Estimating 3D elastic moduli of rock from 2D thin-section images using differential effective medium theory

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ABSTRACT

Standard digital rock physics (DRP) has been extensively used to compute rock physical parameters such as permeability and elastic moduli. Digital images are captured using 3D microcomputed tomography scanners that are not widely available and often come with an excessive cost and expensive computation. Alternative DRP methods, however, benefit from the highly available low-cost 2D thin-section images and require a small amount of computer memory use and CPU. We have developed another alternative DRP method to compute 3D elastic parameters based on differential effective medium (DEM) theory. Our investigations indicate that the pore aspect ratio (PAR) is the most crucial factor controlling the elastic moduli of rock. Based on digital rock modeling in a dry calcite sample with 20% porosity, the bulk modulus is reduced by 51%, 80.7%, and 96.8% for aspect ratios of 1, 0.2, and 0.05, respectively. Similarly, the shear modulus is reduced by 52%, 73.8%, and 92.8% for the same PARs. These findings confirm the importance of the PAR in wave propagation through porous media. Such an evaluation, however, can be very expensive for 3D images because one requires using several of them for drawing a reliable conclusion. Therefore, we aim to capture the PAR distribution from 2D images. This distribution is, then, used to estimate 3D elastic moduli of sample by DEM equations. Three orthogonal 2D images were used and results indicated that 2D PARs in orthogonal orientations could address pore shapes more effectively. Moreover, a stochastic porous media reconstruction method was also used to generate more scenarios of rock structure and those of which that are not seen in 2D images. Results from Berea sandstone and Grosmont carbonate indicated that using only 2D images our proposed method could effectively estimate 3D elastic moduli of rock samples.

INTRODUCTION

Digital rock physics (DRP) is established based on the computation of numerical properties from rock images. High-resolution microcomputed tomography (micro-CT) scanners have made it possible to obtain a 3D image of the internal structure of rocks, including pore space, fractures, and minerals at a small scale. These images are segmented and, subsequently, numerical algorithms are applied to simulate physical processes (e.g., fluid flow, wave propagation, and electrical current to obtain permeability, elastic moduli, and formation factor) in the digital rock medium. Recently, DRP has been extensively used to compute rock physical parameters, and the results have been compared with laboratory data (Arns et al., 2002; Keehm et al., 2004; Saenger et al., 2004, 2016; Andrä et al., 2013b; Karimpouli and Tahmasebi, 2016; Tahmasebi et al., 2016a, 2017; Karimpouli et al., 2017b).

Although the high-resolution data generated using the imaging tools provide detailed information about the rock structure, there are still numerous limitations in practice. For example, micro-CT scanners are expensive and are therefore not available widely. Moreover, 3D numerical simulations are time consuming and a huge amount of memory as well as high-speed computers are required. To alleviate such limitations, some alternative DRP methods have recently been developed that use the widely available 2D images and still predict the 3D properties reasonably well, e.g., Saxena and Mavko (2016), Saxena et al. (2017), and Karimpouli and Tahmasebi (2016). The main purpose of these methods is to estimate the 3D properties of rocks using just 2D images. For example, Karimpouli and Tahmasebi (2016) reconstruct a rock in three dimensions with a crosscorrelation-based simulation method (CCSIM) (Tahmasebi et al., 2016a; Tahmasebi, 2017) and reproduce velocity-
porosity and permeability-porosity trends. Besides being easier, less expensive, and computationally faster, they show that the results are comparable with the standard DRP. Another alternative method was introduced by Saxena and Mavko (2016). They develop a power-law relation between the 2D and 3D elastic parameters of rock based on a 2D plane strain computation through a thin-section image and empirical relations. They also apply this approach and introduce 2D-to-3D transformations for estimating the permeability using Kozeny-Carman and flow path division approaches (Saxena et al., 2017). In a recent work, Karimpouli et al. (2017a) apply such methods on a real case study and find promising results even in comparison with borehole data for clean samples. Basically, these alternative methods use the available 2D thin-section images that are computationally faster and require much less memory. Finding a representative trend for rock physical parameters such as velocity-porosity and/or permeability-porosity trend is one of the main purposes of using such methods (Dvorkin et al., 2011). These trends, indeed, are often controlled by pore space heterogeneity such as pore size, type, and spatial distribution in all scales. In other words, trends are believed to be scale invariant from microscales (DRP) to macroscales (laboratory or well data) (Dvorkin et al., 2011).

