

Homework # 4  
Math 5200-Spring 2009  
Due date March 5th 2009.

1. (ex22, pp94)
2. (ex3, pp95)
3. Let  $(X, \mathcal{M}, \mu)$  be a measure space and  $\{f_n\}_n$  a sequence of measurable functions that converges to  $f$   $\mu$ -a.e. If  $f \in L^1(X)$ , show that

$$\lim_{n \rightarrow \infty} \int_X |f_n| d\mu = \int_X |f| d\mu \text{ implies } \lim_{n \rightarrow \infty} \int_X |f_n - f| d\mu = 0,$$

and that the conclusion may fail if  $f$  is not integrable.

4. Let  $(X, \mathcal{M}, \mu)$  be a measure space and  $\{f_n\}_n$  be a sequence of non-increasing nonnegative measurable functions which converges to  $f$ . Show that

$$\lim_{n \rightarrow \infty} \int_X f_n d\mu = \int_X f d\mu,$$

provided that  $f_1 \in L^1(X)$ , and that the conclusion may fail if  $f_1 \notin L^1(X)$ .