UW Steamboat Engineering

ME/ESE 2020 – Department of Mechanical Engineering
University of Wyoming, Laramie, WY 82071

January 15, 2010

Mr. Richard (Dick) Tate Wourq, CEO and Principle
UW Steamboat Engineering
EN2046 North Hall Way
Laramie, WY 82071

Dear Mr. Wourq:

Subject: Aerodynamic significance pre-lab for the Great Dodge Viper 1:36-Scale Pullback Car Rally

The attached pre-lab, entitled ***Dodge Viper GTS-R Aerodynamic Drag Significance Pre-lab***, gives the background and procedures for experimentally determining whether aerodynamic drag will need to be considered in the analysis for the Great Dodge Viper Rally contract entered into by UW Steamboat Engineering with Gamesters Desktop Rallies, £td, on January 9, 2010. The preliminary conclusion from sample calculations is that aerodynamic drag will not be a significant force.

Sincerely,

Ima Student

Ima Student, Junior Engineer

Attachments:

Appendix A – Modeling of aerodynamic drag and acceleration forces on KiNSMART™ Dodge Viper GTS-R 1:36 scale model pull-back cars

Appendix B – Procedures for experimentally finding parameters for calculating aerodynamic drag and the constant acceleration force for KiNSMART™ Dodge Viper GTS-R 1:36 scale model pull-back cars

DATE: 1/15/2010

TO: Dick Tate Wourq, Principle
UW Steamboat Engineering

FROM: Ima Student

SUBJECT: **DODGE VIPER GTS-R AERODYNAMIC DRAG SIGNIFICANCE PRE-LAB**

Gamesters Desktop Rally, ₤td. (**GDR₤**) is developing a desktop rally game using KiNSMART™ Dodge Viper GTS-R 1:36 scale model pull-back race cars, Item No. KT5039D. On January 9, 2010 **GDR₤** contracted with UW Steamboat Engineering (**UWSE**) to analyze and optimize the final leg of the rally course. Modeling for the final leg identified three conservative forces and three non-conservative forces acting on the car.

Aerodynamic drag is one of the non-conservative forces, but it is expected to be insignificant as the car velocities are expected to be low. If the aerodynamic drag is not a significant force, it can be neglected in subsequent analyses. This pre-lab gives experimental procedures to determine whether aerodynamic drag is or is not a significant force compared to other conservative and non-conservative forces acting on the pull-back car. Approval is requested to proceed with this experiment.

Experiment Overview

The general procedure to determine the significance of aerodynamic drag on the Dodge Viper GTS-R pull-back car will be to compare a calculated drag with a constant acceleration force. The calculated drag will be that on a 2-D flat plate with area equivalent to the measured projected frontal area of the car at the speed obtained by the car on a flat, level surface at the end of a powered run. The estimated constant acceleration force will be the force required to accelerate the car to that same speed. Negligible aerodynamic drag is a drag at least an order of magnitude less than the constant acceleration force. If this condition is not met, the less conservative drag on a sphere of equivalent projected area will be calculated. If the drag force is still not at least an order of magnitude smaller than the acceleration force, the aerodynamic drag is significant and will be included in subsequent Dodge Viper GTS-R dynamic analyses.

Aerodynamic Drag and Acceleration Force Modeling

Aerodynamic Drag. Aerodynamic drag is proportional to the projected area of an object, a coefficient of drag that is dependent upon the shape of the object and the Reynolds number of the flow of air around that object, the density of the air, and the relative speed squared between the object and the air. The aerodynamic drag force, F_D, in terms of the measured parameters of this experiment, is (Roberson, *et al.*, 1997, p. 431-438):

$$F_D = C_D A_p \rho \frac{V_a^2}{2} = C_D \frac{W_{np} A_r}{n W_r} \rho \frac{\left(\dfrac{2L_p}{t}\right)^2}{2} \tag{1}$$

where the projected area of the car, $A_{p,}$ is calculated from the ratio of the weight of *n* projection cutouts, W_{np}, divided by *n* to the weight of a reference cutout, W_{np}, times the area of the reference cutout, A_r. C_D is a non-dimensional drag coefficient, ρ is the air density, and the maximum speed of the car, V_a, is calculated from twice the length of the powered run, L_p, divided by the time for the powered run, *t*.

Acceleration Force. Acceleration forces are calculated using Newton's Second Law, i.e. $F=ma$. The acceleration force, F_a, on the pull-back car in terms of the measured parameters of this experiment is (Tipler, 1991, p. 81):

$$F_a = m_c a = \frac{2 m_c L_p}{t^2} \qquad (2)$$

where m_c is the mass of the car and the average acceleration, *a*, is calculated by dividing the speed, V_a, by *t*.

Detailed derivations of Equations 1 and 2 as well as uncertainty calculations for these values are shown in *Appendix A – Modeling of aerodynamic drag and acceleration forces on KiNSMART™ Dodge Viper GTS-R 1:36 scale model pull-back cars*.

