UNIVERSITY OF WYOMING

SCHOOL OF ENERGY RESOURCES
CENTER FOR ECONOMIC GEOLOGY RESEARCH

WORKING PAPERS

NO. 3

Working Paper Title:

Porosity from sonic: a linear approximation to the Raiga-Clemenceau equation.

Yuri Ganshin and J. Fred McLaughlin | 2021

CENTER FOR ECONOMIC GEOLOGY RESEARCH WORKING PAPERS

DIRECTOR: DR. J. FRED MCLAUGHLIN

About the University of Wyoming School of Energy Resources' (SER) Center for Economic Geology Research (CEGR) Working Paper Series

CEGR, which is one of SER's Centers of Excellence, investigates solutions to the challenges in Wyoming's fossil fuel and mineral industries. CEGR research projects explore opportunities to use Wyoming's distinctive geology and resources in order to develop those opportunities, diversify Wyoming's economy, and to maintain competitiveness in a low-carbon fossil energy future.

The objective of the CEGR Working Paper Series is to share "works in progress". The authors of the papers are solely responsible for the content of their contributions. The working papers can be found at https://www.uwyo.edu/cegr/research-projects/publications/

For more information about CEGR, please visit the CEGR web site at www.uwyo.edu/cegr.

University of Wyoming | School of Energy Resources Center for Economic Geology Research

Porosity from sonic: a linear approximation to the Raiga-Clemenceau equation.

By Yuri Ganshin and Fred McLaughlin
University of Wyoming
yganshin@uwyo.edu

Abstract

Porosity represents the rock's capacity to trap fluid. It is the main quality indicator of subsurface reservoirs used for hydrocarbon production, and also, for CO_2 storage. The knowledge of porosity distribution within a reservoir is a required input to establish the reservoir static and dynamic models. However, the quantitative evaluation of porosity is challenging especially during reservoir characterization. Many factors such as mineralogical composition, type and amount of cement, grain shape and packing pattern, and rock compaction would affect its value. A reliable quantitative evaluation of porosity requires integrating rock physics, well logs and core data. At present, a large number of rigorous, analytical and semi-empirical models exists that provide relations among velocity, porosity, and pore-fluid compressibility. Reviews of such models are given, for example by Mavko et al. (2009) and Saxena et al. (2018). However, many earth science practitioners are still in need for a simple, yet reliable empirical relationship capable to compete with rigorous physics-oriented models.

Equation

$$\phi = C \times (1 - t_m/t)$$

is one the most popular among petrophysicists, which is used to estimate porosity ϕ from sonic logs or from seismic velocity measurements. In the equation, t is the observed interval transit time (inverse velocity), t_m is the matrix interval transit time, and parameter C is an arbitrary constant. A number of authors name this equation as the Raymer-Hunt-Gardner (RHG) equation with the value of parameter $C = 5/8$ (e.g., Asquith and Krygowski, 2004). The online AAPG Wiki open access resource (*https://wiki.aapg.org/Standard_interpretation*) indicates that the value of C can vary between 0.625 and 0.70 with the most widely accepted value of 0.67. Aminzadeh and Dasgupta (2013) provide the range of parameter C values from 0.4 to 0.8. The aim of

this publication is to examine a connection of equation $\phi = C \times (1 - t_m/t)$ with the one originally published by Raymer et al. (1980), and to bring an understanding of the parameter C and the range of its possible values.

The sonic log is type of a porosity log that measures interval transit time (Δt, DT, or simply t) of the compressional waves traveling through the formation along the axis of the borehole. The interval transit time (designated t in this study) is dependent upon lithology and rock porosity (ϕ). A formation's matrix interval transit time (t_m) must be known to derive sonic porosity either by chart or by a variety of proposed equations (Saxena et al., 2018).

Equation of the form

$$\phi = C \times \left(1 - \frac{t_m}{t}\right) \quad (1)$$

is among most frequently used models of predicting porosity from sonic measurements. Noting that compressional velocity of wave propagation $V = 1000000/t$, this equation can be likewise written as

$$\phi = C \times \left(1 - \frac{V}{V_m}\right) \quad (2)$$

where V_m is velocity of the compressional waves in the solid-phase material (matrix).

