Context Rich Differential Equations in the Engineering Core and SCALE-UP without Studios

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Abstract

Student-Centered Active Learning Environments for Undergraduate Programs (SCALE-UP) are instructional models utilizing studio environments to support active and collaborative learning. While reasons for engaging student cohorts in this way are well documented, impediments may exist that keep an instructor from implementing this educational modality, e.g., classroom space, emphasis on lecturing, lack of appropriate activities. That said, if the average student learns best by defining meaning through experiential learning, then one possible way of mitigating SCALE-UP overhead is to leverage student learning groups outside of the classroom environment. To support the adoption of behaviors typical to students participating in a functioning studio environment, some instruction must be dedicated to educating students on those soft-skills supporting group function. In Honors Differential Equations at the Colorado School of Mines, students were asked to participate in group work, outside of scheduled classroom time, on context-rich problems emphasizing discussion, technical writing, and exploration of their own attitudes and values. In fact, contextualization of the mathematical content provides an excellent scaffold for these skill-building exercises. To support group function, students were asked to complete tasks based on curated instructional content focused on problem-solving strategies, meeting structure and design, and decision-making. Additionally, to motivate individual reflection, students completed free-write exercises after reviewing material discussing metacognition, learning preferences and their relationship with learning, aspects of neuroscience relating to cognitive function, and mindsets. In this paper, we review the structure of Honors Differential Equations at Mines, some initial analyses of student work, and discuss whether the practice is useful and/or well-tolerated. Assuming both are true, the resources and workflows defined by this work may provide a good first-step for educators interested in SCALE-UP but lacking the resources necessary for its implementation.

1 Introduction - Motivation and Background

The authors of ”How Learning Works,” define learning as a *process* leading to a *change*, bestowed by *experience*, which increases the potential for improved performance and future

learning. The department of Applied Mathematics and Statistics at the Colorado School of Mines (Mines) is exploring a variety of pedagogical strategies to (i) encourage active learning in core mathematics courses and (ii) make the content of these courses more engaging and relevant to our students. Honors Differential Equations at Mines is delivered to student groups ranging from 20 to 50 students each semester and operates independently of the large-lecture cohort. Students enrolling in the Honors variant typically do so after reviewing literature about course structure and expectations. Those who are intrinsically motivated to study mathematics and/or are seeking degrees in mathematically intensive fields are encouraged to take the course. The expectation is that of an enriched curriculum and assessments based on learning experiences occurring in individual/group settings with grades that heavily weight organized reflection. Though the Honors Differential Equations content largely mirrors that of a standard sophomore-level introduction to differential equations, it delivers this content in a context-rich collaborative environment that emphasizes metacognitive processes and higher-order learning outcomes establishing significantly different behaviors than those supported by worksheet/exam assessments. Specifically, in Honors Differential Equations students are assigned several projects requiring the creation of technical reports analyzing mathematical models. Here student groups typically engaged with the instructor in an inquiry, apprenticeship model similar to studio-based learning. Furthermore, since nearly all content can be motivated through mathematical models of dynamical systems which are also amenable to significant symbolic study, it is possible to simply extend learning outcomes of the standard course, which typically focus on procedural solution techniques. Lastly, through mathematical modeling, it is possible to incorporate the formal/informal communication and mathematical literacy necessary for modeling making possible a variety of experiences designed to engage students at various levels of cognition. As an example, asking students to analyze the equation $v' = 1 - v - \epsilon v^2$ where $\epsilon > 0$ in the contextualization of drag forces, allows students to respond with an *explanation* of the applicable solution techniques or an *evaluation* of the whether the quadratic term is relevant. In this paper, we review the curriculum of Honors Differential Equations and Mines and their methodologies and reflect on their current state.

