A Note on the Relationship Between Mates' Puzzle and Frege's Puzzle

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Abstract

In this note I argue that, relative to certain largely uncontroversial background conditions, any instance of Mates' Puzzle is equivalent to some instance of Frege's Puzzle. If correct, this result is surprising. For, barring the radical move of rejecting the possibility of synonymous expressions in a language *tout court*, it shows that there is no strictly lexical solution to at least some instances of Frege's Puzzle. This forces the hand of theorists who wish to provide a semantic (rather than pragmatic) solution to Frege's Puzzle. The only option open will be modify in one way or another the standard formulation of semantic compositionality.

1. Introduction. In this note I will argue that, relative to certain largely uncontroversial background assumptions, some instance of Frege's Puzzle concerning the substitution of co-referential singular terms can be derived from any instance of Mates' Puzzle concerning the unrestricted substitution of synonymous expressions.[1] Conversely, it will be clear that the argument can equally well be run in reverse so that any instance of Mates' Puzzle may be derived from some instance of Frege's Puzzle. Thus, relative to the indicated background conditions, any instance of Mates' Puzzle is equivalent to some instance of Frege's Puzzle.

If correct, this result is nontrivial. For, while a great deal of philosophical work has gone into finding a solution to Frege's Puzzle, many of the resulting theories fail to generalize

[1] Frege observed that redundant identity statements involving singular terms (i.e., statements of the form 'a = a') have different doxastic properties than true non-redundant identity statements (i.e., statements of the form 'a = b', where 'a' and 'b' are co-referential singular terms). In particular, while the former are trivial truths of logic (and, hence, almost universally believed), the latter are typically nontrivial, perhaps even expressing empirically significant discoveries (and, hence, frequently disbelieved). Frege's insights may be extended. For it follows from these considerations that co-referential terms will not generally be substitutable salve veritate into contexts *reporting* beliefs. That is, even though the sentence 'x believes that a = a' is true, the sentence 'x believes that a = b' may not be true.

As is well known, Frege proposed to solve this puzzle by positing a distinct kind of semantic value for singular terms. According to Frege, even though 'a' and 'b' both *refer* to the same object, they nevertheless differ in their *sense* (*Sinn*). The sense of a term may be thought of as a descriptive concept or mode-of-presentation of the referent. Thus, according to Frege, the failure of co-referential terms to substitute *salve veritate* is accounted for by convincingly to Mates' Puzzle (Frege's own solution being a case in point). Indeed, by the very nature of Mates' Puzzle, with its assumption of synonymy, *any* solution to Frege's Puzzle grounded entirely in lexical semantics will fail to generalize to Mates' Puzzle. This strongly suggests that Frege's Puzzle, like Mates' Puzzle, puts pressure on standard principles of semantic compositionality and not, as is often assumed, on the specification of word meaning.[2] Consequently, if the argument succeeds, it suggests that various deep-rooted assumptions concerning the structure of language may need to be rethought (see Bealer 1993; Taschek 1995; Moffett 2002).

As a preliminary step, I will make the now standard assumption that 'that'-clauses are singular terms that denote propositions (though I will rely on no specific account of propositions themselves). This thesis is extremely well supported. For example, the following argument is intuitively valid:

Everything x knows, x believes
x knows that p

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Therefore, x believes that p.

Arguably, the best explanation for the validity of such arguments presupposes that 'that'-clauses are singular terms (Schiffer 1996; Bealer, 1998).[3] For a detailed discussion of this issue, the interested reader should consult Bealer (1982), Bealer & Mönnich (1989), and Chierchia (1982).

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the fact that such terms differ in their associated senses. Given this solution, Frege seems to be committed to the claim that if two terms have the same sense (and, hence, are synonymous), they should substitute *salve veritate*.

[2] At least insofar as one takes Frege's Puzzle to be a semantic puzzle at all. Of course, some philosophers have argued that the correct solution to Frege's Puzzle involves discourse pragmatics (see, e.g., Bealer 1982; Salmon 1986; Soames 1987). Though I disagree that Frege's Puzzle can be given an adequate pragmatic solution, those philosophers who do prefer this approach should welcome the present result.

[3] If the quantification expressed in the first premise of this argument is objectual, the view that 'that'-clauses are singular terms follows as a matter of course. And there are good reasons for thinking that quantification over 'that'-clauses in English is objectual. For example, consider the following English sentence: Some propositions are not expressible in English. Arguably this sentence is true. If so, then quantification in English cannot be substitutional since, by hypothesis, there will be no true substitution instance that could instantiate the existential quantifier (because, *ex hypothesi*, there could be no such English sentence). Moreover, given the relatively uncontroversial

2. Mates' Puzzle. In a now classic paper, Mates (1952) argued that the following two claims are inconsistent:

> (I) Any adequate definition of synonymy will have the consequence that any two expressions are synonymous in a language L iff they may be interchanged *salve veritate* in each sentence of L in which they occur.

and

> (II) There exists an adequate definition of synonymy.

