Modern Introductory Quantum Mechanics with Interpretation

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by Dr. David R Thayer (Author)

This is a novel quantum mechanics textbook which is appropriate for a one-semester course in all university physics undergraduate programs. In addition to covering the important quantum mechanics topics, from experimental foundations, to theoretical development, the textbook emphasizes the important interpretation issue throughout. In order for the student to grasp the quantum mechanics topics most simply, the pedagogical approach emphasizes 1D analysis. Finally, as an important aid to all students, the textbook contains more than 150 pages of completely solved quantum mechanics problems.

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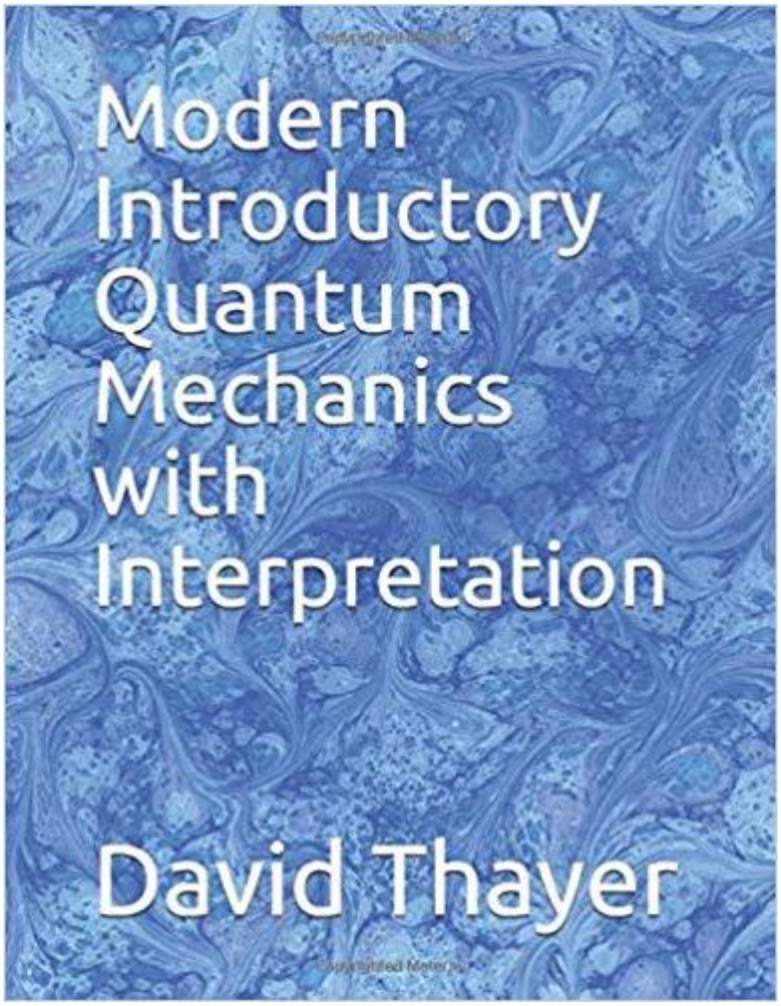
Modern Introductory Quantum Mechanics with Interpretation
David Thayer

Dr. Thayer received his PhD in Plasma Physics at MIT in 1983, and had many years of research experience at the Institute for Fusion Studies (UT – Austin, TX), at LBNL (Berkeley, CA), and at SAIC (San Diego, CA), prior to joining the faculty of the University of Wyoming (UW) in 2000 to focus on high quality physics instruction (quantum mechanics, E&M, classical mechanics, mathematical physics, plasma physics, …), as well as to continue his research interests in the areas of chaos & nonlinear dynamics, and in quantum mechanical foundations. Most importantly, associated with this textbook, Dr. Thayer has taught the introductory one-semester course in quantum mechanics at UW for the past twenty years.

This quantum mechanics textbook is appropriate for a one-semester course in all university physics undergraduate programs. In addition to covering the important quantum mechanics topics, from experimental foundations, to theoretical development, the textbook emphasizes the important interpretation issue throughout. In order for the student to grasp the quantum mechanics topics most simply, the pedagogical approach emphasizes 1D analysis. As an important aid to all students, the textbook contains an extensive set of completely solved quantum mechanics problems covering all the important topics.

