9th Annual Rocky Mountain Discrete Mathematics Days

University of Wyoming

August 1-2, 2005

Support provided by the National Science Foundation

9th Annual Rocky Mountain Discrete Mathematics Days
University of Wyoming
August 1-2, 2005

All talks take place in Engineering 1062.

Schedule for Monday, August 1

12:45 –	1:10	**Registration** *Engineering 1062*
1:15 –	2:10	Willem Haemers *Recent results on spectral characterizations of graphs*
2:20 –	2:45	Leslie Hogben *Minimum Rank and Maximum Eigenvalue Multiplicity of Symmetric Tree Sign Patterns*
2:50 –	3:20	**Break**
3:20 –	3:45	Wayne Barrett *Graphs whose minimal rank is two*
3:55 –	4:20	Jason Grout *Graphs whose minimal rank is two: the finite field case*
4:30 –	4:55	Benson Joeris *Linear-time recognition of Helly circular-arc graphs*
5:05 –	5:30	Sylvia Hobart *Cayley and Ceilidh: Group Theory applied to Engish Country Dance; or, What I did on my summer vacation*
6:15 –		**Dinner** *The Altitude*, 320 S. 2nd Street

Schedule for Tuesday, August 2

9:30 –10:25	Chris Godsil *Type-II Matrices*
10:30 –11:00	**Break**
11:00 –11:25	Bill Cheriwitzo *Open Problems on Sharply focused and Hyperfocused Arcs*
11:35 –12:00	Koen Thas *Recent Progress on Elation Generalized Quadrangles*
12:10—12:35	Eric Moorhouse *Planes, nets and webs*
12:40 – 2:00	**Lunch, Ross Hall 352**
2:00 – 2:25	Ryan Martin *The editing distance in graphs*
2:35 – 3:00	Kathy Fraughnaugh *Maximal Clique partitions of Line Graphs*
3:10 – 3:35	Bryan Shader *Spectrally transverse graphs*

Abstracts of Talks

Recent results on spectral characterizations of graphs.

Willem H. Haemers
Tilburg University, The Netherlands

Abstract; A few years ago, Edwin van Dam and I wrote the paper "Which graphs are determined by their spectrum" (Linear Alg. Appl. 373 (2003), 241-272). In there, a number of problems concerning characterizations of graphs by their spectrum were stated. In the meantime several of these problems were attacked and (partly) solved. Some achievements are:

- New uniqueness results for strongly regular graphs.
- New graphs cospectral, but non-isomorphic, to distance-regular graphs.
- A new method for proving that a graph is determined by its spectrum.
- Construction of non-regular graphs, cospectral to regular graphs with respect to some generalized adjacency matrix (not the Seidel matrix).

The aim of the talk is to survey these developments.

Minimum Rank and Maximum Eigenvalue Multiplicity of Symmetric Tree Sign Patterns

Leslie Hogben
Iowa State University

Abstract A graph (or sign pattern) can be used to describe a family of matrices by the zero-nonzero pattern of entries (signs of entries). Algorithms will be given for computation of:

1. Minimum rank of matrices associated with a tree sign pattern or tree allowing loops.
2. Maximum multiplicity of an eigenvalue of symmetric matrices associated with symmetric tree sign pattern or tree allowing loops.
3. Rational matrices realizing the above values.

This is joint work with DeAlba, Hardy, Hentzel, Wangsness)

Graphs whose minimal rank is two

Wayne Barrett
Brigham Young University

Abstract: Let F be a field, $G = (V, E)$ be an undirected graph on n vertices, and let $S(F, G)$ be the set of all symmetric $n \times n$ matrices whose nonzero off-diagonal entries occur in exactly the positions corresponding to the edges of G, i.e., for $i \neq j$, $a_{ij} \neq 0 \iff ij \in E$. Let $\text{mr}(F, G)$ be the minimum rank over all matrices in $S(F, G)$. It is easy to see that $\text{mr}(F, G) = 1$ if and only if G is the union of a clique with at least 2 vertices and an independent set.

A characterization of those graphs for which $\text{mr}(F, G) \leq 2$ depends on whether or not F is infinite and whether or not $\text{char}\, F = 2$. We present two characterizations of such graphs in the case that F is infinite and $\text{char}\, F \neq 2$.