Procedures

The experimental parameters that must be found to calculate the aerodynamic drag, $F_{D,}$ and the constant acceleration force, $F_{a,}$ of the pull-back car are the projected frontal area of the car, the powered distance the car travels, and the time for the car to accelerate through the powered distance. The project frontal area of the car, $A_{p,}$ will be found by the ratio of the weight of a paper cutout of the projected frontal area of the car to the weight of a known area of paper of the same paper stock as the projected frontal area paper cutout.

The powered run distance, L_p, for the car will be found by fully winding the spring motor of the pullback car and then slowly allowing the car to travel the distance required to unwind the spring motor. The speed of the car in this procedure must be slow enough that insignificant amounts of kinetic energy are developed in the car. The time for the car to accelerate through this powered run distance will be measured by timing a fully wound car from a standing stop through the previously determined powered run distance.

Detailed procedures are given in *Appendix B – Procedures for experimentally finding parameters for calculating aerodynamic drag and the constant acceleration force for KiNSMART™ Dodge Viper GTS-R 1:36 scale model pull-back cars*.

Sample Calculation Results

Assuming the following parameters:

- powered run distance, L_p, – 72.0 inches,
- powered run traverse time, *t*, – 1.00 seconds,
- projected frontal area of the car, A_p, – 0.0013 square meters,

- air density – 1 kilogram per cubic meter at the elevation of Laramie, Wyoming,
- mass of the car of 0.1000 kilograms, and
- drag coefficient – 2 for a 2-D flat plate,

the drag force, F_D, was calculated as 0.017±0.002 Newtons and the constant acceleration force, F_a, was calculated as 0.366± 0.007 Newtons at the 95% confidence level. Detailed sample calculations are shown in *Appendix C – Sample calculations for experimentally finding aerodynamic drag and acceleration forces for KiNSMART™ Dodge Viper GTS-R 1:36 scale model pull-back cars*. The worksheet for collecting the experimental data with the sample data calculations is shown in *Appendix D – Worksheet, with sample values, for laboratory aerodynamic drag and acceleration force calculations for KiNSMART™ Dodge Viper GTS-R 1:36 scale model pull-back cars*.

Conclusions

Since the calculated drag force, using realistic conservative values, is lower by more than an order of magnitude than the constant acceleration force, the aerodynamic drag force will likely be an insignificant force in the Great Dodge Viper Rally analyses. Even though the uncertainty of the aerodynamic drag is about 12% compared to about 2% for the acceleration force, the maximum aerodynamic force, 0.017 Newtons, is still more than an order of magnitude lower than the minimum acceleration force, 0.366 Newtons. Experimental values must be obtained to verify this conclusion. However, the difference is expected to be even greater, since the actual coefficient of drag from the streamlined Dodge Viper GTS-R 1:36 scale model pull-back car will be much lower than that assumed, i.e. the actual drag coefficient will be about one quarter of the drag coefficient for a flat plate. The assumed acceleration is thought to be somewhat non-conservative, making this analysis highly sensitive to errors in acceleration. Speed is proportional to acceleration, and drag is proportional to the speed squared. If the actual measured speed proves to be significantly higher than the speed assumed in the sample calculations, experimental data could show the aerodynamic force is significant and must be considered.

References

Roberson, John A. and Crowe, Clayton T., 1997, "Engineering Fluid Mechanics," 6th Edition, John Wiley & Sons, New York, NY.

Tipler, Paul A., 1991, "Physics for Scientist and Engineers," 3rd Edition, Worth Publishers, New York, NY.

Appendix A – Modeling of aerodynamic drag and acceleration forces on KiNSMART™ Dodge Viper GTS-R 1:36 scale model pull-back cars

List of Variables

a	Acceleration of car (m/s^2)
A_p	Projected frontal area of a single car (m^2)
A_r	Reference area of paper (m^2)
C_D	Drag coefficient of the car
F_D	Aerodynamic drag for on the car (N)
L_p	Powered distance of a car run (m)
L_r	Length of reference paper rectangle (m)
L_w	Width of reference paper rectangle (m)
m_c	Mass of car (kg)
n	Number of items
R_e	Reynolds Number
s_a	Standard error of analog measurements (Units same as measurement units)
s_d	Standard error of digital measurements (Units same as measurement units)
smu_a	Small measuring unit of analog instrument (Units same as measurement units)
smu_d	Small measuring unit of digital instrument (Units same as measurement units)
t	Time (s)
$t_{\%ci}$	Student's *t* probability for the selected confidence interval percentage
U_a	Uncertainty of an analog measured value (Units same as measurement units)
U_c	Uncertainty of a calculated value (Units same as calculated value)
U_d	Uncertainty of a digital measured value (Units same as measurement units)
U_{Fa}	Uncertainty of acceleration force (N)
U_{FD}	Uncertainty of aerodynamic drag (N)
V_a	Relative speed between the air and the car (m/s)
W_{np}	Weight of *n* paper cutouts of the frontal projection of the car (g)
W_r	Weight of the reference paper area (g)
ρ	Air density (1.0 kg/m^3 – at elevation of Laramie, Wyoming)
$\tau_{\%ci}$	Gaussian probability for the selected confidence interval percentage