The Raymer-Hunt-Gardner (RHG) algorithm (Raymer et al., 1980) was designed to predict P-wave velocity (V) given the velocity of the compressional waves in the solid (V_m) and in the fluid (V_f) and the porosity of the rock. The equation has different formulations for different porosity ranges; for simplicity we only focus on the porosity interval [0, 0.37], where the equation for V can be written as

$$V = f(\phi) = (1 - \phi)^2 V_m + \phi V_f \quad (3)$$

The following should be considered when using the RHG relation (Mavko et al., 2009):

- the method is empirical;
- the rock is isotropic with homogeneous mineralogy;
- all solids making up the rock matrix have the same velocity, V_m;
- the rock is fluid-saturated;

- the relation should work best at high enough effective pressure, usually of the order of 30 MPa
- the relation works well for consolidated low-to-medium and high-porosity (<37%) cemented sandstones
- the relation works best with primary porosity. Secondary or vuggy porosity tends to underpredict the velocity and the porosity.

From mathematical point of view, the equation 3 presents a *nonlinear* transformation of porosity values into the rock velocity. Let us approximate velocity function 3 with a simpler, *linear* function; that is, we would like to find a function $g(\phi)$, for which

- relevant calculations can be performed as on function $f(\phi)$;
- the function g is close to f, in the sense that the outcome of the calculation performed on g gives useful information about velocity described by f.

It is Taylor's theorem that allows one to approximate a given function via polynomials of the form

$$P_N(x) = \sum_{n=0}^{N} \frac{f^{(n)}(a)}{n!}(x-a)^n \tag{4}$$

where P_N is the Taylor polynomial of degree N associated to f at the point a. (Ole Christensen and Khadija Christensen, 2005). Observe that for $N = 1$, equation 4 turns to

$$P_1(x) = f(a) + f'(a)(x-a) \tag{5}$$

which is the equation for the tangent line of f at the point a.

Correspondingly, the Raymer-Hunt-Gardner quadratic equation 3 can be approximated with a first-order polynomial as:

$$V = f(\phi) \approx P_1(\phi) = f(a) + f'(a)(\phi - a) \tag{6}$$

That is, at any arbitrary porosity value a, we use our knowledge of what $f(a)$ and $f'(a)$ looks like in order to approximate $f(\phi)$. The important point is that this Taylor polynomial approximates $f(\phi)$ well only for a small range of porosity values ϕ near a.

Let us now estimate $f(a)$ and $f'(a)$ using the function form of the RHG equation (3).

$$f(a) = (1-a)^2 V_m + aV_f = V_m - 2aV_m + a^2 V_m + aV_f \tag{7}$$

$$f'(a) = V_f - 2V_m(1-a) = V_f + 2aV_m - 2V_m \quad (8)$$

After inserting the last two equations into the polynomial equation 6, and simplifying the resultant expression, we have:

$$V = f(\phi) \approx P_1(\phi) = (1-a^2)V_m - (2V_m(1-a) - V_f)\phi \quad (9)$$

Expression 9 represents the slope-intercept form of a linear equation $f(\phi) = b + m\phi$, where slope

$$m = 2V_m(1-a) - V_f \quad (10)$$

and intercept

$$b = (1-a^2)V_m \quad (11)$$

To compute the parameter C, let us solve equation 2 for variable V and present the result in the slope-intercept form, just like we did it in expression 9.

$$V = V_m - \frac{V_m}{C}\phi \quad (12)$$

Equation 12 will become identical to equation 9 in case when intercept and slope values in both equations become equal to each other, that is intercept b from equation 11 should be equal to V_m, and the slope value should be equal to both expressions as follows:

$$m = 2V_m(1-a) - V_f = \frac{V_m}{C} \quad (13)$$

From the above equation, the parameter C can be expressed as

$$C = \frac{V_m}{2V_m(1-a)-V_f} \quad (14)$$

Note dependence of C on three variables, P-wave velocity in the solid phase V_m, velocity in the fluid phase V_f, and porosity value a where the slope m was estimated. The matrix velocity variations in a pretty broad range from $V_m = 18{,}000$ ft/s to $V_m = 19{,}500$ ft/s (characteristic to sandstone matrix) do not cause any significant effect on parameter C, while about the same change in the fluid phase velocities cause much bigger magnitude of parameter C fluctuations (Figure 1). The sensitivity analysis of equation 14 indicate increase of parameter C value with decreasing value of V_m and increasing value of V_f, so that the matrix- and fluid-phase velocities produce the opposite effect on the parameter C. However, the choice of porosity value a has the biggest effect on parameter C among all other variables (Figure 1).

