Math TLC-Algebra-2009
HW #1 Due: Monday, June 29
Submit by e-mailing a PDF file to MathTLCAlgebra@gmail.com

For each of the following binary operations, determine (a) if the operation is commutative, (b) if the operation is associative, (c) if the operation has an identity. In addition, if the operation has an identity find the inverse of your favorite non-identity element.

1. Let $\mathbb{Z}$ be the set of all integers. For each $a, b \in \mathbb{Z}$, we define $a*b = a+b+ab$.
2. Let $\mathbb{Z}$ be the set of all integers. For each $a, b \in \mathbb{Z}$, we define $a * b = a^2b$.
3. Let X be a set and let $P(X)$ be the set of all subsets of X. For each $A, B \in P(X)$, we define $A\Delta B$ to be the set of all elements of x that are in exactly one of A or B.
4. Let A consists of all functions $f : \mathbb{R} \to \mathbb{R}$ of the form

   $$f(x) = ax + b,$$

   where a and b are real numbers with $a \neq 0$. For each $f, g \in A$, we define $f \circ g$ to be the usual composition of functions.