Aerodynamic Drag and Acceleration Force Modeling

Parameter Analysis. Aerodynamic drag is proportional to the projected area of an object, a coefficient of drag that is dependent upon the shape of the object and the Reynolds number of the flow of air around that object, the density of the air, and the speed squared of the object relative to the air. The aerodynamic drag force F_D on KiNSMART™ Dodge Viper GTS-R 1:36 scale model pull-back cars is (Roberson, *et al.*, 1997, p. 431-438):

$$F_D = C_D A_p \rho \frac{V_a^2}{2} \tag{A1}$$

where C_D is a non-dimensional drag coefficient, A_p is the projected area of the car, ρ is the air density, and V_a is the speed of the air relative to the car.

The projected area of the pull-back car, A_p, will be determined experimentally, as will the maximum expected speed of the car, V_a. A_p will be determined by the ratio of weights of paper projections to a known paper area:

$$A_p = \frac{W_{np} A_r}{n W_r} \tag{A2}$$

where n is the number of projected area cutouts, W_{np} is the total weight of all projected area cutouts, W_r is the weight of the reference rectangle, and A_r is the area of the reference rectangle. The area of the reference rectangle paper is the length of the rectangle, L_r, times the width of the rectangle, L_w:

$$A_r = L_r L_w \tag{A3}$$

Substituting the right hand side (RHS) of Equation A3 into Equation A2 for A_r gives:

$$A_p = \frac{W_{np} L_r L_w}{n W_r} \tag{A4}$$

Maximum speed, V_a, will be estimated by timing the traverse of the car through the powered distance, L_p, after pulling back the car to the maximum possible pull-back distance. Assuming constant acceleration, a, from a standing stop, the distance traversed is given by (Tipler, 1991, p. 30-31):

$$L_p = \frac{at^2}{2} \tag{A5}$$

where t is the time for the car to traverse the powered distance of the run. Substituting V_a for at in Equation A5 gives (Tipler, 1991, p. 30-31):

$$L_p = \frac{V_a t}{2} \tag{A6}$$

Solving Equation A6 for V_a then gives V_a as a function of L_p and t.

$$V_a = \frac{2L_p}{t} \tag{A7}$$

Substituting the RHS of Equation A4 for A_p and the RHS of Equation A7 for V_a into Equation A1 gives:

$$F_D = C_D \frac{W_{np} L_r L_w}{n W_r} \rho \frac{\left(\frac{2L_p}{t}\right)^2}{2} = 2C_D \frac{W_{np} L_r L_w}{n W_r} \rho \left(\frac{L_p}{t}\right)^2 \quad \text{(A8)}$$

The average acceleration of the car can be found by rearranging Equation A5:

$$a = \frac{2L_p}{t^2} \quad \text{(A9)}$$

and the acceleration force, F_a, can be estimated using Newton's Second Law and substituting Equation A9 for a, which then gives (Tipler, 1991, p. 81):

$$F_a = m_c a = \frac{2m_c L_p}{t^2} \quad \text{(A10)}$$

where m_c is the mass of the pull-back car.

Uncertainty Analysis. The uncertainty of the direct measurements, U_a for analog instruments and U_d for digital instruments, will be determined from the maximum of statistical or measurement uncertainties according to Equation A11 for analog measurements (*Experimental Errors*, 2010, Equation 17):

$$U_a = \max\left(\frac{t_{\%ci} s_a}{\sqrt{n}}, \frac{\tau_{\%ci} \mathrm{smu}_a}{\sqrt{12}}\right) \quad \text{(A11)}$$

And Equation A12 for measurements with digital instruments (*Experimental Errors*, 2010, Equation 17):

$$U_d = \max\left(\frac{t_{\%ci} s_d}{\sqrt{n}}, \frac{\tau_{\%ci} \mathrm{smu}_d}{2}\right) \quad \text{(A12)}$$

where n is the number of measurements in a sample; smu_a and smu_d are the smallest measuring unit of analog and digital instruments respectively; s_a and s_d are sample standard deviations of the analog and digital measurements respectively; $t_{\%ci}$ is Student's t probability for the selected confidence interval percentage for $n-1$ degrees of freedom; and $\tau_{\%ci}$ is the Gaussian probability for the selected confidence interval percentage. The direct measurements in this particular experiment include L_p, L_r, and L_w, measured with analog measuring instruments, and m_c, t, W_{np}, and W_r, measured with digital measuring instruments.

