**Units, Unit Cancellation and Approximation**

Or… Furlongs, Fortnights and Smoot Units, Oh My!

**Lesson Objective:**

Students will learn about the importance of assigning units to numbers, how to convert between sets of units, and use estimation skills to calculate the approximate time required for a bird to poop on a person lying on the ground.

Students will work with exponents and numbers in scientific notation (MA11.1.1), use units to solve problems, work with both U.S. customary units and the metric system (MA11.3.2), and solve indirect measurement problems (MA11.3.5). Students will devise their own units of measure and estimate the size of an object in the classroom, describing it in both common units and their new unit (MA11.1.3).

**Background Information:**

Length, mass, time and temperature are the basic types of building blocks for all units of measure.

Length can be measured in specific units that range from very small to very large. Examples include, nanometers (nm), micrometers (μm), inches (in), centimeters (cm), feet (ft), yards (yd), meters (m), miles (mi) and kilometers (km).

Examples of units of mass include ounces (oz), grams (g), pound mass (lbm), kilograms (kg) and tons. The unit of pounds (lbs) we commonly use to measure weight are actually the unit pound force (lbf).

Time is commonly measured in units of seconds (s), minutes (min), hours (hr) and days.

Temperature is measured in units of Fahrenheit (°F), Celcius (°C) and Kelvin (K). Kelvin is a common unit for many scientific temperature measurements.

Multiple units are multiples or fractions of base units which make more convenient units of measure. For the base unit of time, multiple units are minutes, hours and milliseconds. Minutes are multiples of 60 seconds, and hours are multiples of minutes. Milliseconds are thousandths of a second. 3 years is an easier measurement to say and understand than 94,608,000 seconds.

Derived units or compound units are built from multiplying or dividing many base units. For example, area of a square is measured as length of a side times length of a side, resulting in a final unit of length2. The volume of a cube is the length of a side times the length of a side times the length of a side, resulting in a final measurement of length3. Velocity is measured as length/time, and we are familiar with velocity being measured as miles/hour (mph), or meters/second (m/s).

**Vocabulary:**

Base units – the fundamental units and building blocks for multiple and derived units. Base units are mass, length, time and temperature.

Multiple units – multiples or fractions of base units. For example, thousands of meters are kilometers (km). Thousandths of a meter are measured as millimeters (mm).

Derived units – units built by multiplying or dividing base or multiple units. These can also be referred to as compound units. Examples of derived units are velocity, being length/time, and measured as meters/second or miles/hour.

Reciprocal – divide the quantity by 1. (e.g. the reciprocal of 3 is 1/3)

**Materials needed for the lesson and activity:**

Pencil

Paper

Calculator

Worksheet

**The lesson:**

Units are weird. We need them to tell us important things about the numbers and values they are assigned to. They are can seem intuitive, like seconds, minutes or meters, or arbitrary like a foot, a slug or a fortnight. Units can be somewhat confusing, such as with the ton. A ton can be a long ton, which is 2,240 lb or 1,016 kg used in British Commonwealth countries, a short ton which is 2,000 lb or 907.2 kg used in U.S., or a tonne which is 1,000 kg (International System of Units). They are all near 1000 kg, but specifying a long or short ton can make a huge difference in total mass if you are filling a whole train with a 100 cars with coal.

We can make up units for convenience, and they work as long as there is a standard for reference. In the U.S. system of measurement, we are familiar with feet and yards, which come from the British standard units. Today you can still see the standards for the units of measure at the Royal Observatory, Greenwich, England. Anyone familiar with the prime meridian and Greenwich mean time? This is also measured from the same location.



<http://www.panoramio.com/photo/46789298>

A unit of convenience could be a building or prominent structure. For example, the Empire State Building is one Empire State Building tall (or 1,250 feet). The Burj Khalifa, the world’s tallest skyscraper, is 2.178 Empire State Buildings tall (or 2,722 feet). The tallest building in Wyoming is White Hall on the University campus, and is 0.16 Empire State Buildings tall (or 200 feet).



A unit of convenience can also be a friend. In challenge to measure the length of the Harvard Bridge without the use of a tape measure or other standard measuring device in 1958, Oliver Smoot was used a unit of standard measure. The bridge was found to be 364.4 smoots ± 1 ear. Smoot was 5’ 7” or 1.70 m, and by converting from smoots to meters, we get a bridge length of 619.5 m. The markings are still on the bridge.



A few standard systems of measure are used today. In the U.S., we use the U.S. system of measure. Standard lengths are inches, foot, yard and mile, standard masses are ounce and pound mass, and temperature is in Fahrenheit.