The SCALE-UP model is an active-learning environment widely propagated in the Physics Undergraduate Research community by Beichner et al.[1] from North Carolina State University. It is rooted in active learning efforts in the Physics community such as the Comprehensive Unified Physics Learning Environment (CUPLE) studio model at the University of Maryland [2,3] and Technology-Enabled Active Learning (TEAL) model at MIT.[4] SCALE-UP has been implemented with significant positive impacts on the Mines campus in the core physics curriculum under the moniker of "Studio Physics" since 2001 as described by Furtak and Ohno[5] and Kohl and Kuo.[6] Our other work at this conference describes the SCALE-UP of multivariate calculus at Mines.[7] In this paper, we discuss how one can apply the philosophic principles of SCALE-UP to a lecture-based introduction to ordinary differential equations even without access to a shared studio space. Specifically, we review the enriched curricular aspects of differential equations that encourage active learning through student reflection and collaborative group work in the setting of a context-rich curriculum. Our goal is to support students in their identification of prior knowledge on which the course content is built and provide them opportunities to grow as self-aware learners. Blending these pedagogical strategies with a traditional SCALE-UP space provides even more opportunities for enrichment since it allows for both the use of technology

and a single shared meeting for groups and instructional staff. While this is not our setting, we note that our strategies for nurturing metacognition through reflection and differentiated learning with feedback fits well with the existing SCALE-UP multivariate calculus at Mines.

In the following, we discuss the methodology of Honors Differential Equations at Mines with a review of previous iterations leading to the current course design, and notable course design elements are presented. Briefly, our current design uses mathematical modeling to motivate the study of differential equations and challenges students to engage the material via reflective inquiry. We conclude with a review of student comments indicating support for these new curriculum elements.

2 Methodology - Supporting Knowledge Construction Through Mathematical Literacy and Promoting Self-Awareness

The guidelines of Universal Design for Learning (UDL) compel educators to focus on the *what*, *why*, and *how* of learning.[8] Since differential equations appear naturally as mathematical models, often untidy ones, the curricular material can be motivated by appealing to the STEM interests of the students. In addition, models contextualizing the mathematics allows for multiple means of engagement, representation, and expression. For example, in the previous drag problem, students can communicate their learning with a coherent blend of mathematical derivation, written commentary/narrative, and/or modern web-enabled media. In Honors Differential Equations students are provided options for demonstrating comprehension, executive function, and self-regulation. Since students choose how they build content knowledge, we have seen great growth in self-regulation due to the natural challenges of modeling. In particular, there are multiple project-length assignments which creates opportunities for students to grow their project planning and time-management skills while they practice self-regulation to perform under the cycles of stress associated with projects.

The Guidelines for Assessment and Instruction in Mathematical Modeling Education, provide the following working definition: “mathematical modeling is a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena.” Context-rich problems are often presented as stories taking place in an everyday context which mathematical modeling naturally accomplishes. Here the student is put into a real-world scenario where they play the role of a modeler, working individually or as a member of a team, tasked to distill a large ill-defined problem into a tractable question which is then the basis for resource identification and scientific communication.[9] The context-rich problems of Honors Differential Equations require both existing and constructed knowledge to solve. Students who start their analyses promptly tend to reach out to peers and instructional staff requesting resources/support and, in doing so, demonstrate the flexibility characterizing expert-level behavior toward solving the complex problems.[10] If active learning requires students to simultaneously think about the work done and the purpose behind it, then curriculum emphasizing mathematical modeling provides a fine scaffolding for building this enriched environment.[11]

While undergraduate students tend to be comfortable with the what and why of learning, they tend to be less comfortable with “the how.” This is especially true when confronted with

ill-defined problems or those requiring more than communication of an algorithmic procedure. To support students having difficulty with the aspects of self-regulation, the modeling content of Honors Differential Equations is supplemented with reflective free-write exercises on metacognition, the controversies of learning styles, growth versus fixed mindsets, the scientific basis for learning, and project-planning. In keeping with the Engineering Learning Methodology advocated by the Trefny Innovative Instruction Center at Mines[12] detailed in Figure 1, we now describe the history of Honors Differential Equations at Mines and the novel pedagogical practices associated with its current implementation which is supported by the work of our colleagues in the Physics department who actively participated in the acquisition of external funding from the Office of Naval Research in support of our efforts, as well as a handful of additional SCALE-UP efforts on campus.