The general uncertainty, U_c, of calculated values, F_D and F_a in this particular experiment, are calculated by a Taylor Series expansion approximation of the propagation of errors (*Propagation of Errors*, 2010, Equation 7):

$$U_c = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial R}{\partial x_i} U_{x_i} \bigg|_{\bar{x}_1, \bar{x}_2, \cdots, \bar{x}_n} \right)^2} \quad \text{(A13)}$$

where n is the number of parameters which comprise the calculated function R, and $\bar{x}_i$ is the mean value for any particular one of the n parameters in function R. Assuming the drag coefficient, C_D, the density of air, ρ, the constant 2. and the number of cutouts, n, are all without uncertainty, the particular equation for the uncertainty of the calculated value F_D, U_{FD}, from Equation A8 is:

$$U_{F_D} = \sqrt{\begin{array}{l}\left(\frac{\partial F_D}{\partial W_{np}} U_{W_{np}}\right)^2 + \left(\frac{\partial F_D}{\partial W_r} U_{W_r}\right)^2 + \left(\frac{\partial F_D}{\partial L_r} U_{L_r}\right)^2 + \\ \left(\frac{\partial F_D}{\partial L_w} U_{L_w}\right)^2 + \left(\frac{\partial F_D}{\partial L_p} U_{L_p}\right)^2 + \left(\frac{\partial F_D}{\partial t} U_t\right)^2\end{array}} \quad \text{(A14)}$$

The partial differential equations for F_D contained in U_{FD} are:

$$\frac{\partial F_D}{\partial W_{np}} = 2C_D \frac{L_r L_w}{nW_r} \rho \frac{L_p^2}{t^2} \quad \text{(A15)}$$

$$\frac{\partial F_D}{\partial W_r} = -2C_D \frac{W_{np} L_r L_w}{nW_r^2} \rho \frac{L_p^2}{t^2} \quad \text{(A16)}$$

$$\frac{\partial F_D}{\partial L_r} = 2C_D \frac{W_{np} L_w}{nW_r} \rho \frac{L_p^2}{t^2} \quad \text{(A17)}$$

$$\frac{\partial F_D}{\partial L_w} = 2C_D \frac{W_{np} L_r}{nW_r} \rho \frac{L_p^2}{t^2} \quad \text{(A18)}$$

$$\frac{\partial F_D}{\partial L_p} = 4C_D \frac{W_{np} L_r L_w}{nW_r} \rho \frac{L_p}{t^2} \quad \text{(A19)}$$

$$\frac{\partial F_D}{\partial t} = -4C_D \frac{W_{np} L_r L_w}{nW_r} \rho \frac{L_p^2}{t^3} \quad \text{(A20)}$$

Substituting the RHS of Equations A15 through A20 into Equation A12 for $\frac{\partial F_D}{\partial W_{np}}$, $\frac{\partial F_D}{\partial W_r}$, $\frac{\partial F_D}{\partial L_r}$, $\frac{\partial F_D}{\partial L_w}$, $\frac{\partial F_D}{\partial L_p}$, and $\frac{\partial F_D}{\partial t}$ respectively gives:

$$U_{F_D} = \sqrt{\begin{array}{l}\left(2C_D \frac{L_r L_w}{nW_r} \rho \frac{L_p^2}{t^2} U_{W_{np}}\right)^2 + \left(-2C_D \frac{W_{np} L_r L_w}{nW_r^2} \rho \frac{L_p^2}{t^2} U_{W_r}\right)^2 + \\ \left(2C_D \frac{W_{np} L_w}{nW_r} \rho \frac{L_p^2}{t^2} U_{A_r}\right)^2 + \left(2C_D \frac{W_{np} L_r}{nW_r} \rho \frac{L_p^2}{t^2} U_{A_r}\right)^2 + \\ \left(4C_D \frac{W_{np} L_r L_w}{nW_r} \rho \frac{L_p}{t^2} U_{L_p}\right)^2 + \left(-4C_D \frac{W_{np} L_r L_w}{nW_r} \rho \frac{L_p^2}{t^3} U_t\right)^2\end{array}} \quad \text{(A21)}$$

Assuming the constant 2 is without error, the particular equation for the uncertainty of the calculated acceleration force value F_a, U_{Fa}, from Equation A10 is:

$$U_{F_a} = \sqrt{\left(\frac{\partial F_a}{\partial m_c} U_{m_c}\right)^2 + \left(\frac{\partial F_D}{\partial L_p} U_{L_p}\right)^2 + \left(\frac{\partial F_D}{\partial t} U_t\right)^2} \qquad \text{(A22)}$$

