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PARTICIPANT RESEARCH ESSAY FOR QRaMM RESEARCH TEAM

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Introduction

In my research, I examine secondary students' reasoning about rate of change—an intensive quantity specifying a multiplicative relationship between quantities. Given the complexity of rate of change, reasoning about rate of change is a challenging, multifaceted activity. Drawing on intuitive ideas of rate of change, constructing and interpreting specific amounts of rate of change, making informal qualitative comparisons between rates of change, and constructing and interpreting representations of rate of change are four aspects suggested by extant literature.

The following research question has guided my work: How do secondary students reason about changing quantities when interacting with mathematical tasks involving multiple representations of constant and varying rates of change?

In the statement that follows, I elaborate on my research examining secondary students' quantitative reasoning about rate of change and articulate my agenda for moving forward with my research.

Examining Students' Reasoning about Rate of Change

In my research, I examine secondary students' reasoning about constant and varying rate of change across different tasks that involve multiple types of dynamic and static representations. Researchers considering constant rate of change have investigated students' determination and interpretation of specific amounts of change (e.g., Lobato & Siebert, 2002). Researchers considering varying rate of change have investigated how students used informal qualitative comparisons to attend to variation in the intensity and/or direction of change (e.g., Stroup, 2002). In my work, I bring together both strands of research, examining secondary students' reasoning about constant and varying rate of change by attending to their determination and interpretation of specific amounts of change as well as to their informal qualitative comparisons of variation in intensity and/or direction of change. I consider mathematical reasoning to be a form of mathematical thinking situated within a context, involving purposeful activity on the part of the one reasoning and interaction between the one reasoning and the (mathematical) environment. When considering reasoning about rate of change, I take a broad perspective, including reasoning about changing quantities involved in a rate of change without stipulating that the reasoning includes either making multiplicative comparisons between changing quantities or focusing on the multiplicative comparison as a single quantity.

For my dissertation study (Johnson, 2010) I examined how six high school students who had not taken calculus reasoned about changing quantities when interacting with mathematical tasks involving multiple representations of constant and varying rates of change. I conducted a series of five individual, task-based interviews with each student. Using the method of constant comparison, I traced students' reasoning from their explanations, written work and gestures to develop characterizations of their reasoning. Next, I analyzed how students combined covariational (Carlson et al., 2002), transformational (Simon, 1996), and proportional (Lamon, 2007) reasoning when reasoning about rate of change. During the series of task-based interviews, students encountered the same set of tasks, but each of the students reasoned in different ways, utilizing different combinations of covariational, transformational, and proportional reasoning. Interestingly, students' ways of reasoning and combinations of different types of reasoning remained consistent across the tasks and interviews, even though the tasks involved different contexts and different types of representations of mathematical quantities and relationships between quantities.

Results from my dissertation study include characterizations of students' reasoning about constant and varying rates of change. The characterizations provide examples of how students reason quantitatively both when determining and interpreting specific amounts of change as well as when making informal qualitative comparisons of variation in intensity and/or direction of change. In addition, the characterizations give evidence of how informal ways of reasoning about rate of change can be conducive to more formal ways of reasoning about rate of change. For example, one characterization serves as a model of reasoning supporting the activity of attending to variation in the intensity of change occurring in the same direction. Another characterization serves as a model of reasoning supporting the activity of constructing an intensive quantity to measure the intensity of a change.

The characterizations of students' reasoning informed the development of a tool for representing different combinations of covariational, transformational, and proportional reasoning involved in reasoning about rate of change. Prior to my dissertation study, transformational reasoning had been hypothetically related to covariational reasoning (Carlson et al., 2002) but combining the different forms of reasoning had not been empirically investigated. In addition, students' combination of different, yet compatible forms of reasoning had not been studied. Students' combination of different forms of reasoning when reasoning about rate of change is notable because it gives insight into the ways in which students make sense of changing quantities involved in rates of change.

Fostering the Development of Students' Reasoning about Rate of Change

I consider my dissertation study to be the beginning of a program of research that I plan to develop over the next five to ten years. I articulate three interrelated strands that I will develop as I move forward with my research.

The first strand involves students' developing understanding of rate of change. The characterizations of students' reasoning and students' combinations of different forms of reasoning resulting from my dissertation study will inform my examination of students' development of ways of reasoning that support a deep understanding of rate of change. I will utilize design experiment methodology to conduct a teaching experiment examining how high school students develop ways of reasoning that support a deep understanding of rate of change. I will design sequences of instructional tasks to be used with small groups of high school students. I anticipate that results from this research will include characterizations of students' ways of reasoning that support a deep understanding of rate of change.

The second strand involves prospective teachers' attention to students' reasoning about rate of change. Students reason differently and those different ways of reasoning have different affordances in terms of supporting a deep understanding of rate of change. Given differences in students' reasoning, it is important for prospective teachers to develop awareness of students' different ways of reasoning and understanding of what those different ways of reasoning could afford. Using tasks and video from my dissertation study I will develop instructional materials for prospective teachers and investigate how those prospective teachers develop awareness of and distinguish between different ways of student reasoning about rate of change. I will also examine interaction between prospective teachers' own reasoning about rate of change and their attention to students' reasoning about rate of change.

The third strand involves the area of teachers' fostering of students' development of ways of reasoning that support a deep understanding of rate of change. To further the third strand, I will design and implement professional development experiences for teachers centering on instruction that fosters students' reasoning about rate of change. The characterizations of students' reasoning, resulting from the first strand, and the ways in which prospective teachers attended to and understood affordances of students' reasoning, resulting from the second strand, will inform the design and implementation of the professional development. After the teachers participate in the professional development, I will then examine how the teachers' classroom instruction fosters students' development of ways of reasoning that support a deep understanding of rate of change.