In the International Systems of measure (SI) is used by the rest of the world besides the U.S. Standard length is in meters, mass in kilograms and temperature is Celsius. For both the US and SI systems, the standard unit for time is seconds.

A system devised from archaic and impractical units is the Furlong/Firkin/Fortnight system (FFF System). A furlong is 660 ft or about 1/8 of a mile (also measured as 201.2 m, or 1/5 of a kilometer). The unit of mass is a firkin, being 40.8 kg or 90 lbs and the unit of time is a fortnight, or 14 days (1,209,600 s).

**Example 1: Units and unit cancellation**

In the US, we measure fuel economy in miles/gallon. This can be written as gallons/mile, its reciprocal. The reciprocal form (gallons/mile) [has its advantages](http://wheels.blogs.nytimes.com/2008/06/20/the-illusion-of-miles-per-gallon/) and is popular in Europe, where fuel economy is expressed as liters per 100 kilometers.



It’s a bit easier to think of gallons/mile so let’s do that. What are gallons? Gallons are a unit of volume and volume can be expressed in the base unit of length3. What about miles? What do they measure? Miles are a unit of length. So, gallons/miles can be expressed in base units of length3/length. This simplifies to length2. What is another way to describe length2? Length squared is also known as area.



So the reciprocal of gas mileage can be measured as area. That’s kind of strange because we when we travel we think of the covering distance and not in area. Do we think of traveling between towns in acres? That doesn’t seem intuitive.

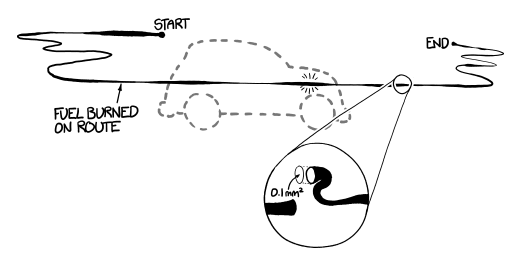
To make this more concrete, let’s say a car gets 20 miles/gallon on a trip. This is [about 0.12 mm2](http://www.wolframalpha.com/input/?i=1%2F%2820+mile%2Fgallon%29+to+mm%5E2). So what does this mean physically? If you took all the gas you burned on a trip and stretched it out into a thin tube along your route, 0.12 mm2 would be the cross-sectional area of that tube.



So what if your fuel economy went up and your car got 40 miles/gallon. What would the cross-sectional area be? It would come out to 0.06 mm2. This means the amount of fuel used for a given length is less. That starts to make sense intuitively. But still,unit cancellation is weird.



For 20 miles/gallon:



<http://what-if.xkcd.com/11/>

**Units, Unit Cancellation and Approximation**

**Adapted from the What if? Blog: [Droppings via xkcd.com](http://what-if.xkcd.com/11/)**

**The question:**

If you went outside and lay down on your back with your mouth open, how long would you have to wait until a bird pooped in it?

—Adrienne Olson

OK, how do we start?

We need some approximations. First, how many birds are there on the Earth? From an academic paper titled “How many birds are there?” published in 1996 in the journal *Biodiversity and Conservation* assumes there are 300 billion birds in the world. So we need to know how many birds there are in the world, so how big is the world?

The radius of earth is 6371 km. What is the formula for the area of a sphere? Asphere = 4πr2. So the area of the surface of the earth is:



This assumes birds are randomly distributed across the entire surface of the Earth, even though we know they aren’t.

The next thing we need to estimate is how frequently a birds poop. Let’s estimate that a bird poops once an hour and that there is one poop per bird.



Let’s say birds sleep 8 hours a day, so that the time that it can poop would be 24 hours – 8 hours = 16 hours/day.



Finally we need to introduce the size of the mouth and to be able to cancel poop. To do that, we estimate:

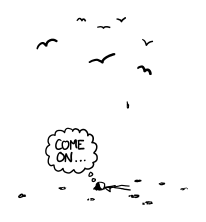


To put it all together, and determine the time we get:



To get the time in sensible units, we take the reciprocal:





Since your mouth is much smaller than the rest of your body, it is more likely that you would be pooped on many times before a bird pooped in your mouth.

And really, unit cancellation is weird. But if you ever get stuck on a physics test or word problem, cancelling units can help lead you down a path to solve the problem, even if you have no idea initially of what you are doing.

This were adopted and adapted from:

<http://what-if.xkcd.com/11/>

**Conclusive review of material:**

look at the lesson objective and what the students should get out of the lesson

**Any follow up:**

<http://www.what-if.xkcd.com/11/>

<http://wheels.blogs.nytimes.com/2008/06/20/the-illusion-of-miles-per-gallon/>