The partial differential equations for F_a contained in U_{Fa} are:

$$\frac{\partial F_a}{\partial m_c} = 2\frac{L_p}{t^2} \qquad \text{(A23)}$$

$$\frac{\partial F_a}{\partial L_p} = 2\frac{m_c}{t^2} \qquad \text{(A24)}$$

$$\frac{\partial F_D}{\partial t} = -4\frac{m_c L_p}{t^3} \qquad \text{(A25)}$$

Substituting the RHS of Equations A23 through A25 into Equation A22 for $\frac{\partial F_a}{\partial m_c}$, $\frac{\partial F_a}{\partial L_p}$, and $\frac{\partial F_a}{\partial t}$ respectively gives:

$$U_{F_a} = \sqrt{\left(2\frac{L_p}{t^2} U_{m_c}\right)^2 + \left(2\frac{m_c}{t^2} U_{L_p}\right)^2 + \left(-4\frac{m_c L_p}{t^3} U_t\right)^2} \qquad \text{(A26)}$$

Appendix A References

Roberson, John A. and Crowe, Clayton T., 1997, "Engineering Fluid Mechanics," 6th Edition, John Wiley & Sons, New York, NY.

Tipler, Paul A., 1991, "Physics for Scientist and Engineers," 3rd Edition, Worth Publishers, New York, NY.

"Experimental Errors," (Equation 17), Retrieved January 6, 2010 from http://wwweng.uwyo.edu/classes/meref/Experimental_Errors.pdf.

"Propagation of Errors," (Equation 7), Retrieved January 6, 2010 from http://wwweng.uwyo.edu/classes/meref/Propagatin_of_Errors.pdf.

Appendix B – Procedures for experimentally finding parameters for calculating aerodynamic drag and the constant acceleration force for KiNSMART™ Dodge Viper GTS-R 1:36 scale model pull-back cars

To calculate and then compare the aerodynamic drag and the constant acceleration force of KiNSMART™ Dodge Viper GTS-R 1:36 scale model pull-back cars, three parameters must be experimentally determined: the projected frontal area of the car, the powered run distance the car travels, and the time for the car to accelerate through the powered run distance. The projected frontal area of the car will be calculated from the ratio of weights of a known area of paper to the weight of a paper cutout of the projected frontal area of the car made from the same paper as the known area paper. The powered distance for the car will be found by fully winding the spring motor of the pullback car and then slowly allowing the car to travel the distance required to unwind the spring motor. The time for the car to accelerate through the powered distance will be measured by timing a fully wound car accelerating from a standing stop through the previously determined powered run distance.

Procedures

Projected Frontal Area. If the laboratory day has bright sunlight, the projected frontal area of the car will be determined from the solar shadow of the car by performing steps 1a through 3a, assuming the rays of sunlight are straight and parallel. A diagram for implementing the solar shadow procedure is shown in Figure B-1.

1a. Hold a clipboard with a sheet of engineering composition paper perpendicular to the rays of the sun, hold the Dodge Viper GTS-R 1:36 scale model pull-back car parallel to the rays of the sun, and trace around the shadow cast by the car.
2a. Move to a clear spot on the paper and repeat Step 1 *n* times, *n* being a minimum of ten.
3a. Cut out all the car frontal profiles with scissors.

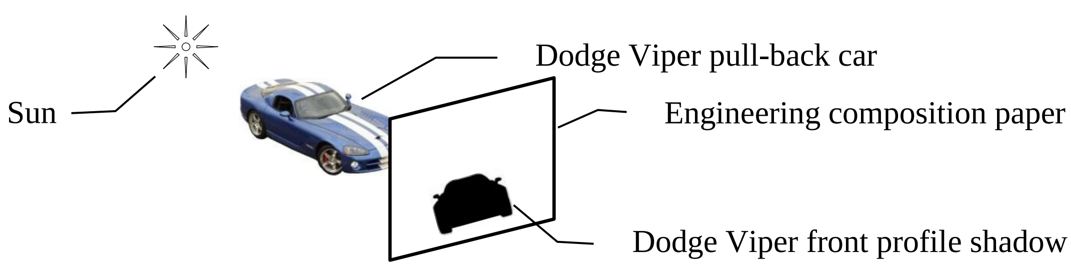

Figure B-1 Layout Diagram for determining Dodge Viper projected frontal area from solar shadows

If the laboratory day is overcast, the projected frontal area of the car will be determined by digitizing the projected outline with a right triangle as detailed in steps 1b through 3b. A diagram for implementing the right triangle procedure is shown in Figure B-2.

1b. Hold the car against a vertical surface and perpendicular to a sheet of engineering composition paper. Plot points on the paper by placing a right triangle against the car as shown in Figure B-2 by marking the location of the 90° corner of the right triangle. Plot points every ¼ inch or so, or use closer spacing as needed to define small details. Connect the points to form the outline of the frontal projection of the car.
2b. Move to a clear spot on the paper and repeat Step 1 *n* times, *n* being a minimum of ten.
3b. Cut out all the car frontal profiles with scissors.