In this study, a new approach is introduced to estimate the elastic parameters of rock using 2D images. Theoretically speaking, estimation of the effective elastic moduli of rock depends on (Mavko et al., 2009) (1) the volume fractions of the individual components, (2) the elastic properties of each component, and (3) the geometric details of the components such as shapes, size, and spatial distributions. Among all, pore type, which is usually expressed by the pore aspect ratio (PAR) — the ratio of smallest to largest radius of pore space, strongly affects seismic-wave velocities, especially in carbonate rocks (Eberli et al., 2004). For example, for a given porosity, mineral composition and fluid type, velocity variation is due to changes in pore shape (i.e., PAR) (Anselmetti and Eberli, 1999; Assera et al., 2003; Karimpouli and Malehmir, 2015; Karimpouli and Tahmasebi, 2017). Sun (2004) finds that the variation of velocity caused by different PARs could be up to 2.5 km/s or even more, which highlights the importance of the PAR for interpretation and evaluation of rock seismic responses.

To consider the PAR effect theoretically, several methods based on effective medium theory have been developed. For example, Kuster and Toksöz (1974) introduce a formulation based on a long-wavelength first-order scattering theory that evaluates the effective elastic parameters of a medium containing different pore types. The self-consistent approximation (O’Connell and Budiantsky, 1974) uses mathematical solutions for elastic deformation of a single inclusion with a specific shape and approximates the interaction of inclusions by replacing the background medium with the as-yet-unknown effective medium (Mavko et al., 2009). In differential effective medium (DEM) theory (Berryman, 1992), models are built by incrementally adding inclusions with varying PARs to a matrix.

On the experimental side, some studies have been carried out to determine the effect of PAR on wave velocities in rocks based on constructing real rock models with known pore structure and studying their effect using laboratory measurements. For example, Ass’ad et al. (1992) embed penny-shaped rubber disks into an epoxy resin matrix. Rathore et al. (1995), and similarly Tillotson et al. (2012), entrench disc and penny-shaped aluminum foils into a sand and silica cement matrix, respectively, and leach them out by acids to form cracks with known geometry. Recently, Wang et al. (2015) use soft penny-shaped silicone disks with physical properties similar to those of water and expandable polystyrene balls into clean crystalline grains of carbonate cuttings as the matrix to build samples with different PARs and sizes. Although the experimental studies are realistic, controlling some parameters such as porosity value, embedding a range of PARs, and even constructing similar samples is not straightforward. Alternatively, in this study, we use digital rock modeling (DRM) (Garboczi and Day, 1995; Roberts and Garboczi, 2000) to explore the effect of pore geometry and shape on bulk and shear moduli of digital samples. In this method, synthetic rocks are considered as a digital image with distinct phases for porosity and minerals. This flexible method allows one to construct a synthetic model containing porosity and/or minerals with any arbitrary shape, size, orientation, and distribution. This model can be solved with a finite-element algorithm to compute the linear elastic properties (Garboczi and Day, 1995; Andrä et al., 2013b). Roberts and Garboczi (2000) describe how DRM could analyze and help in our understanding of the actual behavior of porous materials.

In this paper, we use the DRM to first show that PAR is the main geometric factor affecting elastic properties. Second, we use PAR distributions, extracted from the original and reconstructed 2D images, and DEM equations to approximate the aforementioned properties. Many studies showed that DEM equations are appropriate, in practice, for a wide range of pore shapes from cracks and fractures to ball shape porosities in sandstones (Mukerji et al., 1995; Arns et al., 2002; Saenger et al., 2004) and carbonate (Xu and Payne, 2009; Neto et al., 2015; Wang et al., 2015).