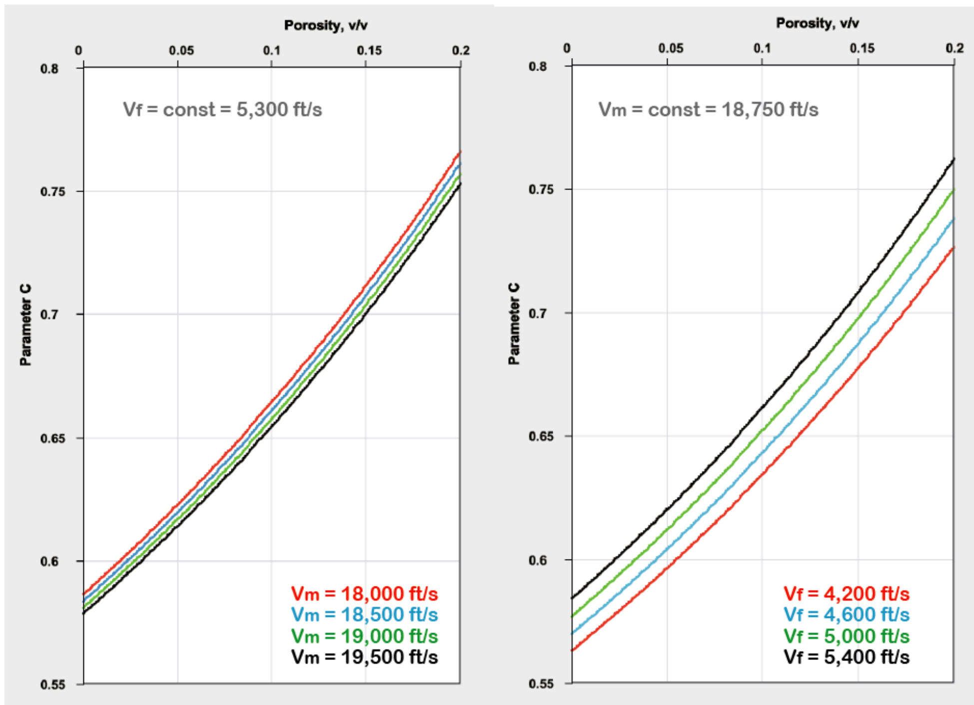

Figure 1. Sensitivity of parameter C to the matrix velocity variations (left panel, fluid velocity is fixed at 5,300 ft/s) and to the fluid velocity variations (right panel with fixed matrix velocity at 18,750 ft/s).

The results of sensitivity analysis of parameter C indicate that a special care should be taken when selecting a point for a tangent line approximation of the velocity-porosity transform 3. The point with porosity value a should be within the range of the measured porosity values. That is, when the measured porosities are distributed over a considerable extent (e.g., good quality reservoirs) the parameter C values should be larger those selected for low-porosity measurements (e.g., tight sands). The proper choice of parameter C is the price one should pay for using linear approximation 2 instead of more accurate Raymer-Hunt-Gardner transform 3. Recall to the Taylor's theorem, a first-order polynomial approximation will produce good fitting results only for a small range of porosity values ϕ near a. We illustrated this fact in Table 1 by computing the goodness-of-fit for different range of the "measured" or, better to say, fitted porosities. The goodness-of-fit columns of table 1 summarize the discrepancy between porosity values calculated with the RHG equation 3 and corresponding values derived from its linear approximations (equation 2) with parameter C calculated from equation 14.