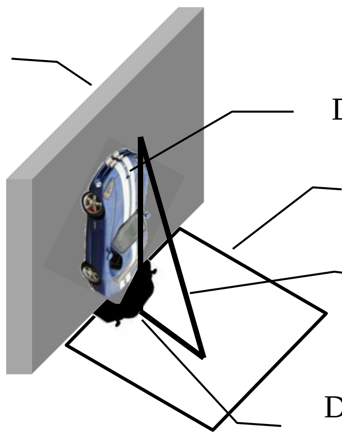

Figure B-2 Layout Diagram for determining Dodge Viper projected frontal area using a right triangle

Once the car outlines are completed, the projected frontal area of the car is determined by a weight ratio of the profile cutouts to a known area of paper. The procedure is given with steps 4 through 9.

4. Cut out a rectangle of known area from engineering composition paper of the same paper weight as the paper used for the projected frontal area of the car cutouts. A suggested size is a length, L_r, of 10 inches, and width, L_w, of 5 inches, giving a reference area, A_r, of 50 square inches. Measure the length and width of the reference paper cutout with a Great Neck 1250Y tape measure.
5. Record L_r and L_w in the laboratory worksheet shown in Appendix D
6. Calibrate the AccuLab VI-1200 electronic scale.
7. Weigh all *n* of the car outline cutouts together on the AccuLab VI-1200 electronic scale and record the weight in the Appendix D laboratory worksheet.
8. Weigh the 10-inch wide by 5-inch high reference paper rectangle on the AccuLab VI-1200 electronic scale and record the weight in the Appendix D laboratory worksheet.
9. Calculate the uncertainties of the weights of the *n* frontal area cutouts, U_{Wnp}, and the reference cutout, U_{Wr}, using Equation A12. Calculate the uncertainties of the lengths of the reference rectangle, L_r, and the width of the reference rectangle, L_w, using Equation A11. These calculations are to be made in the Appendix D laboratory worksheet.

Maximum Speed. Measure the powered run distance, L_p, of the car by:

1. placing a strip of masking tape on the floor as a starting point,

2. pulling the Dodge Viper GTS-R 1:36 scale model pull-back car backwards until a first ratcheting click is heard, indicating the spring in the car is fully wound,
3. picking up the car while holding the rear wheels of the car to prevent motion,
4. placing the car so the rear bumper of the car is even with the forward edge of the strip of tape,
5. placing an object, such as one's finger, in front of the car to prevent motion,
6. releasing the wheels of the car,
7. slowly moving the object to allow the car to move forward until the car no longer moves,
8. measuring the distance from front edge of the tape to the rear bumper of the car with a Great Neck 1250Y tape measure,
9. recording the results in the Appendix D worksheet,
10. repeating Steps 1 through 9 a minimum of ten times, and
11. calculating and recording in the Appendix D worksheet the uncertainty of L_p using Equation A11.

The time, t, for the powered run distance of the car will be measured by:

1. placing a strip of masking tape on the floor so the forward edge is at distance L_p from the forward edge of the starting point tape,
2. pulling the Dodge Viper GTS-R 1:36 scale model pull-back car backwards until a first ratcheting click is heard, indicating the spring in the car is fully wound,
3. placing the car with rear bumper even with the forward edge of the tape,
4. releasing the car and simultaneously starting a NuLine SM10532 stop-watch timer from zero,
5. stopping the stop-watch timer when the rear bumper of the car passes the forward edge of the masking tape at distance L_p from the forward edge of the starting point tape,
6. recording the results in the Appendix D worksheet, and
7. repeating Steps 1 through 6 a minimum of ten times.
8. calculating and recording in the Appendix D worksheet the uncertainty of t using Equation A12.

The maximum speed is then estimated from Equation A7, assuming constant acceleration as estimated with Equation A9. The assumption of constant acceleration is not quite accurate, since the torque from the spring motor of the car will decrease somewhat as it unwinds. However, such an assumption will conservatively predict the aerodynamic drag, since the predicted maximum speed will be greater than what will actually be achieved. The acceleration force is not conservatively estimated; since the estimated constant acceleration will be greater than the actual acceleration when the pull-back car reaches maximum speed. The use of a drag coefficient for a flat plate should more than compensate for the fact that the acceleration force is not conservatively estimated.

