RESEARCH ON MODELS & MODELING
AND IMPLICATIONS FOR COMMON CORE STATE CURRICULUM STANDARDS

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There are a number of current resources that provide information on models and model perspectives (MMP). For a modeling-related review of the literature on mathematics learning and problem solving, see Lehrer & Lesh (2012). For links to monographs and books associated with the International Community for Teaching Modeling & Applications, see Lesh, Haines, Galbraith & Hurford (2010). For an introduction to how MMP has become a central theme in engineering education, see Deifus-dux, Bowman, & Zawojewski (2009); and, for an introduction to the usefulness of MMP in teacher education see Zawojewski, Chamberlin, Hjalmarsen & Lewis (2008). Or, in the National Council of Teachers of Mathematics’ most recent Handbook of Research in Mathematics Education, the chapter on problem solving (Lesh & Zawojewski, 2007) describes how, compared with traditional approaches to problem solving and metacognition, MMP provides much more powerful and effective ways to help learners become: (a) better problem solvers, and (b) better able to use mathematics in real life situations beyond school. Also, another recent book, titled Foundations for the Future in Mathematics Education (Lesh, Hamilton & Kaput, 2007), describes a research agenda that is aimed at showing why MMP should play a central role in any curriculum standards documents that claim to be future-oriented. Similarly, another earlier book, titled Authentic Assessment in Mathematics Education (Lesh & Lamon, 1992) describes a variety of reasons why model-eliciting activities (MEAs), of the type described by Chamberlin & Coxbill in this book, make it possible to operationally define (i.e., measure) a broad range of deeper and higher-order achievements that are not possible to document and assess using standardized tests which are based on psychometric theory. And finally, MMP is strongly represented in three recent books on research design in STEM education (Kelly & Lesh, 2001; Kelly, Lesh, & Baec, 2007; Lesh, 2007)

After recognizing that research on models & modeling has become such an active and productive area within mathematics education research, one might imagine that advocates of models & modeling should be very pleased to see that modeling is listed as one of the highlighted practices in the USA’s newest Common Core State Curriculum Standards (CCSCS). But, notice that the CCSCS’s conception of modeling reduces it to “applying mathematics that (students have already been taught) to solve problems arising in everyday life.” So, this impoverished conception of modeling completely ignores the perspectives that: (a) models for mathematizing experiences should be strong candidates for being, in themselves, among the most important goals of the K-16 mathematics curriculum, (b) the development of these models represents an important part of what it means to develop “conceptual” understandings of even traditionally emphasized curriculum goals, and (c) proficiency at model development is directly related to measurable conceptions of many of the higher-order competencies that the CCSC Standards fail to “operationally define” in ways that are measurable. Consequently, in this brief commentary, I’ll briefly describe a variety of reasons why MMP should be expected to make large differences in mathematics education policy documents such as the newest Common Core State Curriculum Standards (CCSCS).
The CCSC Standards claim to focus on future-oriented goals. In a technology-based age of information, sophisticated mathematics tools are now being used by many more people than in the past; and, the results can be seen simply by looking in newspapers such as USA Today – in sections ranging from sports to business. Yet, because the accepted way of producing standards documents is to use political consensus-building processes in which the main people whose voices are heard are mathematics teacher educators and university-based mathematicians, and because people in fields that are increasingly heavy users of mathematics are ignored (such as engineers, business managers, and medical decision-makers), it is not surprising that almost nothing is said in the CCSC Standards which could not have been said 40 years ago. In particular, in the CCSC Standards, the apparent main purpose of each mathematics course appears to be to prepare students for the next mathematics course. No research was done to investigate: (a) the nature of new kinds of problem solving situations that students are likely to encounter beyond school, (b) new levels and types of understandings that are needed in the preceding situations, (c) how the preceding understandings develop, and (d) how development can be facilitated, documented, and assessed for the preceding kinds of understandings (Lesh, Hamilton, & Kaput, 2007). For example:

- In useful topic areas like statistics, it has been known for many years that Bayesian procedures (which are based on computational models) are much more powerful than traditionally-used analytic procedures (which are based on normal distributions and on other Calculus-based models which seldom actually fit the situations they are used to describe). Yet, in mathematics departments, computational model development continues to be largely ignored (Lesh, 2004).

- For people who are heavy users of mathematics and technology, one of the most common kinds of mathematical problems that they face are those in which operational definitions are developed to measure quantities that cannot be measured directly. Yet, there are almost no such problems in mathematical textbooks or tests (Lesh, 2007).

- Almost the entire K-16 mathematics curriculum is restricted to the consideration of “problems” that can be described adequately using only a single function going in one direction – with no conflicting constraints (such as low costs and high quality), no feedback loops, no second-order effects, and (until after Calculus has been taught) no problems that involve like maximization, minimization, or stabilization (Lesh, 2007).

- Almost no problems occur in the K-16 mathematics curriculum that involve several layers of interacting trends of the type which occur commonly in hyphenated-sciences studying the complex behaviors of different kinds of biological, economic, or social eco-systems (Lesh, 2007).