Porosity a, **v/v**	***Intercept Error, %***	***C***	***Goodness-of-Fit,*** r^2 $\phi = 0 - 0.15$	***Goodness-of-Fit,*** r^2 $\phi = 0 - 0.25$	***Goodness-of-Fit,*** r^2 $\phi = 0 - 0.35$
0.000	*0.00*	*0.5823*	*0.9862*	*0.9479*	*0.8740*
0.025	*0.06*	*0.5998*	*0.9958*	*0.9716*	*0.9162*
0.050	*0.25*	*0.6183*	*0.9992*	*0.9875*	*0.9490*
0.075	*0.56*	*0.6380*	*0.9964*	*0.9956*	*0.9726*
0.100	*1.00*	*0.6591*	*0.9873*	*0.9960*	*0.9871*
0.125	*1.56*	*0.6815*	*0.9722*	*0.9887*	*0.9922*
0.150	*2.25*	*0.7056*	*0.9507*	*0.9736*	*0.9880*
0.175	*3.06*	*0.7314*	*0.9231*	*0.9507*	*0.9747*
0.200	*4.00*	*0.7591*	*0.8894*	*0.9202*	*0.9521*

Table 1. The Goodness-of-Fit to the quadratic Raymer-Hunt-Gardner equation as a function of parameter C and the range of porosity values used for linear approximation. The following velocity values of the solid (matrix) and fluid phases were used for calculations: $V_m = 18{,}750\ ft/s$ and $V_f = 5{,}300\ ft/s$. The Goodness-of-Fit was summarized separately within three different porosity segments: 0.0 – 0.15, 0.0 – 0.25, and 0.0-0.35 volumetric fractions.

In other words, it describes how well the linear models (with different C-values) fit the nonlinear RHG equation. The calculations in table 1 were performed for the sandstone lithology ($V_m = 18{,}750$ ft/s and $V_f = 5{,}300$ ft/s) and for different porosity segments (1) $\phi = 0 - 0.15$, (2) $\phi = 0 - 0.25$, and (3) $\phi = 0 - 0.35$. For the first porosity segment, the best-fitting straight line approximation results from the slope-intercept measurements at point with 0.05 porosity value. In this case, the parameter C=0.6183. For a broader range of approximated porosities from 0 to 0.25 volumetric fractions, we obtain the parameter C=0.6591 corresponding to porosity $a = 0.10$. Finally, for the broadest possible range of fitted porosities, we obtained the largest value of C=0.6815 at point with porosity a=0.125. A graphical comparison between the quadratic RHG velocity-porosity transform and its linear approximations at different porosity points is shown in figure 2. We used the same velocity parameters for graphing as those used for calculations in Table 1. It is apparent from the figure that the larger value of parameter C provides a better approximation for a higher porosity range, while the smaller C-values better describe velocity-porosity relationship over a small range of porosity measurements around $\phi = 0$.