MODELING OF EFFECTIVE ELASTIC MODULI

DEM theory

In DEM theory, two-phase composites are modeled by incrementally adding inclusions of one phase (e.g., porosity) to the other phase (matrix). When the porosity is zero, the matrix is considered as phase 1. Porosity is iteratively added as phase 2 with a known geometry until it reaches the desired value. In each iteration, effective bulk and shear moduli are computed and, then, used as a background module for the next iteration (Mavko et al., 2009). The coupled system of ordinary differential equations for the effective bulk and shear moduli, namely, $K^{*}$ and $\mu^{*}$, are (Berryman, 1992)

\[
(1 - y) \frac{d}{dy} [K^{*}(y)] = (K_2 - K^*)P^{(2)}(y),
\]

and

\[
(1 - y) \frac{d}{dy} [\mu^{*}(y)] = (\mu_2 - \mu^*)Q^{(2)}(y),
\]

with initial conditions $K^{*}(0) = K_1$ and $\mu^{*}(0) = \mu_1$, where $K_1$ and $\mu_1$ are the bulk and shear moduli of the initial matrix (phase 1), $K_2$ and $\mu_2$ are the bulk and shear moduli of the incrementally added porosity (phase 2), and $y$ is the concentration of phase 2. For fluid inclusions and voids, $y$ is equal to porosity $\phi$. The terms $P$ and $Q$ are geometric factors that are described by Mavko et al. (2009), and the superscript $^{(2)}$ on $P$ and $Q$ indicates that these terms are for the inclusion of phase 2 (porosity) in the background medium with effective moduli of $K^{*}$ and $\mu^{*}$.
Digital rock modeling

A digital rock often refers to a 2D or 3D numerical matrix with arrays labeled as pore or minerals. Relevant values are allocated to each phase representing their physical properties. Physical simulations such as flow transport, electrical current flow, and elastic deformation can be numerically computed through this model, which result in permeability, resistivity, and elastic-moduli/elastic-wave velocity, respectively (Andrá et al., 2013b). A real digital rock is normally produced using direct imaging. Some image processing algorithms are applied for segmentation to usually obtain a binary image with two phases of pore and mineral (Andrá et al., 2013a). However, the synthetic digital sample can be generated numerically in a predefined pattern of pore and mineral phases. By using DRM, one can generate various synthetic 2D or 3D samples with any pore and/or mineral shape, size, orientation, and distribution. The overlap between different phases can also be controlled under the dry and saturated conditions. However, discretization still remains a prominent challenge. In other words, the model size must be large enough, with respect to the largest feature (e.g., pore or mineral phase), to detract the discretization effect on the containing shape. For example, having a pore with a small PAR is not possible unless an appropriately large model is used. Some examples of such synthetic rocks with known pore-size distributions are shown in Figure 1.

EFFECTS OF GEOMETRIC DETAILS OF INCLUSIONS ON ELASTIC MODULI

As mentioned before, we aim to explore the effect of pore geometry on the elastic properties using synthetic models. Geometric parameters are pore shape, size, orientation, and spatial distribution. Among them, pore size, orientation, and spatial distribution are assumed to be random in our generated models, which is acceptable in most cases of the real samples. However, pore orientation could be discussed in anisotropic media. We, here, just explore the effects of pore shape (or PAR) and, thus, effects of other parameters are neglected by computing average properties of several generated samples.

We used the PAR as a known criterion for generating different pore shapes in a model. Accordingly, the PAR values of 1, 0.1, and 0.01 are considered as ball, discs, and penny shape or crack porosity. We developed an in-house 2D and 3D code to produce digital models with desired pore geometry (see Figure 1). Computation time is one of the main limiting factors for studying 3D models. The runtime for 3D models is much higher than 2D ones. Therefore, because a sensitivity analysis is an aim of this study, the 2D models are studied and the results are extended to three dimensions using an alternative DRP method introduced by Saxena and Mavko (2016). Then, the effect of pore structure is studied in three dimensions.

Three different models, such as, A, B, and C with PAR of 1, 0.2, and 0.05, respectively, were generated with porosities of 5%, 10%, 15%, and 20%. Table 1 summarizes characteristics of these models. For having porosities with the same size, but different PAR, large radii of pores were fixed and, subsequently, small radii of pores were accordingly computed using the PAR values. Figure 2 illustrates some of these synthetic models. Pores were considered to have a random size, distribution, and orientation, which is realistic and common in a real rock. To extend the generality of our proposed method, three models were generated in each case (i.e., for a specific PAR and porosity value), and an average value was considered as the result.

