1. Let $m(t) = t^2 - t - 1 \in \mathbb{Q}[t]$. Then $m(t)$ is irreducible in $\mathbb{Q}[t]$. Consider the quotient ring $F = \mathbb{Q}[t]/\langle m(t) \rangle$, which is a field. Consider the elements $\alpha = 2t + 3 \in \langle m(t) \rangle$ and $\beta = 3t - 1 \in \langle m(t) \rangle$ in $F$. Evaluate each of the following, and express in standard form as $(*)t + (*) + \langle m(t) \rangle$ where each ‘(*)’ is a rational number:

(a) $\alpha + \beta$
(b) $\alpha \beta$
(c) $\beta^{-1}$
(d) $\alpha / \beta = \alpha \beta^{-1}$
(e) $\alpha^5 \beta^3$

Use MAPLE® where appropriate.

2. How many homomorphisms are there from $\mathbb{Z}_6$ to $\mathbb{Z}_8$? Justify your answer.

3. Let $F = \{0, 1, \alpha, \beta\}$ be a field with exactly four elements. Determine the addition and multiplication tables for $F$. (Note: The handout Review: Basic Notation and Properties of the Integers gives addition and multiplication tables for $\mathbb{Z}_4$. However, $\mathbb{Z}_4$ is not a field.)

4. Let $F$ be a finite field with $q$ elements. Determine the number of monic irreducible polynomials in $F[t]$:
(a) of degree 1;
(b) of degree 2;
(c) of degree 3.

Hint: One possible approach is to count the number of reducible monic polynomials of a given degree, and subtract from the total number of monic polynomials of that degree.