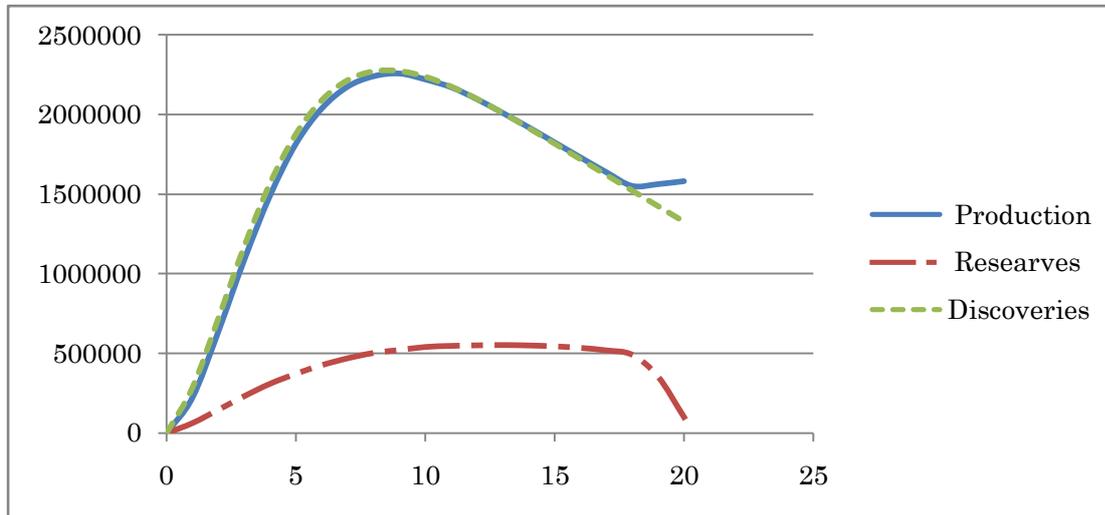


Figure 6. Time paths for reserve, discovery and production (cf × 10<sup>6</sup>).

The cost of reclamation linearly corresponds to the additions of depth. The present value of cumulative reclamation costs are about 386 million dollars. According to the assumption, the depth of the average well is 3,500 feet; about 27,400 wells are drilled until the terminal state, which is calculated from the cumulative well depth. The present value of reclamation cost per well is 14,085 dollars. The estimated reclamation cost is smaller than the actual reclamation cost as shown in table 1; however, if 14,085 dollars is posted as an environmental bond to each well, it is closer to the actual reclamation cost than the actual bond per well.

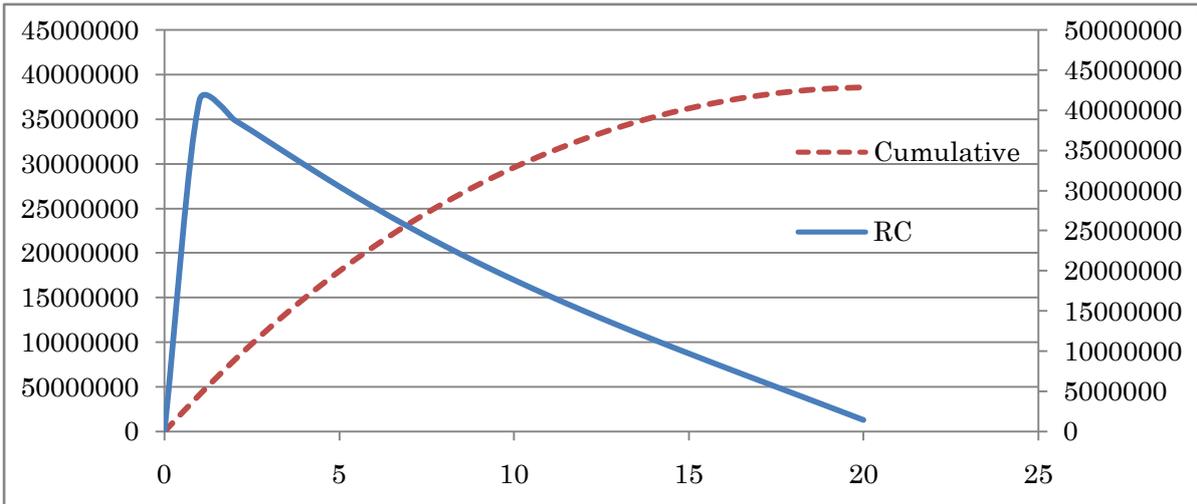


Figure 7. Time paths for current and cumulative reclamation cost (dollar  $\times 10^3$ )

## 5. Conclusion

In this paper we presented analytical and numerical solutions for a dynamic model of profit maximization for oil and gas development with reclamation costs and bonding requirement included. The proposed models are variants to both Pindyck (1978) and Deacon (1993). The analytical solutions represented in the phase diagrams suggest that an optimal bond can be chosen that both allows resource extraction and reclamation. The two policy goals are not at cross purposes. Moreover, internalizing reclamation costs into the decision framework of the firm causes an increase in in-place reserves to well ratios. So not only does society gain by restoring ecosystem services lost as a result of the reclamation and restoration process, but it slows resource exploitation.