In each model, calcite elastic properties \((K = 68.3, \mu = 28.4 \text{ GPa})\) were allocated to the mineral phase. For the pore phase, air properties \((K = \mu = 0)\) were assigned to mimic the dry rock condition. To have a fluid-saturated condition, fluid properties can be used instead of air. Then, using the method proposed by Garboczi and Day (1995), effective elastic parameters of each model were numerically calculated by a finite-element linear elastic algorithm. Figure 3 illustrates the results of the bulk and shear moduli of synthetic rock samples by DRM (solid lines). As a validation, the elastic properties of these models were also computed using DEM equations (the dashed lines). Figure 3 shows that the results by these two methods are highly comparable. However, the power of DRM is to compute physical properties of models with a mixture of arbitrary pores and mineral shapes, which is not the case in this study.

According to these results, in each pore type (or PAR), elastic moduli generally decrease nonlinearly with increasing the porosity. For example, in a dry rock with 20% porosity, the bulk and shear moduli are reduced to 45% and 38% for the PAR of 1, 68% and 50% for the PAR of 0.2, and 93% and 82% for the PAR of 0.05, compared with a sample with zero porosity. This implies that the reduction rate of elastic moduli also increases with declining PAR.

Table 1. Characteristics of the generated models to study the PAR effect.

<table>
<thead>
<tr>
<th>Model</th>
<th>PAR</th>
<th>Pore shape</th>
<th>Major and minor pore radii (pixel)</th>
<th>Porosity value (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>Ball</td>
<td>20, 20</td>
<td>5, 10, 15, and 20</td>
</tr>
<tr>
<td>B</td>
<td>0.2</td>
<td>Disc</td>
<td>20, 4</td>
<td>5, 10, 15, and 20</td>
</tr>
<tr>
<td>C</td>
<td>0.05</td>
<td>Penny</td>
<td>20, 1</td>
<td>5, 10, 15, and 20</td>
</tr>
</tbody>
</table>

Figure 1. Some examples of (a) 2D and (b and c) 3D synthetic digital rock model with a specific pore size and geometry distribution.

Figure 2. Examples of the generated models (a) A, (b) B, and (c) C with PARs of 1, 0.2, and 0.05, respectively (see Table 1).
Moving from a ball to a disc shape, and also to the penny-shape pore type, makes the rock softer, which gives rise to a reduction in the wave velocity of the sample.

ESTIMATING ELASTIC MODULI OF ROCK FROM 2D IMAGES

Based on the results shown in the previous section, one can observe that the elastic parameters of rock are highly sensitive to PAR distribution. Therefore, the main idea in this study is to reproduce the distribution of PAR values from 2D images and, finally, compute the elastic parameters of the 3D medium using DEM equations. Here, the question is: How comparable are 2D and 3D PARs? In addition, How do we construct the distribution of PAR values from 2D images? In fact, instead of using one value for describing the PAR of a 3D pore, three PAR values obtained from 2D orthogonal sections of pores can be used, which in turn is more informative to explain an arbitrary pore shape. According to Figure 4, in the 3D case, the PAR is defined as $\frac{C}{A}$, which does not contain any information along the $y$-axis ($B$). On the other hand, by using three PAR values for 2D cases ($\frac{B}{A}, \frac{C}{A}, \frac{C}{B}$), the pore shape can be described more effectively.

In this study, three orthogonal 2D images are used in any case (see Figure 5a). However, one may discuss that in Figure 4, 2D PARs come from one pore, whereas, in reality they are not accessible for all pores in just three orthogonal 2D images. This may be a true statement, but due to the implemented statistical basis in this approach, other PAR values will likely be considered using simulated 2D images and frequency histogram of PAR. To add more variance to PAR values, the CCSIM is used to reproduce those probable pores that are not seen in these 2D images, but may exist in the real rock sample (Figure 5b). In each 2D image, individual pores can be distinguished, and the corresponding PARs are computed. Using the PAR values obtained from three 2D images and several simulated images, a distribution of PARs are reproduced (see Figure 5c). The DEM equations are then solved by applying PARs to estimate the 3D elastic parameters of a rock sample (Figure 5e). A simple graphical flowchart is shown in Figure 5.

CCSIM

The conditional reconstruction method used in this paper is based on the CCSIM algorithm (Tahmasebi and Sahimi, 2012, 2016; Tahmasebi et al., 2016b). In this algorithm, a 1D raster path starting from one corner of the simulation grid and ending in the other corner is implemented. First, a random pattern is selected from the initial digital image and inserted in the simulation grid. To keep the similarity of the current pattern with the next one, an overlap region is used for finding a similar candidate pattern. At the end, a cross-correlation function is applied. The more candidates that exist, the more variable the realization will be. Due to the complexity of the used images, in this study, we used five candidates.