Key attributes:
- Textbook is novel, as it approaches the quantum interpretation issues through the notion of deterministic chaos.

- Textbook is specifically designed as a self-contained one-semester course for junior or senior level undergraduate physics students.

- Textbook contains all the important subject content and mathematical details necessary for a comprehensive introduction to quantum mechanics.

- Textbook is strategically designed for the physics student to grasp all the important quantum mechanical concepts as quickly as is possible, with emphasis on 1D quantum mechanics.

- Textbook has more than 150 pages of completed solved problems, included as an appendix.

- Textbook starts in section I with a detailed description of all the significant experiments which led up to the development of quantum mechanical theory.

- Textbook continues with a very efficient overview of the key components of quantum mechanical theory.
- Textbook demonstrates the notion of unbounded and bounded quantum systems most simply using 1D analysis, prior to extending the concepts to 3D.

- Textbook develops the foundational uncertainty principle simply as a Fourier transform pair constraint consequence of wave mechanics.

- Textbook leaves more advanced topics, such as a thorough understanding of the Bell inequality, until section II, where it is shown that the spin correlation experiments can be understood without the need for a nonlocal construct.

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MODERN INTRODUCTORY QUANTUM MECHANICS with INTERPRETATION

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PREFACE

The quantum world of the small (objects of atomic size or smaller), is an extremely foreign concept to most beginning physicists. The concepts that are useful in describing the quantum domain of nature are in clear contrast to the ones that are used to describe classical physics, appropriate for understanding objects in our everyday lives. Typically, the most common application of classical physics is through the use of Newton's equations of motion for massive particles, as well as through the use of Maxwell's equations for the electric and magnetic fields, and for the associated electromagnetic waves. Even though these classical physics concepts can be applied to high speed particles (approaching the speed of light), by using a relativistic approach, the key classical physics assumption is of precise particle trajectories (in a deterministic sense). However, in the quantum domain, this is not the case; in fact, it is fundamentally impossible to simultaneously know both the precise position of a quantum particle and its precise momentum, which is normally assumed for a classical particle.

Classical physics concepts are the ones that we are comfortable with when we describe the motion of relatively sizeable objects (such as a baseball or even a grain of sand) that are found in our everyday lives. Unfortunately, it turns out that by using these classical physics concepts, we are ill-prepared when we try to describe the motion of very small objects (such as an electron), that are found in the quantum domain of nature. Furthermore, the issue of constructing an adequate description of objects in the quantum domain is much deeper than one might expect. Specifically, in classical physics we have found it useful to describe an object that has a small spatial extent as being classified as a particle, and in contrast to this, we have also found it useful to describe a collection of particles, which are correlated and interact over a sizeable spatial extent, as forming a wave (for example, used in the description of a ripple on the surface of a pond of water). However, in many situations in the quantum domain, we have had to resort to a bizarre combination of these concepts, referred to as the wave/particle duality. This is even true for the description of a seemingly simple object, such as an electron; in some situations it behaves like a point particle, and in other situations it behaves as a spatially extended wave. Shockingly, this is only the "tip of the iceberg" with respect to the quantum mechanical interpretation issue.

The most telling quantum interpretation issue is exemplified by the long-standing argument that occurred between two of the founders of quantum mechanics, Bohr and Einstein, which was brought to the forefront in the 1935 EPR publication (to be detailed in this textbook). To describe this argument most simply, Bohr believed (which is denoted as the Copenhagen interpretation of quantum mechanics) that an underlying quantum reality does not actually exist until a measurement occurs on the quantum system; however, in marked contrast to this, Einstein believed that there is an objective quantum reality prior to measurement. To expand on this Einstein ontological notion, he assumed that there must be an element of reality which exists prior to measurement, if it can be predicted with certainty prior to a confirming measurement taking place. In order to further emphasize that the quantum domain is unusual, these two opposing viewpoints will be explored in detail throughout this textbook, with the conclusion that a modern description of quantum mechanics must include an admixture of both of these concepts, as neither one by itself is correct. Ultimately, this discussion will lead to the most important topic on the completeness of quantum mechanics.