The empirical model based upon Section 1 estimated a bond rate appropriate for oil and gas firms in the Rocky Mountain West. Parameters were obtained through econometric

estimation and an optimal bonding rate per well was calculated in GAMS. The optimal bond rate was estimated to be \$14,805 per well, which is close to actual data on the average cost of reclamation per well for orphaned wells in Wyoming.

The analysis only considers bonding and reclamation from the firm perspective. Future research will incorporate agency objectives and public good changes, the loss and then resumption of ecosystem services. Additionally an analysis of when reclamation occurs or the rate of reclamation is another need. Agencies are now pushing for interim reclamation, or partial reclamation during the production period. Finally, another area of inquiry is analyzing the effectiveness of maximum allowable disturbed areas that encourage firms to reclaim as much as they can so that they can develop in other areas.

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## APPENDIX: GAMS Code for the dynamic programming model

OPTION NLP = MINOS;

### Sets

t            Time periods                            /0\*20/  
tfirst(t)    First period of time  
tlast(t)     Last period of time;  
tfirst(t) = yes\$(ord(t) eq 1);  
tlast(t) = yes\$(ord(t) eq card(t));

### Scalars

delta    discount rate                            /0.05/  
p0      initial market (\$ per Bcf)               /3500/  
R0      initial oil reserve (Bcf)                 /20/  
w0      initial well depth                        /10/  
  q0     initial oil production (Bcf)             /150/  
k0      average depth(Thousand feet per well)   /3.5/  
kt      wells drilled                              /0/  
e      e = 2.71828183                            /2.71828183/  
x0      initial amt of total addition             /0/  
m      R1                                         /998.008/  
A    disc for constant                           /1/  
alpha   disc for lnw                             /1.4106/  
alpha2   disc for x                             /-0.000000060541/  
beta    C2                                         /49.24/  
beta2   C2                                         /0.0015117/  
gamma1   RC w(t)                                /5800.4/  
PI\_t    profit at t                                /0/  
year    periods                                   /0/  
RC      Reclamation Cost                        /0/;

### Variables

Z      Profit;

### Positive Variables

R(t)    amount of oil reserve  
w(t)    depth of wells  
k(t)    wells drilled  
x(t)    total addition of oil  
p(t)    price of oil

$q(t)$  oil output  
 $disc(t)$  discovery  
 $Pt(t)$  Profit at  $t$   
 $DPt(t)$  Profit at  $t$   
 $tw(t)$  Total depth;

Equations

$SEQ\_R(t)$  State eq of R  
 $SEQ\_x(t)$  State eq of x  
 $SEQ\_tw(t)$  State eq of tw  
 $EQ\_p(t)$  Eq of p  
 $EQ\_disc(t)$  Eq of discovery  
 Profit profit function;

$$EQ\_disc(t).. \text{disc}(t) = e = A * (tw(t)**alpha) * e**(alpha2*x(t));$$

$$SEQ\_R(t).. R(t) = e = R0$(tfirst(t)) + (R(t-1) + disc(t) - q(t)) $(not tfirst(t));$$

$$SEQ\_x(t).. x(t) = e = x0$(tfirst(t)) + (x(t-1) + disc(t)) $(not tfirst(t));$$

$$SEQ\_tw(t).. tw(t) = e = w0$(tfirst(t)) + (tw(t-1) + w(t))$(not tfirst(t));$$

$$EQ\_p(t).. p(t) = e = p0$(tfirst(t)) + (p(t-1)*(1+delta/4)) $(not tfirst(t));$$

\*Boundary conditions

$$w.lo(t) = 0.001;$$

$$R.lo(t) = 0.001;$$

$$q.lo(t) = 0.001;$$

$$p.lo(t) = 0.001;$$

\*Initial values of choice variables

$$w.fx(tfirst(t)) = w0;$$

$$q.fx(tfirst(t)) = q0;$$

model pindyck /all/;

\*NLP solution

solve pindyck using nlp maximizing Z;

DISPLAY w.l,q.l,R.l,p.l, disc.l, Pt.l, DPt.l;