PAR computation

For using the PAR equations, an ellipse is fitted to the pore space. PAR is defined as the ratio of the smallest to the largest radius of the ellipse. As illustrated in Figure 6, for a digital rock image with an arbitrarily shaped porosity, the porosity (or here we call it mass) and mineral phases are considered to be one and zero, respectively. The center of porosity (or mass of the image) ($i_c, j_c$) (Figure 7) can be obtained using the moments of the image along each axis divided by the total mass of the image (Corke, 2017):

$$i_c = \frac{M_{10}}{M_{00}}, \quad j_c = \frac{M_{01}}{M_{00}},$$

(3)

where $M_{10}$ and $M_{01}$ are the moment of the image around the $i$ and $j$ axes (Figure 7) and $M_{00}$ is the total mass of the image.

Figure 3. Crossplot of (a) bulk and (b) shear moduli of different models A (circle), B (square), and C (diamond) with different PARs in dry (solid line) and water-saturated (dashed line) conditions.

Figure 4. There are one and three PARs in the 3D and 2D cases, respectively. It is clear that three PARs obtained from orthogonal sections can characterize the pore shape more realistically.
Subsequently, the inertia matrix of the image can be further defined as (Corke, 2017)

$$IM = \begin{bmatrix} \tau_{02} & \tau_{11} \\ \tau_{11} & \tau_{20} \end{bmatrix},$$ (4)

where $\tau_{02}$ and $\tau_{20}$ are the central second-order moment (moments of inertia) and $\tau_{11}$ is the axial second-order moment (product of inertia). The equivalent ellipse (Figure 7) is the ellipse that has the same inertia matrix as the porosity. The eigenvalues and eigenvectors of IM are related to the radii of the ellipse and the orientation of its major and minor axes. The radii of the equivalent ellipse are (Figure 7)

$$A = 2 \sqrt{\frac{\gamma_1}{M_{00}}}, \quad B = 2 \sqrt{\frac{\gamma_2}{M_{00}}},$$ (5)

where $\gamma_1$ and $\gamma_2$ are the eigenvalues of IM. PAR of the equivalent ellipse (or porosity) is calculated using

$$PAR = \frac{B}{A}. \quad (6)$$

**Inserting the PAR distribution in DEM equations**

Each pore type is expressed by a value of PAR. Then, the PAR distribution represents the frequency of each pore type with a specific PAR. Therefore, by dividing the PAR range (i.e., [0, 1]) by “n” parts (bin number of histogram), the fractional porosity of each bin value (PAR₁,…,PARₙ) relative to total porosity ($\phi$) can be computed by the PAR frequencies ($f₁,…,fₙ$). Using these values, the model starts with phase 1 (or a host mineral) and the proportional porosity of $f₁ \times \phi$ is incrementally added with PAR of PAR₁, for the first part (or pore type). The term PAR₁ is used for calculating the geometric factors ($P’$ and $Q’$ in equations 1 and 2). Then, the effective elastic properties of the medium ($K’₁, \mu’₁$) are computed using DEM equations (equations 1 and 2). In the second step, these values are considered as background properties (or the host phase) and the proportional porosity of $f₂ \times \phi$ is incrementally added with PAR of PAR₂. This procedure is continued until all parts are added and the final effective elastic properties ($K’₃D, \mu’₃D$) are computed.

**DIGITAL ROCK SAMPLES**

Andrá et al. (2013a) introduce a set of benchmark digital rock samples such as Berea and Fontainebleau sandstone and Grosmont carbonate. In this study, we used the segmented samples of Berea sandstone and Grosmont carbonate to evaluate the proposed workflow.

**Berea sandstone**

Berea sandstone is mostly composed of quartz grains with a minor content of clay, K-feldspar, ankerite, and zircon. Petrography studies, as well as microprobe results, have demonstrated an isotropic solid matrix in this sample (Madonna et al., 2012). The connected laboratory porosity of the used sample is approximately 20% with permeability between 200 and 500 mD. Andrá et al. (2013b)

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**Figure 5. Flowchart of the proposed method.**
show that the average computed values of the bulk and shear moduli of a $700 \times 700 \times 1024$ digital sample are 19.7 and 21.5 GPa, respectively. In this study, a $400 \times 400 \times 400$ sample is extracted from the original digital sample ($1024^3$) (see Figure 8a) (Andrä et al., 2013b). The porosity of this sample is 20%, and its computed bulk and shear moduli are 19.1 and 20.1 GPa, respectively.