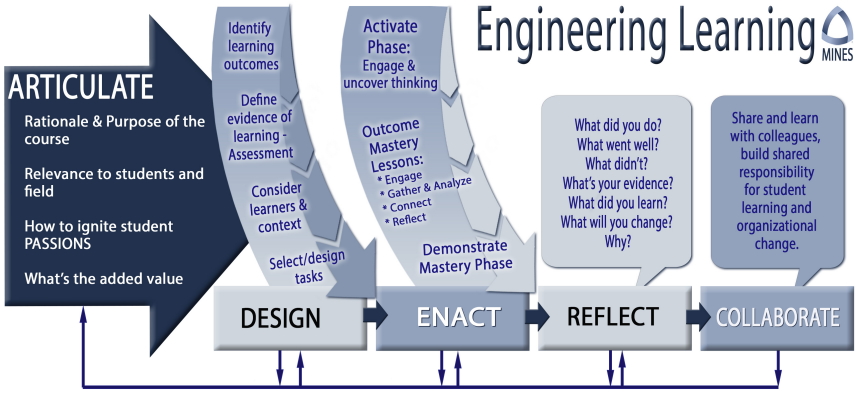

Figure 1: Diagram of the Engineering Learning Process at Mines

2.1 Articulate Phase - Rationale, Relevance, and Added Value

In his 1997 address to the MAA, Gian-Carlo Rota predicted that "The sophomore course in differential equations will never be reformed. It will die of natural death, and it will be replaced by several shorter courses that will deal with realistic aspects of differential equations." With the advanced but wonderfully accessible text of Strogatz,[13] which brought nonlinear dynamics to the undergraduate curriculum, and the advent of computer programs capable of rendering computer visualizations of ordinary differential equations and their solutions, the demise of the introductory course may have been averted. In the case of differential equations, instructional and curricular reform seem intertwined. The realism championed by Rota can be found in modeling. With the utility of the internet, complicated models are readily found, and the very act of distilling their open and messy forms leads to authentic activities well-suited to collaborative and active learning environments.

Honors Differential Equations at Mines, over the past ten-years, has gone through three phases. In the first, students were introduced to the geometric principles laid out by Strogatz through the text of Blanchard Devaney and Hall.[14] While this text has serious drawbacks, we were able to gain insight into how these concepts could be delivered at the introductory level. Moreover, it was shown that advanced and highly nonlinear models, (e.g., studying bifurcations in the dynamics of

a bead constrained to move along a circular path in a highly viscous ambient background), could be made into course projects. In the second phase, weekly problem sets covering algorithmic tasks were discarded in favor of reflective journals built around prompts delivered through a course blog. These blog posts either recapitulated lecture content or extended the content to mathematical models or deeper symbolic analysis. While the transition to the third and current implementation is subtle, the prior deliveries left us with some clear lessons:

1. On average, students found the models motivating and, in the best cases, models generated ah-ha moments where the relevance of our mathematical language burst to life
2. Students required structured practice on non-contextualized example problems
3. While models certainly add value and ignite passion, the questions which they raise can become overwhelming

The current form of Honors Differential Equations seeks to retain the value added by the modeling practice and, at the same time, provide support for students finding their balance between mastery of algorithmic procedures and exploration of open-ended modeling problems.

2.2 Design - Course Goals and Learning Objectives

When considering course design, we asked ourselves what skills may be important to students during the remainder of their college career and what do we want them to remember twenty years from now? In our opinion, a growth mindset will serve them well, both in and out of the classroom. Additionally, the unreasonable relevance of our mathematical language in describing our natural world is something we hope they share with others even two-decades from now.

In Table 1, we define the learning outcomes of the course and compare them to the large-lecture implementation of differential equations at Mines. What we see here is that all outcomes in 225 are mirrored in 235 with the addition of content associated with nonlinear systems and an emphasis on modeling which pushes several outcomes into the highest levels of Bloom's Taxonomy associated with the cognitive domain, see Figure 2.[15] To meet our three and twenty-year goals we emphasize the challenge of applying mathematics contextualized in the natural world and make clear to students the opportunity for personal growth it represents. In terms of course objectives, this can occur if all students in the course are asked to evaluate the fitness of models and to recommend modifications.