Mates' Adequacy Constraint (I) is extremely plausible. For suppose that there were some sentence S that did not conform to it. Then the truth conditions of S would not be determined by the semantic values of S's atomic constituents plus logical form. That is, the truth conditions of S would not be determined by its compositional truth theory (on one standard construal of this notion).[4] It appears, therefore, that the Adequacy Constraint is at least as well motivated as the principle of a compositional truth theory for natural languages, and the latter is pretty well motivated indeed.

So if Mates' argument is successful, it looks as though we are either going to have to give up the concept of synonymy or we are going to have to abandon the traditional view of a compositional truth theory. Neither option is particularly appealing.

Here is Mates' argument. Choose two expressions of the language that are synonymous according to some chosen explication. For the sake of fixing ideas, let us suppose the chosen

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claim that 'that'-clauses are syntactic constituents, even if the quantification in such arguments is substitutional, the most natural classification of 'that'-clauses is still as singular terms–though perhaps vacuous ones.

[4] Here's why. Let $S = S(\xi)$ and let $S' = S(\xi')$, where S' arises from S by substituting some expression ξ' synonymous (according to some chosen explication of this concept) with ξ into S. We may suppose that the logical form of $S(\xi)$ is the same as the logical form of $S(\xi')$. I will abbreviate this sameness of logical form as follows: $\Delta(S) = \Delta(S')$. Finally, let $V[S(\xi)]$ be the truth value of $S(\xi)$. Now suppose that this is a case for which Mates' Adequacy Constraint fails. Then we will have $V[S(\xi')] \neq V[S(\xi)]$. But by hypothesis we have it that $[\![S(\ )]\!] = [\![S(\ )]\!]$, $\Delta(S) = \Delta(S')$ and $[\![\xi]\!] = [\![\xi']\!]$. That is, the meaning of the matrix sentence S( ) = the meaning of the matrix sentence S( ), the logical form of S = the logical form of S′, and by hypothesis the meaning of ξ = the meaning of ξ'. It follows that $V[S(\xi)]$ is not determined by the meanings of the constituent expressions in S plus logical form.

expressions are the predicates 'recursive' and 'computable'. Now consider the seeming truism that

(1) Nobody doubts that whoever believes that *f* is recursive, believes that *f* is recursive.

(for some particular function *f*). On our background assumptions we can conclude from (1) that

(2) Nobody doubts that whoever believes that *f* is recursive, believes that *f* is computable.

But surely (2) can be false even though (1) is true. Notice, moreover, that (1) and (2) are entirely neutral on the question of whether or not everyone who believes that *f* is recursive also believes that *f* is computable. Even if we grant this principle, it may still be reasonable to doubt it. But if people do reasonably doubt it, then sentence pairs such as (1) and (2) will differ in truth-value. And this is enough to show that synonymous terms cannot always be substituted one for the other *salve veritate*.

So we must either give up (I) and apparently with it the hope of a compositional truth theory, or we must be mistaken about the synonymy of the expressions 'recursive' and 'computable'. However, since these predicates were chosen according to an *arbitrary* explication of the concept of synonymy, it follows that there can be no adequate explication of this concept. That is, we must give up (II).

Mates' Puzzle is particularly disturbing from the perspective of formal semantics because it admits the kind of solution Frege gave to his puzzle concerning the failure of co-referential terms to substitute *salve veritate* only at the cost of abandoning the notion of synonymy altogether. For the predicates 'recursive' and 'computable' were assumed to be synonymous and, therefore, could not have differed in their semantic values–specifically, they could not have differed in any hypothesized associated senses. For this reason, theorists who favor some version of a neo-Fregean semantic theory appear to be committed to the claim that Mates' Puzzle is different in kind from Frege's Puzzle.[5] I will now endeavor to show that this contention is mistaken by arguing that relative to certain background conditions the two puzzles are equivalent. If correct, this equivalence shows that various, lexically motivated approaches to substitution failure in natural language simply do not provide a uniform solution (unless, of course, they deny outright the possibility of synonymous expressions).

3. Deriving Frege's Puzzle. In order to derive an instance of Frege's Puzzle from Mates' Puzzle, we need only make the observation that if somebody, say x, doubts that whoever believes that *f* is recursive, believes that *f* is computable, they must on reflection doubt that the proposition that *f* is recursive is the same proposition as the proposition that *f* is computable. For this person doubts that the proposition that *f* is recursive has the property of being believed by another person y while the proposition that *f* is computable lacks this property. Thus, the falsity of (2) implies the falsity of (3):

(3) Nobody doubts that the proposition that *f* is recursive = the proposition that *f* is computable.

Thus, whatever grounds we have for rejecting (2) will give us equally good grounds for rejecting (3). In addition, (3) is intuitively false. Now consider the corresponding statement (4).

(4) Nobody doubts that the proposition that *f* is recursive = the proposition that *f* is recursive.