1. The graphs having 6 specific forbidden subgraphs (graphs that do not occur as induced subgraphs).
2. The graphs whose complements have the form
   $G^c = (K_{s_1} \cup K_{s_2} \cup K_{p_1,q_1} \cup \cdots \cup K_{p_k,q_k}) \vee K_r$,
   for nonnegative integers $s_1, s_2, k, p_1, q_1, p_2, q_2, \ldots, p_k, q_k, r$.

This is joint work with Hein van der Holst and Raphael Loewy

Graphs whose minimal rank is two: the finite field case

Jason Grout
Brigham Young University

Abstract: Given a finite field F and an undirected graph G on n vertices, let $S(F, G)$ be the set of all symmetric $n \times n$ matrices over F whose nonzero off-diagonal entries occur in exactly the positions corresponding to the edges of G. Let $\mathrm{mr}(F, G)$ be the minimum rank of all matrices in $S(F, G)$. The graphs for which $\mathrm{mr}(F, G) \leq 2$ have been characterized using forbidden subgraphs and also by their general structure. It has been announced that there are only a finite number of forbidden subgraphs characterizing the set $\mathrm{mr}(F, G) \leq k$ for any nonnegative k.

We present a preliminary report on an algorithm for characterizing graphs such that $\mathrm{mr}(F, G) \leq k$ for nonnegative k. We also present our current results from the algorithm for $\mathrm{mr}(F_2, G) \leq 3$, including both a structure characterization and a forbidden subgraph characterization.

This is joint work with Wayne Barrett and Don March

Linear-time recognition of Helly circular-arc graphs

Benson Joeris
Colorado State University

Abstract: A set of intervals on a line has the "Helly property," which means that if each pair of them intersects, they have a collective nonempty intersection. Unfortunately, the same is not true, in general, of arcs on the circle, and this makes circular-arc graphs much harder to work with as combinatorial objects than interval graphs.

In 1974, Gavril described the class of "Helly circular-arc graphs," which are the the intersection graphs of sets of circular arcs that obey the Helly property. They are properly contained in the class of circular-arc graphs, properly contain the class of interval graphs, and share many of the nice combinatorial properties of interval graphs.

We improve the known time bounds for recognizing this class from $O(n^3)$ to linear, give a forbidden subgraph characterization of those circular-arc graphs that are Helly, and give related algorithmic results on sets of arcs on the circle.

The talk is based on joint work with Ross McConnell, Colorado State, and Jeremy Spinrad, Vanderbilt.

Cayley and Ceilidh: Group Theory applied to Engish Country Dance;
or, What I did on my summer vacation

Sylvia A. Hobart
University of Wyoming

Abstract. Think Jane Austin, and couples lined up on the dance floor moving in stately patterns. These dances are still danced (every week, in some cities), and there are hundreds of dances, some just composed and others dating back 300 years.

The figures of the dance involve a permutation of the positions of the dancers in the line of couples. We will look at these from the point of view of group theory.

(I should point out that Ceilidh is prounounced "Kaylay".)

Type-II Matrices

Chris Godsil
University of Waterloo

Abstract: If A and B are $m \times n$ complex matrices, their *Schur product* $A \circ B$ is the entrywise product of A and B, that is, it is the $m \times n$ matrix with

$$(A \circ B)_{i,j} = A_{i,j} B_{i,j}.$$

The all-ones matrix J is the identity for this product. If no entry of A is zero, there is a unique matrix $A^{(-)}$ such that

$$A \circ A^{(-)} = J;$$

we call $A^{(-)}$ the *Schur inverse* of A. If A is $v \times v$, we say that A is a *type-II* matrix if

$$AA^{(-)T} = vI$$

(where $A^{(-)T}$ is the transpose of $A^{(-)}$).

Hence if A is type II, then $A^{-1} = \frac{1}{v} A^{(-)T}$, which is easily computed, and so these matrices might be viewed as a gift to students in linear algebra courses. However the real reason we are interested in them is that certain type-II matrices were used by Vaughan Jones to construct so-called *spin models*, which provide useful invariants of links and knots.

In my talk I will show how type-II matrices arise in connection with a number of interesting combinatorial and geometric problems, and hence are comparatively common. I will also develop some of their theory, which leads to the surprising conclusion that finding new spin models is a graph-theoretical problem.

Open Problems on Sharply focused and Hyperfocused Arcs

Bill Cheriwitzo
University of Colorado–Denver

Abstract: A *sharply focused k-arc* in a Desarguesian plane is a set of k points, no three collinear, such that on some line of the plane (exterior to the arc) all the secants of the arc meet in a set of k points (the focus set). A hyperfocused arc is a k-arc whose secants meet some line in a set of $k-1$ points. These point sets can be used in the construction of a geometric solution to a secret sharing scheme problem, but I will not have enough time to discuss this application.