Even after recognizing the preceding realities, one still might ask: “Aren’t the same basic ideas and skills needed for the preceding kinds of situations?” The answer is: “No! Even when the same basic ideas and skills are important, the levels and types of understandings are often quite different. For example, in the primary grades (K-2), the CCSC Standards recommend focusing almost exclusively on problem solving situations that involve only counts of discrete sets of objects. Yet, if we look outside of school classrooms, and if we ask “What kind of situations should K-2 children be able to describe using numbers?” then it’s clear that young children need to be able to use numbers to describe locations (addresses, positions, or coordinates), composite units (units of units), actions (exchanges, transformations), continuous measurements (lengths, distances, areas), quantities that have both a magnitude and a direction
(vectors), signed numbers (negative numbers), exchange rates or ratios, and a wide range of “ness” quantities (orange-ness, rough-ness, sweet-ness), “per” quantities (raisons per cookie), “ity” quantities (dens-ity, probabil-ity) or accumulating quantities. Furthermore, many of these situations involve information that is given in concrete, graphic, or tabular forms – so that representational fluency is highlighted (Lehrer & Lesh, 2012). And, many also involve maximization, minimization, stabilization, compensation, equalization, or other issues that traditionally have been thought of as needing to be postponed until after students have been taught Calculus.

Are the preceding kinds of problems accessible to average ability primary school children? Yes!!! However, our results also suggest that such achievements are unlikely to occur unless most of the design “specs” for MEAs are satisfied (Lesh, Kelly, Hole, Hoover, Post, 2000). Among other things, these design “specs” state that: (a) children must clearly recognize the need for the kind of thinking that is desired, (b) their sense-making must be based on extensions of their own personal knowledge gained through “real life” experiences, and (c) their thinking must be expressed in the form of artifacts or tools whose usefulness (power, sharability, reusability) can be tested by students themselves (Kelly, Lesh & Baec, 2009). For example, Figures 1 and 2 show different model-eliciting activities (MEAs) that are simulations of “real life” situations – but that are based on children’s stories (because children’s realities are not necessarily the same as for adults). … The primary criteria that we used to assess the “realness” of tasks were: (a) Do the children try to make sense of the problem using their own “real life” experiences – instead of simply trying to do what they believe some authority (e.g., their teacher) considers to be correct (even if it doesn’t make sense to the children)? (b) When the children are aware of several different ways of thinking about a given problem, are they themselves able to assess the strengths and weaknesses of these alternatives – without needing to ask their teacher or some other authority? When the preceding two criteria are satisfied, results of our work have shown consistently that most average ability children are able to go from “first-draft thinking” to “Nth-draft thinking” without heavy guidance from an outside authority. Furthermore, even though their first-draft responses might have been unimpressive, their Nth-draft responses are often very impressive indeed.

Figures 1 and 2 show a collection of problems that are all based on a story about Beauregard Frog in Sugar Swamp – where “proper hops” always go from one lily pad to the next in vertical or horizontal directions (not diagonals). A third MEA is based on a story about a horse named Isabelle who loves to eat apples in the shade of apple trees. The children’s task is to write a letter to Tom, Isabelle’s owner, describing how, for a given orchard, the largest number of trees can be enclosed inside a fence of a given length - where the fence is a string of soda straws on a loop of string. A fourth MEA is based on a story about Fussy Rugbugs (whose homes are represented as colored post-it notes). The children’s task is to describe how to locate the most rugbug homes within a given loop of string – when the rugbugs insist that their rugs must be put together so they “just touch” but “never overlap”.

The CCSC Standards claim to focus on “conceptual” understandings. Yet, the only goals that are “operationally defined” in ways that are measurable are declarative statements (facts) and condition-action rules (skills). … Make sense of problems. Reason abstractly. Construct viable arguments. Model with mathematics. Use appropriate tools strategically. Attend to precision. Look for structure. Look for regularity. … What does it mean to “understand” any of these practices? How can these understandings be documented and assessed? After characterizing such goals using lofty-sounding-but-exceedingly-vague-language, the CCSC Standards reduce all such goals to lists of facts and skills. But, in MMP research and development, the mastery of nearly all such goals is considered to be closely associated with the development of a toolkit filled with powerful, sharable, and reusable models associated with the most important
(a) Where should Beauregard Frog choose for his home lily pad – so that the sum of the distances to his three friends pads is a small as possible?

(b) When children record the total distances from each lily pad in Sugar Swamp, Patterns emerged about distances from one to several other home pads.

In follow-up teacher-led discussions, students gave “thumb signals” (up, down, left, right) to help teachers find a best place to locate Beauregard’s pad.

(a) Put a mark (X) to show the location of a possible home pad for Beauregard. Then, put another marker 5 hops away from Beauregard. The child’s task to find all of the other pads that are exactly 5 hops from Beauregard. (note: This same basic task can be repeated for other locations of X and other numbers of hops.