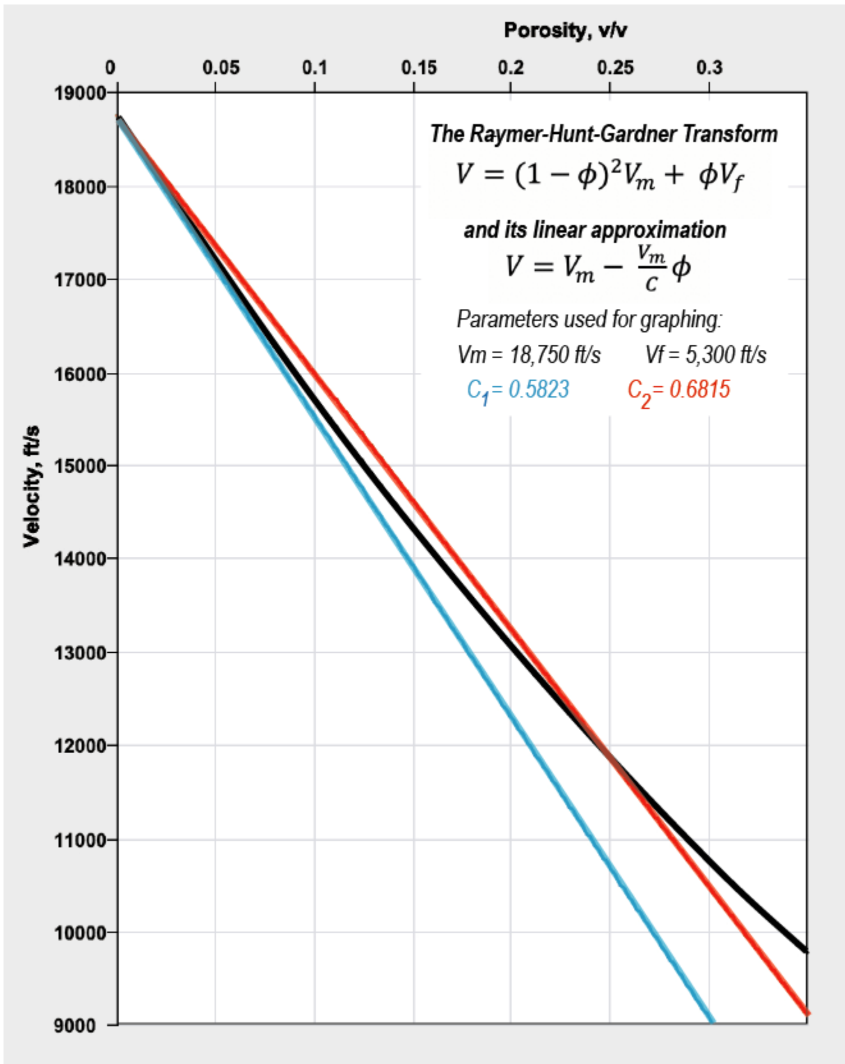

Figure 2. The Raymer-Hunt-Gardner velocity-porosity transform (black line) and its linear approximations computed for different values of C (C_1=0.5823 for the blue line and C_2=0.6815 for the red line). Sandstone matrix with V_m=18,750 ft/s and V_f=5,300 ft/s.

Note that the presented C-values were obtained for the fixed values of $V_m = 18{,}750\ ft/s$ and $V_f = 5{,}300\ ft/s$. Though these velocity values are typical for a variety of sandstone formations, the site-specific matrix and fluid velocity values should be considered when making porosity predictions based on the actual measurements.

There is one more factor to consider when choosing parameter C. It is the shift of velocity intercept value b from V_m that becomes progressively higher with increasing value of porosity a and parameter C, correspondingly. In table 2 this shift is designated as the intercept error that can be estimated from equation 11 as follows:

$$Intercept\ Error = 1 - \frac{b}{V_m} = 1 - 1 + a^2 = a^2$$

Therefore, the larger values of parameter C are associated with some decrease in the quality of fit that is seen in table2 when the goodness-of-fit starts to decline rapidly for porosity values above 0.15 volumetric fractions. That is why, we do not recommend choosing the parameter C values above 0.70.

Raiga-Clemenceau et al. (1988), in a search for some more fundamental velocity-porosity relationship and utilizing experimental data published by Raymer et al. (1980), proposed the equation of the form

$$\phi = 1 - \left(\frac{t_m}{t}\right)^{1/x} \qquad (15)$$

Where x is an exponent specific to the matrix lithology. Table 2 lists the following parameters related to the matrix natures.

Matrix	t_m, μsec/ft	x
Silica	55.5	1.60
Calcite	47.6	1.76
Dolomite	43.5	2.00

Table 2. Matrix-specific parameters to be used with Equation 2. Modified from Raiga-Clemenceau et al., (1988).

As pointed out by Raiga-Clemenceau and his co-authors, equation 15 is functionally similar to the relation between electrical resistivity and porosity and leads to a similar concept of formation factor. Therefore, the authors named the transit time-porosity transform (15) as the Acoustic Formation Factor (AFF) equation. The equation lacks mathematical complexity of theoretical equations based on rock mechanics, but still is physically meaningful and shows good agreement with experimental data in a great variety of cases.