After a brief review of the subject and known results I will pose some interesting open problems and indicate the progress that is being made on them.

Recent Progress on Elation Generalized Quadrangles

Koen Thas
Ghent University, Belgium

Abstract: The *Moufang condition* is one of the central group theoretical conditions in Incidence Geometry, and was introduced by Jacques Tits in order to classify spherical buildings of rank at least 3.
About ten years ago, Norbert Knarr studied buildings of rank 2 and type B_2, i.e. *generalized quadrangles*, satisfying one of the Moufang conditions locally at one point. He then posed the fundamental question whether the group generated by the root-elations with root containing that point is sharply transitive on the points opposite it, that is, whether that point is an elation point for that group.

In this talk, we want to lecture on this question, its recent solution by the speaker, and corollaries.

Planes, nets and webs
G. Eric Moorhouse
University of Wyoming

Abstract: The main open questions in the study of finite projective planes concern the possible orders of finite planes, and the question of whether planes of prime order are necessarily the classical ones. A promising approach to both questions relies on conjectured bounds for ranks of finite nets (i.e. rank of the incidence matrix over a field of positive characteristic). The conjectured rank bounds for (finite) nets agree with the known rank bounds for (infinite) webs, but different mathematical tools are required in the finite case. In 1991 I verified the conjectured rank bounds for 3-nets (those having 3 parallel classes of lines) using loop theory. I will describe recent progress in the case of 4-nets, using the method of exponential sums.

The editing distance in graphs
Ryan Martin
Iowa State University

Abstract: An edge-operation on a graph G is defined to be either the deletion of an existing edge or the addition of a nonexisting edge. Given a family of graphs $\mathcal{G}$, the editing distance from G to $\mathcal{G}$ is the smallest number of edge-operations needed to modify G into a graph from $\mathcal{G}$. In this paper, we fix a graph H and consider $\mathrm{Forb}(n, H)$, the set of all graphs on n vertices that have no induced copy of H. We provide bounds for the maximum over all n-vertex graphs G of the editing distance from G to $\mathrm{Forb}(n, H)$, using an invariant we call the **binary chromatic number** of the graph H. We give asymptotically tight bounds for that distance when H is self-complementary and exact results for several small graphs H. Joint work with Maria Axenovich and André Kézdy.

Maximal Clique partitions of Line Graphs
Kathy Fraughnaugh
University of Colorado-Denver

Abstract: A maximal clique partition of a graph $G = (V, E)$ is a family of maximal complete subgraphs of V that partitions E. The smallest size among all maximal clique partitions of G is the maximal clique partition number $\mathrm{mcp}(G)$. The line graph $L(G)$ of G is the graph whose vertices correspond to the edges of G where two edges are adjacent in $L(G)$ whenever they are incident in G. Not every line graph has a maximal clique partition. We find necessary and sufficient conditions for the existence of a maximal clique partition in $L(G)$. When $L(G)$ has a maximal clique partition, we show that this partition is unique and find $\mathrm{mcp}(L(G))$.

This is joint work with Chariya Uiyyasathian, Chulalongkorn University, Bangkok, Thailand.

Spectrally transverse graphs
Bryan Shader
University of Wyoming

Abstract: Given a graph G with vertices $1, 2, \ldots, n$, we denote by $\mathcal{S}(G)$ the set of all n by n symmetric matrices $A = [a_{ij}]$ such that $a_{ij} \neq 0$ and $i \neq j$ implies that there is an edge in G from i to j. The graph G is *spectrally transverse* provided O is the only matrix $B = [b_{ij}]$ such that there is a matrix $A \in \mathcal{S}(G)$ with distinct eigenvalues with $[A, B] = O$, and $b_{ii} = 0$ for all $i \neq j$, and $b_{ij} = 0$ whenever $\{i, j\}$ is an edge of G. We give families of spectrally transverse graphs, as well as, examples of induced subgraphs that are forbidden from spectrally transverse graphs. The notion of spectrally transverse graphs is motivated by studying the interesection of the isospectral surface of a matrix A with the class $\mathcal{S}(G)$. When G is spectrally transverse, such an intersection (for all $A \in \mathcal{S}(G)$ with distinct eigenvalues) is necessarily a surface.

This is joint work with Justin Christian, Ken Driessel, and Chi-Kwong Li.