Grosmont carbonate

Grosmont carbonate is composed of dolomite and karst breccia. The laboratory results show that its porosity is approximately 21% with a permeability between 150 and 470 mD. Simulation results from Andrä et al. (2013b) indicate that, for a $1024^3$ digital sample, the porosity is 24.7%, and the computed bulk and shear moduli are 23.2 and 11.5 GPa, respectively. In this study, a sample with the size of $200 \times 200 \times 200$ voxels is extracted from the original segmented sample ($1024^3$) (see Figure 8b) (Andrä et al., 2013a). The porosity of this sample is 25.4%, and its computed bulk and shear moduli are 23.1 and 13.4 GPa, respectively.

RESULTS

According to the proposed method, four sets of 2D orthogonal images were arbitrarily extracted from each 3D digital rock samples. Each set contains three 2D images in the x-, y-, and z-directions, as shown in Figure 5a. Requirements of the CCSIM algorithm, such as template size, overlap size, the number of candidates, and other parameters (Tahmasebi, 2017), were adjusted in each set to generate the most reliable reconstructed 2D images. For each 2D image, 10 realizations are reconstructed, which results in $33 (3 + 3 \times 10)$ images for each set.

To find the PAR values in each 2D image, after implementing a median filter, individual pores are detected and separated using a watershed algorithm (Beucher and Meyer, 1992). Then, the eigenvalues of the inertia matrix of each pore are calculated, and the corresponding radii of the equivalent ellipses are obtained, which are then used for calculating the PAR. After calculating the PARs of all 2D images, the corresponding histogram frequency of PARs is computed. Then, the PAR values and their corresponding frequencies are used in the DEM equations and, subsequently, the elastic parameters of the 3D medium of the rock are computed. The average porosity of the three actual images is used as the porosity of the final 3D model.

Figure 9 illustrates the results for Berea sandstone and Grosmont carbonate. In each sample, the actual trend of the rock was produced using subsamples extracted from the original sample (Karimpouli and Tahmasebi, 2017). The size of subsamples was half of the original sample (i.e., $200 \times 100^3$ for Berea and Grosmont, respectively). Thus, eight subsamples are extracted in each case. We applied the finite-element method of Garboczi and Day (1995) to compute the static elastic moduli of the 3D sample and subsamples. This method solves the basic Hooke’s law equations of linear elasticity.
Due to multiple simulation steps, in each set of 2D images, different values are obtained for the elastic moduli in each run of the algorithm. As mentioned, the reconstructed images contain various pore structures, which induces more variability in the ultimate results. This variation changes in each set depends on the pattern of the pore structure of the actual 2D images. According to Figure 9, the bulk modulus is reasonably well-estimated in both cases. However, the shear modulus is slightly higher than the real trend but is still acceptable. The main reason for obtaining such promising results is that the method is based on the distribution of PARs, the key parameter for the evaluation of elastic moduli. A comparison with results computed by Andrä et al. (2013b) also shows how reliable this method is.

**DISCUSSIONS**

There are some critical points during the computation process, which are discussed here. The first point is tiny linear pores (Figure 10). Because the smallest radius of the equivalent ellipse is computed as zero for these pores, their PAR is also zero. These are, in fact, penny-shaped pores or microcracks with PARs of 0.01, which strongly reduce the elastic properties of the rock. Our investigations showed that an empirical value in the range of 0.01–0.02 is reasonable to replace the PARs of these pores. In our cases, values of 0.018 and 0.01 were allocated for Berea sandstone and Grosmont carbonate, respectively (Figure 9).