As brought out in his EPR publication, Einstein believed that the quantum mechanical model is incomplete, which led to the unresolved debate on quantum mechanical interpretation. Once again, Einstein's belief of the incompleteness of quantum mechanics, is in direct contrast to Bohr's opposite opinion on the subject. To make this issue clear, one should note that classical physics is considered to be a complete science, in the sense that the trajectory of classical objects are self-consistent and completely understood. For example, the solution of the Newtonian equation of motion of a charged massive particle, in combination with the associated self-consistent solution of Maxwell's equations, which provide knowledge of the force acting on the charged particle, results in a complete description of the particle motion (which is exhibited in the classical field of plasma physics). However, in the quantum mechanical domain of reality, such a complete description of quantum mechanical objects does not still exist today. Unfortunately, as was proposed by the founders of quantum mechanics, there is an underlying uncertainty principle which constrains our capability of achieving precise and complete quantum knowledge, which has not been violated since the inception of the principle. Although quantum mechanics was developed during a reasonably lengthy (roughly a thirty year) period, starting in approximately 1900, with numerous physicists involved, it is the measure of discreteness (the quantum), associated with a very small number (Planck's constant), which is pervasive throughout all aspects of quantum mechanics, which has led to a dramatic contrast between classical and quantum mechanics. Nevertheless, rather than leaving the reader with a vague understanding of quantum interpretation due to quantum discreteness, as is the case with most previous quantum mechanics textbooks, a clear understanding of quantum interpretation is emphasized in every topic throughout this book.

There have been numerous very useful quantum mechanics textbooks which have been constructed during the past half century; however, just as with the founders of quantum mechanics, the quantum interpretation issue has been sorely lacking in most of these textbooks. As the quantum mechanical interpretation issue has recently (during the past twenty years) come to the forefront of physicists' concerns, it is finally necessary to produce a modern introductory quantum mechanics textbook with interpretation. This is especially true since the Bell inequality spin experiments have promoted the desire to explore nonlocal quantum aspects, such as quantum teleportation and quantum computing. However, as noted in a recent publication (Thayer and Jafari, IJARPS, Vol. **2**, Issue 2, pp. 18-26, 2015), on the spin correlation of singlet state pair particles, it is possible to interpret the unusual spin product expectation value result as being associated with a nonlinear dynamics model which incorporates deterministic chaos, in contrast to having to invoke a far more disturbing nonlocal interpretation. In addition, as the field of nanotechnology continually advances, it will become more and more necessary to educate our physics students with modern quantum mechanics tools in order that they can ultimately achieve a far more advanced and complete quantum mechanics, which can successfully be used on the quantum engineering problems of the future.

This textbook has been constructed to provide the physics student with the most important quantum mechanics tools that have been successfully applied to the quantum domain of nature, which have been very useful over the past century. However, in marked contrast to previous textbooks on this subject, the quantum interpretation issue has been highlighted throughout this book. This is a particularly important approach in order that we are able to train our young physicists to continually look for the future, and ultimately develop a far more useful quantum mechanics than we have today. For example, for those who know a bit about standard quantum mechanics, it should be noted that the subject is always based on simple linear operator theory; however, this is precisely the approach that has led to major problems with quantum interpretation. Specifically, I strongly believe when we are able to get beyond our current simplistic quantum model, and properly include the complicated measurement process in our model, the well-known issue of the need for a mystical wave function collapse to a linear eigenvalue state, upon measurement, will disappear along with our interpretation problems. As an educator, it is with this desire that I construct this "Modern Introductory Quantum Mechanics with Interpretation" textbook for our young physicists.