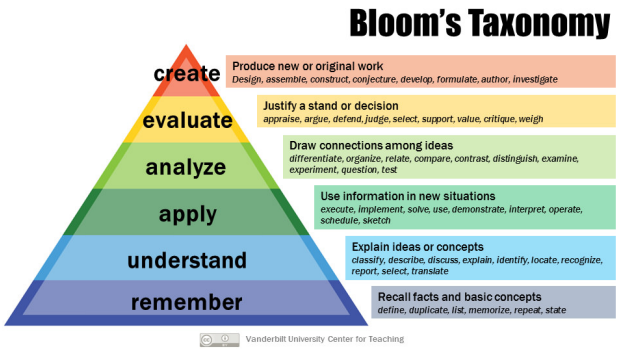

Figure 2: Visualization of Bloom's Taxonomy over the Cognitive Domain[15]

Table 1: Learning Outcomes: Honors Differential Equations vs. Differential Equations at Mines

Honors Differential Equations	**Differential Equations**
1. Construct models associated with natural/physical phenomenon such as biological populations, classical/quantum oscillators. Analyze and explain their predictions. Summarize and critique the models. Recommend and support modifications.	**1.** Investigate the models of physical systems such as exponential and logistic growth, spring-mass systems, and LRC circuits using first and second order differential equations.
2. Solve first and second order linear ordinary differential equations using classical techniques such as Integrating Factors, the Method of Undetermined Coefficients, powerseries and Laplace Transforms, eigenvalues and eigenvectors and interpret the solutions in both phase and state space.	**2.** Solve first and second order linear ordinary differential equations using classical techniques such as Integrating Factors, the Method of Undetermined Coefficients, and Laplace Transforms.
3. Apply the concepts of linearity, superposition, and existence and uniqueness of solutions to solve linear differential equations.	**3.** Solve 2×2 linear, constant-coefficient, homogeneous systems of differential equations using eigenvalues and eigenvectors, and interpret the corresponding phase portraits.
4. Apply the apparatus of linearization, nullclines, conservation and dissipation to analyze linear and nonlinear differential equations.	**4.** Apply the concepts of linearity, superposition, and existence & uniqueness of solutions to solve linear differential equations.

2.3 Enact Phase - Novel Curricular Elements

While the lecture meetings for the Honors Differential Equations course are not remarkably different from those in a typical lecture-only course, there is a growing set of content forming the basis of a progressive student evaluation. In order for students to receive ample practice with the common techniques for solving ordinary differential equations, a "master list" of problems is assigned which must be completed by the end of the semester. Solutions and tutorial videos are provided, and students are also required to make study guides for each solution technique. This way, students will have a memorable training experience with notes, in the event future courses utilize these specific solution procedures.

In addition to a master list of problems, students in the Spring 2019 implementation are asked to:

- Respond to the following questions weekly:
    1. What did you learn this week?

2. What questions remained unclear?
3. If you were the professor, what questions would you ask to find out whether the students understood the most important material of this week?

Adapted from Etkina[16], these questions give the instructor insight into students' metacognitive processes and knowledge creation.

- Reflect and respond to the following prompts at least twice a semester:
  1. Describe an experience (in life or in this class) that you would like to improve upon in the future,
  2. Research and select strategies that you used to overcome the difficult situation, and
  3. Describe how you would improve upon the experience in the future.

  These questions adapted from the work of Matheson et al. and Dounas-Frazer et al.[17,18], are meant to stimulate self-reflection.
- Watch curated media and conduct five-minute free-writes reflecting on the content. There are currently two modules in total, with the third slated for our next iteration:
  1. How do I learn? How can I get better at learning? Why should I care?
     - Index of Learning Styles (ILS) by Richard Felder and Linda Sliverman[19]
     - Learning styles & the importance of critical self-reflection by Tesia Marshik[20]
  2. Mindsets
     - Cisco Junior High School Quiz Adapted from Carol Dweck's Mindset quiz[21]
     - The power of believing that you can improve, Carol Dweck[22]
  3. Effective strategies for learning in math and science
     - "Learning How to Learn", Barbara Oakley[23]
     - Mind-mapping, Pomodoro, and Feynman Techniques

  These materials are focused on empowerment through self-awareness/reflection and are presented by a first-year undergraduate teaching assistant who provides summaries and topics to ponder, leading to a free-write prompt.

In addition to these tasks, students are asked to work in groups on projects that are focused on modeling and choose a final individual project. Here the topics range from continued work with the symbolic language of dynamical systems to work with linear and nonlinear models.

