

Home work # 2
Math 5290-Fall 2008
Due date 09/29/08 in class

1. **(ex9.3 pp61)**. Given (Ω, \mathcal{A}, P) , a null set for P is a subset N of Ω such that there exists a $A \in \mathcal{A}$ satisfying $N \subset A$ and $P(A) = 0$. Let

$$\mathcal{A}' = \left\{ A \cup N; A \in \mathcal{A}, N \in \mathcal{N} \right\}$$

where \mathcal{N} is the set of all null sets. Suppose that $X = Y, a.s.$ where X and Y are two real-valued functions on Ω . Show that

$$X : (\Omega, \mathcal{A}') \mapsto (\mathbb{R}, \mathcal{B})$$

is measurable if and only if

$$Y : (\Omega, \mathcal{A}') \mapsto (\mathbb{R}, \mathcal{B})$$

is measurable.

2. **(ex9.5 pp61)**. Given Ω, \mathcal{A}, P , suppose X is a rv. with $X \geq 0 a.s.$ and $E[X] = 1$.

Define $Q : \mathcal{A} \mapsto \mathbb{R}$ by

$$Q(A) = E[X1_A].$$

Show that Q defines a probability measure on (Ω, \mathcal{A}) .

3. **(ex10.11 pp74)**. Let X and Y be independent r.v.'s with distributions $P(X = 1) = P(Y = 1) = \frac{1}{2}$ and $P(X = -1) = P(Y = -1) = \frac{1}{2}$. Let $Z = XY$. Show that X, Y, Z are pairwise independent but that they are not mutually independent.