Noting that $t = 1000000/V$, the AFF equation can be written in terms of velocity as

$$\phi = 1 - \left(\frac{V}{V_m}\right)^{1/x} \tag{16}$$

And solving this equation for velocity, we'll get an alternative form of the AFF equation

$$V = f(\phi) = V_m(1-\phi)^x \tag{17}$$

Let us approximate the power function of equation 17 with a first-order polynomial utilizing the approach that we used before with the Raymer-Hunt-Gardner quadratic equation. To do this, we must first estimate expressions for $f(a)$ and $f'(a)$ of equation 17.

$$f(a) = V_m(1-a)^x \tag{18}$$

$$f'(a) = -xV_m(1-a)^{x-1} \tag{19}$$

After inserting equations 18 and 19 into the polynomial form 6 and combining like terms

$$V = f(\phi) \approx P_1(\phi) = V_m(1-a)^x \left[1 - \frac{(\phi-a)x}{1-a}\right] \tag{20}$$

And removing out the denominator from the square bracket, we have

$$V \approx V_m(1-a)^{x-1}[1 - a - \phi x + ax] \tag{21}$$

After dividing both sides of equation 21 by parameter V_m and solving it for porosity ϕ, we can simplify the approximated porosity function to

$$\phi = \frac{1}{x}\left(1 - a + ax - (1-a)^{1-x}\frac{V}{V_m}\right) \tag{22}$$

Note that if $a = 0$ and $x = 8/5$ (sandstone matrix), the equation 22 turns into

$$\phi \approx \frac{5}{8}\left(1 - \frac{V}{V_m}\right) \tag{23}$$

which is the form of velocity-porosity relation misnamed as the Raymer-Hunt-Gardner (RHG) by Asquith and Krygowski (2004).

To bring equation 22 to a function form of equation 2, let's define the two new variables, k and, n such as

$$k = 1 - a + ax \tag{24}$$

$$n = (1 - a)^{1-x} \tag{25}$$

After substituting the corresponding expressions in equation 22 with the new variables, we have:

$$\phi = \frac{1}{x}\left(k - n\frac{V}{V_m}\right) \tag{26}$$

The expression 26 turns into a form of equation 2 only if $k = n$ and $C = \frac{k}{x}$. Theoretically, the equality of k to n is only possible if $a = 0$. Considering a general case of $a \neq 0$, we have to admit the loss of accuracy when using equation 2 instead of equation 22. The percentage error formula takes the form of:

$$Percent\ Error = 100 \times \frac{n-k}{k} \tag{27}$$

We can now estimate parameter C of equation 2 based on linear approximation to the AFF equation derived by Raiga-Clemenceau and his co-authors. We do it for different values of porosity a used to calculate derivatives. We also compute the goodness-of-fit for different ranges of the fitted porosities in the same way, we did it in table 1. The results of computation for the sandstone lithology (exponent x = 1.6 = 8/5) are shown in table 3.

Porosity a, **v/v**	***Percent Error, %***	***C***	***Goodness-of-Fit,*** r^2 $\phi = 0 - 0.15$	***Goodness-of-Fit,*** r^2 $\phi = 0 - 0.25$	***Goodness-of-Fit,*** r^2 $\phi = 0 - 0.35$
0.000	*0.00*	*0.6250*	*0.9965*	*0.9872*	*0.9703*
0.025	*1.59*	*0.6344*	*0.9989*	*0.9928*	*0.9798*
0.050	*3.38*	*0.6438*	*0.9998*	*0.9966*	*0.9871*
0.075	*5.38*	*0.6531*	*0.9993*	*0.9986*	*0.9923*
0.100	*7.60*	*0.6625*	*0.9976*	*0.9992*	*0.9959*
0.125	*10.06*	*0.6719*	*0.9947*	*0.9983*	*0.9977*

0.150	*12.78*	*0.6812*	*0.9908*	*0.9961*	*0.9980*
0.175	*15.78*	*0.6906*	*0.9859*	*0.9927*	*0.9969*
0.200	*19.09*	*0.7000*	*0.9802*	*0.9882*	*0.9945*

Table 3. The Goodness-of-Fit to the AFF equation as a function of parameter C and the range of porosity values used for linear approximation. The calculations were done for the sandstone lithology, x = 1.6 with the matrix velocity $V_m = 18{,}750\ ft/s$. The Goodness-of-Fit was summarized separately within three different porosity segments: 0.0 – 0.15, 0.0 – 0.25, and 0.0-0.35 volumetric fractions.