2.4 Reflect Phase - Changes Prompted by Student Comments

Since entering the third phase of development in the Fall of 2017, at the end of each term, we have reviewed grade distributions, student performance on learning outcomes, and student feedback on

end-of-term course evaluations. Due to the nature of the students who choose to enroll in the course, the vast majority of students meet our targets for performance on the learning objectives and the grade distributions are strongly skewed toward the high-end. As a consequence, we have found student responses to the open-ended questions on our end-of-term course evaluations to be the most helpful in identifying improvements in subsequent offerings of the course.

Based on this feedback, we have made the following improvements:

1. To process the significant amount of questions created by individuals each week, we have tasked the student groups to organize and distill the weekly metacognitive questions.
2. To support our higher-order course outcomes, we have tasked the student groups to complete project reports on either open or advanced mathematical models.
3. To strike a better balance between procedural practice and project-based learning, we no longer have groups conduct out-of-class quizzing on solution techniques.

These changes are reflected in the current implementation. To give the reader some sense of what students are reflecting on with the novel curricular elements, we have the following statements made in free-writes and guided reflections:

- I was originally dreading this assignment because every time I hear about learning styles, I feel like it doesn't apply to me. Hearing the TED talk was extremely affirming that there are no scientific results that show that learning styles work.
- (Speaking about being faced with a difficult task/concept) Thinking that I am incapable when I come across something that I struggle with is something that is fairly heavily ingrained in me.

3 Results and Discussion

Reflecting on the grade data since 2014, we find that the more weekly assessments students have to perform, the more the grade distribution skews to the A/B range. Additionally, semesters which emphasize procedural practice tend to have lower average grades. That said, students generally perform well relative to the course outcomes. Since so few students enroll in the course and those that do tend to seek out enriched curriculum at Mines, quantitative metrics are insufficient to demonstrate an improved student experience. Instead, we consider whether these treatments are useful and well-tolerated. To address this, we have selected the following comments from student end-of-year evaluations and various reflections we have collected:

- The master problems and the midterm/final project were extremely helpful for my learning process. The weekly reflections were helpful as well because I had to go back through my notes and review the material to determine what we learned that week.
- I love how we focus mainly on application of ideas in this course, I don't question why we are learning this material but instead can see its direct application through the process of learning it.

- I like that it isn't like a standard math class, like "here's this equation... memorize this... if x then do y... etc." The necessary information is provided, and then there is a big emphasis on making sure you understand it so that it can be applied to the models in following classes.
- The emphasis on the conceptual, while amazingly difficult, also really helps promote a deeper understanding of the class.
- After finishing the last group homework we had, I definitely felt a marked difference between my communication skills at the beginning of the class and by the end of the class.

It appears that emphasizing mathematical modeling and giving students the time to reflect is useful excercise. For those who might try to support active learning through process reflection, it should be noted that through groups it is both possible and important to process the overwhelming amount of raw textual data produced from weekly individual reporting.

While there were no strong opinions contradicting the previous statements, indicating that the treatment is well-tolerated, there are some cautionary points:

- I found it pretty difficult to stay motivated to study and learn without the looming threat of homework and exams.
- Throughout the entire course, I wish I would have started trying to do the differential equations earlier on.

In our opinion, it is not clear how the final point should be addressed. The average student can self-study the basic solution techniques seen in an introductory course in differential equations in roughly one month. Furthermore, the major results of the course do not rely on more than a couple of key models whose solution technique is rather straightforward. So, the instructor is left with a question of whether to conduct in-semester assessments of progress through the master list problems or to permit the student to complete them on their own timeline. This is a personal choice and we note the following statement made by a student during their end-of-term reflection,

- On the midterm reflection I noted that the issue was that I did not act soon enough to solve the master list problems and so I was more lost during those classes. When it came to the latter half of material, I made sure to do the problems as we learned the material and found lecture much more effective.

Acknowledgments

The authors would like to thank the Mines administration as well as our departmental leadership for facilitating our efforts to pilot this redesign. We would also like to thank colleagues from across campus who provided expertise and intellectual support in this redesign, particularly Vince Kuo and Pat Kohl from the Physics Department. This project was supported in part by ONR Grant N00014-15-1-2435.

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