ECON 5350 Midterm Exam – Fall 2009

1. (40 pts) Consider the following bivariate regression model:

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i;\ i = 1, ..., n.$$

Show the equivalence of the matrix and summation forms for the OLS slope estimator, b_1. You do not need to derive the summation form. Hint: The inverse of A is

$$A^{-1} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{-1} = \frac{1}{a_{11}a_{22} - a_{12}a_{21}} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}.$$

2. (30 pts) Consider the Classical multivariate linear regression model:

$$Y = X\beta + \epsilon$$

with two error structures: (i) $\epsilon \sim N(0, \sigma^2 I)$ and (ii) $\epsilon \sim N(0, \sigma^2 \Omega)$, where Ω is a symmetric positive-definite matrix.

(a) Is the ordinary least squares, $b = (X'X)^{-1}(X'Y)$, unbiased for each model? Defend your answer.

(b) Derive the variance of b for each model.

(c) Find the efficient estimator for the partitioned model

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \end{bmatrix},$$

where ϵ_1 and ϵ_2 are mutually independent and have variance-covariance matrices, $\sigma_1^2 I$ and $\sigma_2^2 I$.

3. (20 pts) True or False? If false, explain why.

   (a) High multicollinearity leads to biased coefficient estimates and low t statistics.

   (b) The null hypothesis, $H_0 : \beta_1 + \beta_2 = 1$, can be tested with the following t statistic

$$t = \frac{b_1 + b_2 - 1}{se(b_1) + se(b_2)}.$$

   (c) Consider the following model $y_i = \beta_0 + \beta_1 x_i^* + \epsilon_i$, where $x_i = x_i^* + \nu_i$ is measured with error. The least squares estimate of β_1 is unbiased if $E(\epsilon_i \cdot \nu_i) = 0$.

   (d) The variance of the estimated marginal effect, $\delta = \partial\hat{y}/\partial x$, from the quadratic model $y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \epsilon_i$ is

$$var(\delta) = var(b_1) + 2var(b_2).$$

4. (10 pts) What is the output for the following gauss code?

```
x=zeros(3,3);
for i (1,3,1);
for j (1,i,1);
x[i,j]=(i+j)^2;
endfor;
endfor;
print x;
```