Presumably (4) is true, at least for minimally rational agents. But (4) differs from (3) only in the substitution of the predicate 'computable' for the purportedly synonymous predicate 'recursive'. We now have a variation on Mates' Puzzle that involves propositional identities.

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[5] Mates' Puzzle causes problems for other types of semantic theory as well. For instance, recent advocates of two-dimensional semantics (Chalmers 1996; Jackson 1998) cannot simply extend their treatment of traditional instances of Frege's Puzzle to handle Mates'-style puzzles. For if two terms really were *synonymous* they would agree, minimally, in their primary intensions. Consequently, the standard two-dimensionalist account of Frege-style substitution failure (namely, that in doxastic contexts a 'that'-clause denotes the primary intension of the embedded clause) would fail.

If one understands Frege’s Puzzle broadly as a puzzle about the failure of co-referential terms (whether atomic or complex) to substitute *salve veritate*, then this is also an instance of Frege’s Puzzle. For given our background assumption that ‘that’-clauses are complex singular terms, the failure of the predicate ‘computable’ to substitute into (4) generates a correlative failure of the complex singular term ‘that *f* is computable’ to substitute for the complex singular term ‘that *f* is recursive’ in (4).

Moreover, given a few more reasonable background assumptions, we can derive a version of Frege’s Puzzle that involves only *atomic* co-referential singular terms. To do this, we first suppose that our language contains the atomic singular terms ‘Principle P’ and ‘Principle P′’ for the propositions that *f* is recursive and that *f* is computable, respectively.[6] We then make the following assumptions. For a certain individual x, both of the following are true of x:

(5) x believes that Principle P is that *f* is recursive

(6) x believes that Principle P′ is that *f* is computable

Finally, we assume that x is also the very same individual that made (3) false; that is, the individual who doubts that the proposition that *f* is recursive is the proposition that *f* is computable. Consequently, in addition to (5) and (6), we also have the following:

(7) x doubts that the proposition that *f* is recursive = the proposition that *f* is computable.

But given (5)–(7), we know that x can (and we assume does) make a correct inference that makes (8) true:

(8) x doubts that Principle P is Principle P′.[7]

At the same time, it appears consistent to assume that x is not so confused or irrational as to doubt that Principle P is Principle P. That is, we may assume that the following is false:

(9) x doubts that Principle P is Principle P.

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[6] Thus, the terms ‘Principle P’ and ‘Principle P′’ behave in the same way that, for example, the term ‘The Continuum Hypothesis’ does. Presumably, ‘The Continuum Hypothesis’ is a singular term denoting the proposition that there are no cardinalities strictly between the cardinality of the real numbers and the cardinality of the natural numbers.

[7] Notice that since belief and the other propositional attitudes are not closed under logical consequence, our background assumptions do not commit us to the truth of (8).

Now, generalizing from (8) and (9) respectively we get the following pair of sentences.

(10) Nobody doubts that Principle P is Principle P′.

(11) Nobody doubts that Principle P is Principle P.

Presumably, (11) is true. We have just shown, moreover, that (10) may be false. Thus, this pair of sentences constitutes a classic instance of Frege's original Morning Star/Evening Star puzzle. For (11) differs from (10) only in the substitution of one atomic co-referential singular term (i.e., 'Principle P′') for another (i.e., 'Principle P'). Thus, given only the standard assumption that 'that'-clauses are complex singular terms denoting propositions (plus certain contingent background assumptions), we have derived an instance of Frege's Puzzle from an arbitrary instance of Mates' Puzzle.

It should be clear, moreover, that this very argument can be run with equal felicity in the opposite direction. That is, we assume (10) and (11) as a standard instance of Frege's Puzzle. Then, given our background assumptions, we can derive the falsity of (3). And from here it is a short step to recovering the original instance of Mates' Puzzle.

4. Conclusion. I have argued that some instances of Frege's Puzzle are, at least relative to uncontroversial background assumptions, equivalent to instances of Mates' Puzzle. But since Mates'-style puzzles depend on the failure of synonymous (and not merely co-referential) expressions to substitute *salve veritate,* it follows that at least some instances of Frege's Puzzle involve the failure of synonymous (and not merely co-referential) terms to substitute *salve veritate*. This is surprising. For, barring the radical move of rejecting the possibility of synonymous expressions in a language *tout court*, it shows that there is no strictly lexical solution to at least some instances of Frege's Puzzle.

Moreover, if one wishes to avoid a pragmatic solution to Frege's Puzzle, it seems that the only remaining option is to modify in one way or another the standard formulation of semantic compositionality. This is an inherently dicey option for it requires one to provide a compositional semantics for the language that does *not* depend entirely on the meanings of the atomic expressions of the language plus logical form. Any attempt to do this is at risk of running afoul of Davidson's learnability requirement for natural languages (Davidson 1965). It remains to be seen whether semantic theorists can meet this challenge in a satisfactory way.

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