Similar to results in table1, the C-values calculated at points with porosity value 0.05, 0.10, and 0.125 produce the best-fitting approximations to the AFF equation for the range of fitted porosities 0-0.15, 0.-0.25, and 0-0.35 accordingly. Importantly, the overall goodness-of-fit estimates presented in table 3 for the AFF equation are superior to those presented in table 1 for the RHG equation. This happens despite the fact that the percent of error specified in equation 27 is relatively larger than the intercept error inherent to linear approximations to the RHG equation. An increased quality of straight-line fitting to the AFF line is due to its smaller curvature compared to the RHG curve. In other words, the amount by which the AFF curve deviates from a straight line is smaller. To illustrate this statement, we graphed the AFF equation and its linear approximations in figure 3. We used the same porosity segments for linear approximation as we did in figure 2. The blue line corresponds to the tangent line at zero porosity value, and the red line is the best-fit estimate for a broad-range of fitted porosities from 0.0 to 0.35 volumetric fractions (with the slope estimated at point with porosity $a = 0.125$). Obviously, the larger value of parameter C provides a better approximation for a higher porosity range, while the smaller value of C could be a better choice for a narrow range of porosities with the measured values close to zero (figure 3). Another important conclusion from comparing the results presented in table 1 and table 3, can be derived about the range of values of parameter C. For the range of analyzed porosity values from 0 to 0.2 volume fractions (tables 1 and 3), the corresponding range of parameter C used to approximate the RHG equation is from 0.5823 to 0.7591 (23.2% variation), and to approximate the AFF equation, the C parameter varies from 0.625 to 0.70 (10.7% variation). In other words, when fitting the AFF equation to a straight line, it is safe to use a single value for parameter C (about 0.66) in either equation 1 or equation 2 to get a good quality fit. However, in case of the RHG equation, as it was discussed above, the proper choice of parameter C is not so obvious. From this point of view, we would consider it appropriate to name the velocity-porosity transform 2 as an approximation to the AFF equation but not just the RHG equation.