Finally, in order to help quantum mechanics physics instructors who choose to use this textbook, it is useful to briefly present the organization of the sections that follow. This textbook is designed to cover all of the most important topics of introductory quantum mechanics that are appropriate for undergraduate physics students in order to efficiently master the subject. The teaching material is optimally designed using two sections, which can be covered throughout a one-semester course, where the first section, Elementary Quantum Mechanics, is the most important material to be covered in a one-semester course, and the second section, Advanced Quantum Mechanics Topics, can be added to the one-semester course if time permits. The quantum mechanics textbook is designed to be self-contained, which provides all the subjects that a student needs to fully understand quantum mechanics. The first section starts with a very important chapter (**1** Historical Review & Interpretation Issues), which provides the background and history (covered throughout ten sections, Sect. 1.1 to Sect. 1.10) that is associated with the creation of quantum mechanics (which occurred roughly throughout the period from 1900 to 1930). Although many of the current quantum mechanics textbooks used in a variety of schools do not offer such a comprehensive history of the development of quantum mechanics, I believe that the approach used here is essential from a sound pedagogy perspective, in order that the quantum mechanics student can appreciate how the extremely unusual quantum mechanics theory was developed, especially as it is in marked contrast to classical mechanics theory. Since the majority of quantum mechanics can be understood by simply focusing on one-dimensional mechanics analysis and two-state spin ½ particles, the second chapter (**2** Elementary & Representative Concepts in 1D Quantum Mechanics) briefly covers most of the quantum mechanics topics within this practical limitation, that are provided in Points 1) to 10) at the beginning of Chapter 2, which is essentially all a student needs to know in order to become reasonably proficient in quantum mechanics. More specific details and examples of elementary quantum mechanics are then provided in the following five sections of Chapter 2 (Sect. 2.1 to Sect. 2.5). The first section then continues with two chapters (denoted as: **3** Free Particle Quantum State Analysis; and **4** Bound Particle Quantum State Analysis), which provide the slightly more complicated examples of 1D quantum analysis that are associated with free particles (which includes an introduction to scattering analysis), and then with bound particles (which includes analysis associated with general confining potentials, such as found in the well-known harmonic oscillator problem, which utilizes the quadratic potential, and then in the hydrogen atom problem, which utilizes the Coulomb potential). Although section one ends with a brief analysis of the hydrogen atom, where it is necessary to first reduce the general 3D analysis to the most important representative radial 1D problem, the more difficult concept of rotational angular momentum is included. For the students and physics programs who desire to explore more advanced quantum mechanics topics, the second section can be used as an appropriate continuation of the first section to address a few more informative advanced topics. The second section begins with a chapter on an extensive review of all the most important quantum mechanics topics (**5** Review of General Quantum Mechanics), so that the reader can finally appreciate the current limitation of quantum mechanics, and the associated interpretation problems (such as the unexplained need for a wave function collapse upon measurement), that permeate the simple linear eigenvalue approach to quantum mechanics, which must be rectified in the future, using a more sophisticated quantum model (which might be approached using a nonlinear dissipative/driven quantum system, that naturally includes the needed probabilistic aspects of quantum mechanics, through the inherent deterministic chaos aspects of the more sophisticated model, which will ultimately resolve the interpretation issues that we struggle with today). The second section continues with two important chapters (**6** General Uncertainty Principle and **7** Bell Inequality - Explained), which help to complete the "Modern Introductory Quantum Mechanics with Interpretation" textbook, by providing some additional insight into the deeper structure and limitations of current quantum mechanics. Specifically, the general approach to the uncertainty principle that is presented is based on the Schwartz inequality, which is quite abstract but general, and it is in marked contrast to the derivation of the Heisenberg uncertainty principle, using the much more appealing variational analysis, which is presented in Chapter 1 (and given in detail in Appendix IV); consequently, even for such long standing concepts as the uncertainty principle, it should be clear to the student that interpretation issues with quantum mechanics can continually be rectified. Finally, the full description of the Bell inequality is provided since it demonstrates the unusual correlation that exists between spatially separated spin ½ singlet state particles, which can ultimately be understood without the need to invoke a bizarre nonlocal correlation between the particles, which would have pleased Einstein (as detailed in Appendix V); consequently, this important topic again emphasizes the need to continually improve on quantum interpretation, and speculate on the structure of the more sophisticated quantum models of the future which will rectify all quantum interpretation issues. Ultimately, in order to give the quantum mechanics students practice in applying the quantum mechanics analysis tools, the very extensive Appendix VII provides numerous completely solved quantum mechanics problems that follow (in consecutive order) all the topics provided throughout the textbook.

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