(b) Figure B shows the solution to the task described in figure 4.

(c) Put markers at two points A and B, and child’s task is to find more such points that are the same distance to both A and B. (Note: Some locations for A and B don’t have any points that are the same distance from both of them.)
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concepts in any given course or grade level. In fact, in MMP research, MEAs were designed explicitly to be thought-revealing activities for such achievements. So, at the same time that MEAs promote conceptual change, they also document development in ways that make assessments straightforward. It should not be surprising that models and modeling are closely related to deeper “conceptual” understandings. This is because concepts are tools for conceptualizing (mathematizing, dimensionalizing, coordinatizing, etc.); and, in mathematics, conceptualizing things mathematically means describing or explaining them quantitatively, geometrically, algebraically, and so on. And, in mathematics and science, mathematical descriptions or designs are referred to as models. So, both models & modeling are closely tied to what it means to develop many of the most important types of “conceptual understandings” of “big ideas” in mathematics. Yet, models are not facts, and they are not skills. But, they among the most important types of knowledge that students need to develop in order to be able to use mathematics in real life situations beyond school.

What is a model according to MMP? A model is a system for describing another system for some specific purpose (Lesh & Doerr, 2003). What is the most important distinguishing characteristic that makes mathematical models different than other types of models – such as those used in chemistry, biology, history, or music? Mathematics is the study of structure. So: Mathematical models focus on the structural (or systemic) properties of the systems-as-a-whole that they are used to describe. Sometimes, these properties of the system-as-a-whole are referred to as emergent properties of the system. But, in any case, in mathematics, such properties of systems-as-a-whole include not only properties such as symmetry, invariance, and centrality; they also include properties which often appear as “undefined terms” within the axiom systems that define different kinds of mathematical structures. Examples of such “undefined terms” include “points” and “lines” in the axioms that define Euclidean Geometry; and, they include “identity elements” or “inverse elements” within the axiom systems that define metric spaces or counting numbers (e.g., see Peano’s Postulates).

What does it mean to be an “undefined term” in a mathematical system? It means that all of the mathematical meanings of these terms come from the systems-as-a-whole in which they reside. So, the psychological counterpart of this claim can be seen in Piaget-inspired research showing what children’s thinking is like before their interpretations of these “undefined terms” are based on relevant systems of operations, relations, and patterns. And, this means that model development is closely related to the development of “conceptual” understandings of many of the most important “big ideas” in the mathematics curriculum.

The CCSC Standards claim to emphasize research-based learning progression. Yet, because the only goals that are operationally defined in ways that are measurable are lists of facts and skills, the CCSCS view of learning progressions continues to emphasize reductionist perspectives in which learning and problem solving both occur by: (a) first learning two or more ideas separately, then combining them to form higher-order understandings, or (b) first learning an idea and a problem solving process, then combine them to solve a problems.

Similar to the wave and particle models for describing the behaviors of light in physics, MMP supports two significantly different descriptions of learning and problem solving in school mathematics. On the one hand, Piaget-inspired “constructivists” have produced detailed and extensive descriptions of the conceptual systems that children must develop in order to “think operationally” (or “systemically) about many of the most important “big ideas” in the K-16 mathematics curriculum. They have described many of the most important characteristics of pre-operational thinking; and, they have described intermediate stages between pre-operational and operational thinking. However, ever since Piaget’s original studies of children’s
mathematical thinking, these descriptions of cognitive development have resulted in learning progressions that lead to exceedingly pessimistic views about possibilities for accelerating development. Furthermore, whereas the strength of these learning progressions is that they clarify the importance of structure in mathematics learning, both Vygotsky’s notion of zones of proximal development and Piaget’s notion of decalage imply that: (a) tasks characterized by the same structure are often significantly different in difficulty, and (b) the difficulty of a single task can be changed significantly by changing only seemingly superficial (non structural) characteristics of the task. So, a strength of Piaget-inspired “constructivist” conceptions of cognitive development results from it’s emphasis on structure; and, a weakness is that it gives inadequate attention to the fact that knowledge is organized around experience at least as much as it is organized around abstractions (i.e., cognitive structures). In modern research in the learning sciences, this latter perspective on cognition often is referred to as situated cognition; and, in MMP research focusing on situated forms of conceptual change, significant conceptual changes often occur quite rapidly (Harel & Lesh, 2003; Lesh & Doerr, 2011)

In mathematics, the models that students develop tend to be both situated and structural. That is, structure is an important characteristic of the models that students construct. Yet, conceptual adaptations often occur quite rapidly in situated model development activities (i.e., MEAs) where the conceptual tools that students develop usually are strongly shaped by the context of the tasks — and they often integrate concepts and procedures associated with a variety of textbook topic areas. On the other hand, model development is not just context-specific learning. This is because the models that students develop are expressed as tools which are designed to be powerful, sharable, and reusable. So, the underlying models that students develop tend to be both situated and structural, and both specific and general. (Lesh & Doerr, 2011; Lesh & Yoon, 2004).

References