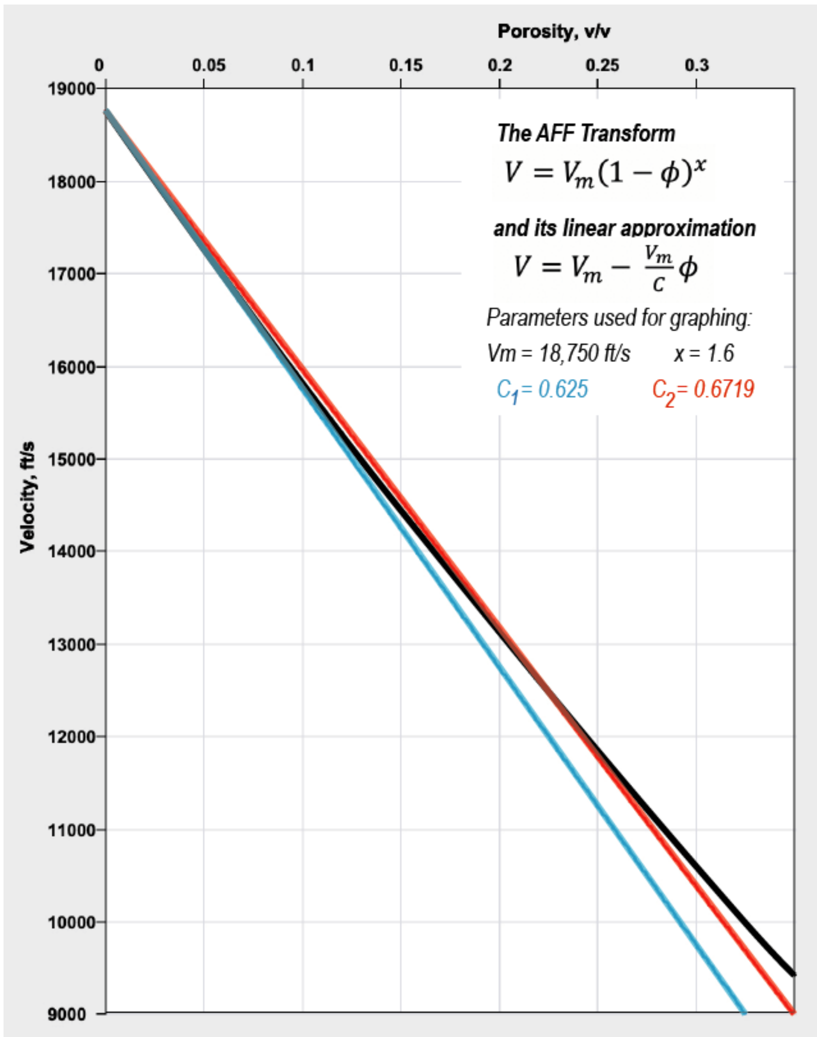

Figure 3. The AFF velocity-porosity transform by Raiga-Clemenceau et al (black line) and its linear approximations computed for different values of C (C_1=0.625 for the blue line and C_2=0.6719 for the red line). Sandstone matrix with V_m=18,750 ft/s and x=1.6.

We also would recommend not to choose values of parameter C outside the range (0.625-0.70) since it will result in velocity-porosity relationship that disagrees with the original AFF equation and lacks its physical meaning of being an acoustic formation factor.

Conclusion

As it is shown in this study, equation of the form $\phi = C(t - t_m)/t$ is a linear approximation to either of the nonlinear velocity-porosity transforms proposed by Raymer et al. (1980) and by Raiga-Clemenceau et al. (1988). However, the equation introduced by Raiga-Clemenceau et al. as the AFF equation, provides a better fit to its linear approximation. Accordingly, we would consider it appropriate to name the velocity-porosity transform $\phi = C(t - t_m)/t$ as an approximation to the AFF equation but not just the RHG equation.

The accuracy of this approximation diminishes with increasing porosity values used to calculate derivatives. This is inherently associated with the assumptions done when deriving the approximated formulas.

The choice of parameter C depends, to a lesser extent on the matrix lithology, and to a larger extent, on the range of modeled porosities. When making a linear approximation to the Raymer's et al. equation, the fluid velocity is another factor that considerably affects the value of parameter C. For a typical range of the sandstone matrix velocity from 18,000 to 19500 ft/s, pore-fluid velocity from 4,000 to 5,500 ft/s, and the range of modeled porosity values from 0 to 0.35 volume fractions, the parameter C value may vary from about 0.55 to 0.75. For the same sandstone matrix and the range of modeled porosity, the parameter C estimated from the AFF equation has significantly smaller range of values, from 0.625 to 0.70. The smaller degree of ambiguity in parameter C, is another reason to name the velocity-porosity transform $\phi = C(t - t_m)/t$ as an approximation to the AFF equation. For the low-porosity sandstones we recommend using C = 0.64, and for a porosity range from 0.0 to about 0.35, we recommend using parameter C = 0.66. Still, it remains unclear, who was the first to derive velocity-porosity transform similar to equations 7 through 9 but definitely, Raymer et al. (1980) have nothing to do with it.

REFERENCES

Fred Aminzadeh, and Shivaji Dasgupta, 2013, Geophysics for Petroleum Engineers, Volume 60, Elsevier Science, 1st Edition, 344p

Asquith, G., and D. Krygowski, 2004, Porosity Logs, in G. Asquith and D. Krygowski, Basic Well Log Analysis: AAPG Methods in Exploration 16, p. 37-76

Ole Christensen, and Khadija L. Christensen, 2005, Approximation Theory: From Taylor Polynomials to Wavelets, Springer-Science+Business Media, 2nd printing

Gary Mavko, Tapan Mukerji, and Jack Dvorkin, 2009, The Rock Physics Handbook: Tools for Seismic Analysis of Porous Media, Cambridge University Press, Second Edition, 511 p.

Raiga-Clemenceau, J., J.P. Martin, and S. Nicoletis, 1988, The Concept of Acoustic Formation Factor for More Accurate Porosity Determination from Sonic Transit Time Data: The Log Analyst, p. 54-60

Raymer, L.L., E.R. Hunt, and J.S. Gardner, 1980, An improved sonic transit time-to-porosity transform: Society of Professional Well Log Analysts, 21st Annual Logging Symposium, Transactions, Paper P

Vimal Saxena, Michel Krief, and Ludmila Adam, 2018, Handbook of Borehole Acoustics and Rock Physics for Reservoir Characterization, Elsevier, First Edition, 484p.