The other point is the bin number of the histogram. To compute the histogram plot, the PAR range is divided by $n$ number of bins and the frequencies are counted in each bin. The center value of each bin is assumed as PAR, which means the larger the bin, the more averaged is the value for the PAR is used. It is inevitable to increase the bin number to have a more accurate computation. For the investigation of this bin number effect, we selected those results, which are very close to original values (obtained from original trend), with a trial and error method in both samples. Then, the bin number was changed along a wide range from 5 to 1000 linearly along PAR values (i.e., [0, 1]). The results displayed in Figure 11 show that with increasing bin number, the results converge to the original value. It should be noted that not all values of PAR need to be divided linearly because their effect is not similar. According to Figure 3, it is implied that the effect of pores from ball to disc shape (PAR of 1 to 0.1) is comparable with pores from disc to penny-shaped pores (PAR of 0.1 to 0.01). Therefore, we divided each of [0, 0.1] and [0.1, 1] into 100 bins and called it nonlinear division. Our results showed that 100 nonlinear divisions can produce the same result as 1000 linearly division but in a shorter time.

The final point is the effect of importing large to small and small to large PAR pores to DEM equations. For multiple inclusion shapes, the effective moduli depend not only on the final volume fractions of the constituents but also on the order in which the incremental additions are made (Mavko et al., 2009). To explore this effect, three 2D orthogonal images from each Berea sandstone and Geomsont carbonate were selected, and the corresponding elastic properties were computed using the proposed method (Figure 5) for 10 iterations. In each iteration, PAR values were arranged to import from (1) small to large and (2) large to a small value during computation. Figure 11 shows these results. According to this figure, when PARs are introduced to DEM equations from large to small values, an approximately 2% and 3% softer sample is obtained relative to the condition when small to large PARs are imported for Berea sandstone and Grosmont carbonate, respectively. Because the effect direction is not significant, one could either use the average of both directions or just use a single direction and neglect the other direction.

The presented results in Figure 4 indicated that among all geometric parameters, the pore shape (or PAR) is the most important parameter that highly affects the elastic moduli. Thus, the core of this method is, indeed, the DEM equations that are solved with PARs obtained from the real and reconstructed media of rocks. Although these PARs are 2D, orthogonal cross sections along three main axes compensate for the lack of PAR information in three dimensions. They could even address the shape of a 3D pore better than one 3D PAR value does. The implemented method in this paper is fast and computationally effective, and it requires a small

Figure 9. Bulk and shear modulus of (a) Berea sandstone and (b) Grosmont carbonate obtained from the proposed alternative DRP method. Results by Andrä et al. (2013b) are from a cropped sample with 400$^3$ pixels from original 1024$^3$ sample. The real trend in each plot is obtained using 3D subsamples extracted from a real image of the rock.

Figure 10. Penny-shaped pores or microcracks. The white and dark cells are the mineral and pore, respectively. The PAR of these kinds of pores should be an empirical value in the range of 0.01–0.02.
Figure 11. Effect of introducing small to large and large to small PAR to DEM equations for (a) Berea sandstone and (b) Grosmont carbonate. Original values were obtained from original trends. A slightly softer sample is obtained when large to small PARs are imported relative to when small to large PARs are introduced.

amount of memory. Two reasons explain the computational efficiency of our proposed method: (1) a fast 2D reconstruction and (2) an efficient analytical computation. Going from 2D samples to 3D samples is not only computationally expensive, but it also requires extensive knowledge and assumptions. Therefore, in this paper, the 2D samples are used directly. Furthermore, performing the computations on a 2D sample can take several hours. In contrast, the proposed method only uses the 2D samples and conducts the computations on the same samples, which only takes around a few seconds (approximately 30 and 20 s for the Berea and Grosmont samples, respectively). Input data are 2D thin-section images, meaning that the proposed method can be used along with or instead of the standard DRP, especially when high-quality 3D images are not available. A notable application, which is expected for this paper, results in pore scale. Because these images are in two dimensions, three orthogonal images were proposed to be used and, therefore, three PARs were used instead of one 3D PAR. Our results showed that three PARs could effectively explain the effect of the pore shape. To reproduce a representative PAR distribution, the CCSIM reconstruction method was used and several realizations were reconstructed for each orthogonal 2D image. This increases the variance of the PAR values and reconstructs those pore shapes that cannot be observed in the actual 2D images but may exist in the real sample.

The obtained results in this paper revealed that the proposed method could efficiently estimate the elastic moduli of Berea sandstone and Grosmont carbonate. The values of the bulk modulus of rocks, predicted by this method, are close to the real trend of the rock samples obtained from 3D images, whereas the shear modulus was slightly overestimated.

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