Appendix C – Sample calculations for experimentally finding aerodynamic drag and acceleration forces for KiNSMART™ Dodge Viper GTS-R 1:36 scale model pull-back cars

Sample Calculation Results

Assuming:

- the weight of ten (*n=10*) KiNSMART™ Dodge Viper GTS-R 1:36 scale model pull-back car projected frontal area cutouts, W_{np}, is 0.0010 kilograms,
- the weight of the corresponding rectangular cutout, W_r, is 0.0025 kilograms,
- the length of the corresponding rectangular cutout, L_r, is 0.254 meters,
- the width of the corresponding rectangular cutout, L_w, is 0.127 meters,
- the powered distance, L_p, of the car of is 1.829 meters (72.0 inches),
- the traverse time of the distance L_p is t, is 1.0 seconds, and
- the mass of the pull-back car, m_c, is 0.1000 kilograms

the estimated aerodynamic force on the car, F_D, using Equation A8 is:

$$F_D = C_D \frac{W_{np} L_r L_w}{n W_r} \rho \frac{\left(\frac{2L_p}{t}\right)^2}{2} = 2\frac{0.0010*0.254*0.127}{10*0.0025} 1 \frac{\left(\frac{2*1.829}{1.00}\right)^2}{2} = 0.017 \text{ N}$$

where the drag coefficient, C_D = 2, the number, n = 10, and the air density, ρ = 1 kg/m^3, are assumed to be without error.

Using Equation A10, the estimated acceleration force on the pull-back car, F_a, required to maintain constant acceleration, a, is:

$$F_a = \frac{2 m_c L_p}{t^2} = 2\frac{0.1000*1.829}{1.00^2} = 0.366 \text{ N}$$

where the constant 2 is assumed to be without error.

Since measurements have not yet been taken, the measurement instrument uncertainties, calculated using Equations A11 and A12 are used to compute the uncertainties of F_D and F_a in the sample calculations. These uncertainties are:

- $U_{W_{np}} = 0.0001 \text{ kg}$
- $U_{W_r} = 0.0001 \text{ kg}$
- $U_{L_r} = 0.001 \text{ m}$
- $U_{L_w} = 0.001 \text{ m}$
- $U_{L_p} = 0.001 \text{ m}$
- $U_t = 0.01 \text{ s}$
- $U_{m_c} = 0.0001 \text{ kg}$

Equation A20 calculates the uncertainty, U_{FD}, of the aerodynamic drag force, F_D:

$$U_{F_D} = \sqrt{\begin{aligned}&\left(2C_D \frac{L_r L_w}{nW_r}\rho\frac{L_p^2}{t^2}U_{W_{np}}\right)^2 + \left(-2C_D \frac{W_{np} L_r L_w}{nW_r^2}\rho\frac{L_p^2}{t^2}U_{W_r}\right)^2 + \\ &\left(2C_D \frac{W_{np} L_w}{nW_r}\rho\frac{L_p^2}{t^2}U_{L_r}\right)^2 + \left(2C_D \frac{W_{np} L_r}{nW_r}\rho\frac{L_p^2}{t^2}U_{L_w}\right)^2 + \\ &\left(4C_D \frac{W_{np} L_r L_w}{nW_r}\rho\frac{L_p}{t^2}U_{L_p}\right)^2 + \left(-4C_D \frac{W_{np} L_r L_w}{nW_r}\rho\frac{L_p^2}{t^3}U_t\right)^2\end{aligned}}$$

$$U_{F_D} = \sqrt{\begin{aligned}&\left(2*2.0\frac{0.254*0.127}{10*0.0025}1.0\frac{1.829^2}{1.00^2}*0.0001\right)^2 + \\ &\left(-2*2.0\frac{0.0010*0.254*0.127}{10*0.0025^2}1.0\frac{1.829^2}{1.00^2}*0.0001\right)^2 + \\ &\left(2*2.0\frac{0.0010*0.127}{10*0.0025}1.0\frac{1.829^2}{1.00^2}*0.001\right)^2 + \\ &\left(2*2.0\frac{0.0010*0.254}{10*0.0025}1.0\frac{1.829^2}{1.00^2}*0.001\right)^2 + \\ &\left(4.0*2.0\frac{0.0010*0.254*0.127}{10*0.0025}1.0\frac{1.829}{1.00^2}*0.001\right)^2 + \\ &\left(-4.0*2.0\frac{0.0010*0.254*0.127}{10*0.0025}1.0\frac{1.829^2}{1.00^3}*0.01\right)^2\end{aligned}}$$

$$U_{F_D} = \sqrt{3.60\text{x}10^{-6}} = 0.002 \text{ N}$$

Equation A25 calculates the uncertainty, U_{Fa}, of the acceleration force, F_a:

$$U_{F_a} = \sqrt{\left(\frac{2L_p}{t^2}U_{m_c}\right)^2 + \left(\frac{2m_c}{t^2}U_{L_p}\right)^2 + \left(-\frac{4m_c L_p}{t^3}U_t\right)^2}$$

$$U_{F_a} = \sqrt{\left(\frac{2*1.829}{1.00^2}0.0001\right)^2 + \left(\frac{2*0.1000}{1.00^2}0.001\right)^2 + \left(-\frac{4*0.1000*1.829}{1.00^3}0.01\right)^2}$$

$$U_{F_a} = \sqrt{0.0000537} = 0.007 \text{ N}$$

From these calculated uncertainties, the maximum aerodynamic force, F_D, is 0.019 Newtons, and the minimum acceleration force, F_a, is 0.359 Newtons, F_D being more than an order of magnitude smaller than F_a.

