Math 4000
HW 4 - Ch 4, 5
Due Tuesday, February 18

(1) We know that, if $s$ is the side of a regular inscribed $n$-gon and $t$ is the side or a regular inscribed $2n$-gon, then

$$t = \sqrt{2 - \sqrt{4 - s^2}}.$$ 

Now, assume that a hexagon is inscribed in a unit circle,. The hexagon’s perimeter is 6, a rough approximation of for the circle’s circumference of $2\pi$, and so $\pi \approx 3.00$. Now use the above through seven doublings, until you have the perimeter of a regular inscribed 768-gon. What is the corresponding approximation of $\pi$ based on these inscribed figures?

(2) Euclid noted that $\frac{C}{D}$ for any circle is always constant. We now call this constant $\pi$. Euclid also gave formulas for area of a circle to be $A = k_2D^2$ and volume of a sphere to be $V = k_3D^3$. Express $k_2$ and $k_3$ in terms of $\pi$.

(3) An equilateral triangle has each side $4x$ units long. Find its area both by the standard formula, $A = \frac{1}{2}bh$ and by Heron’s Formula and verify the results are equal.

(4) Begin with a right triangle with sides $a$, $b$, and hypotenuse $c$. Determine the area two ways - Heron’s Formula, and $A = \frac{1}{2}ab$. Equate and manipulate these equations to deduce the Pythagorean Theorem.