Three intermediate values, A_p, V_a, and a, are calculated as a check on the reasonableness of the calculations of F_D and F_a. A_p is calculated using Equation A4:

$$A_p = \frac{W_{np} L_r L_w}{n W_r} = \frac{0.0010 * 0.254 * 0.127}{10 * 0.0025} = 0.0013\,\text{m}^2$$

V_a is calculated using Equation A7

$$V_a = \frac{2L_p}{t} = \frac{2 * 1.829}{1.00} = 3.66\,\text{m/s}$$

And a is calculated using Equation A9

$$a = \frac{2L_p}{t^2} = \frac{2 * 1.829}{1.00^2} = 3.66\,\text{m/s}^2$$

Appendix D – Worksheet, with sample values, for laboratory aerodynamic drag and acceleration force calculations for KiNSMART™ Dodge Viper GTS-R 1:36 scale model pull-back cars

1:36 Scale Dodge Viper GTS-R Pull Back Car
Aerodynamic Drag Significance Laboratory Worksheet

Class:	ME/ESE 2020	Section:	13	Table:	Z
Engineers:	Ima Student	Adda Person		Date:	2/15/2010

Constantants and Conversion Factors

in2m =	0.0254	m/in	Convert inches to meters	$\tau_{95\%}$ =	1.96		Gaussian distribution 95% probability	
sqin2sqm =	0.000645	m^2/in^2	Convert in^2 to m^2	$t_{95\%\_9DF}$ =	2.262		Students *t* 95% probability - 9 DF	
g2kg	0.001	kg/g	Convert grams to kilograms	smu_{tm} =	0.0016	m	Tape measure	Smallest Measuring Units
ρ =	1.0	kg/m^3	Air density, Laramie Wyoming	smu_s =	0.0001	kg	Scale	
C_D =	2.0		Assumed drag coefficient	smu_{sw} =	0.01	s	Stop watch	
m_{c_g} =	100	g	Mass of car	U_{mc} =	0.0001	kg	Uncertainty of mass of car	
m_c =	0.1	kg		$U_{digital}=T_{au\_95}$*smu/2			Uncertainty of digital measurement	
				$U_{analog}=T_{au\_95}$*smu/SQRT(12)			Uncertainty of analog measurement	

Projected Frontal Area Measurements / **Uncertainties of measured variables**

n =	10						0		Number of projection cutouts
W_{np_g} =	1.0	g	W_{np} =	0.0010	kg	U_{Wnp} =	0.0001	kg	Weight of projection cutouts
W_{r_g} =	2.5	g	W_r =	0.0025	kg	U_{Wr} =	0.0001	kg	weight of reference cutout
L_{r_in} =	10.0	in	L_r =	0.254	m	U_{Lr} =	0.001	m	Length of reference cutout
L_{w_in} =	5.0	in	L_w =	0.127	m	U_{Lw} =	0.001	m	Width of reference cutout

Powered Run Distance and Time Measurements

Powered Distance. L_p			Powered Run Time, *t*		
Trial No.	Distance		Trial No.	Time	
	in			s	
1	72.00		1	1.00	
2	72.00		2	1.00	
3	72.00		3	1.00	
4	72.00		4	1.00	
5	72.00		5	1.00	
6	72.00		6	1.00	
7	72.00		7	1.00	
8	72.00		8	1.00	
9	72.00		9	1.00	
10	72.00		10	1.00	
L_p =	1.829	m	t =	1.00	s
Std Dev =	0.000	m	Std Dev =	0.00	s

Intermediate Values

Projected area					
A_{p_in} =	2.0	in^2			
A_p =	0.0013	m^2	$A_p=W_{np}*L_r*L_w/(n*W_r)$		
Maximum velocity					
V_a =	3.7	m/s	$V_a=2*L_p/t$		
Average Acceleration					
a =	3.7	m/s^2	a=2*Lp/t^2		
smu_{Lp} =	0.002	m	smu_t =	0.01	s
n_{Lp} =	10		n_t =	10	
U_{Lp} =	0.001	m	U_t =	0.01	s

U_{LP}=MAX($t_{\_95}$*s_{Lp}/SQRT(n_{Lp}),$Tau_{\_95}$*smu_{tm}/SQRT(12))

U_t=MAX($t_{\_95}$*s_t/SQRT(n_t),$Tau_{\_95}$*smu_t/SQRT(12))

Estimated Aerodynamic and Acceleration Forces

F_D =	0.017	N	±	0.002	N	$F_D=C_D*W_{np}*L_r*L_w/(n*W_r)*\rho*(2*L_p/t)$^2/2	Aerodynamic Drag
F_a =	0.366	N	±	0.007	N	$F_a=2*m_c*L_p/t$^2	Acceleration Force